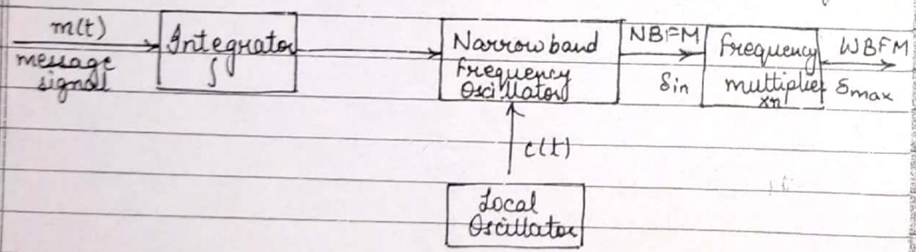
Generation of F.M. signal

- Direct method
  - V.C.O. Method (Voltage controlled oscillator)
  - Varactor diode modulator
  - Reactance Modulator circuit (FET or MOSFET)
- Indirect method (Armstrong method) NBFM

Indirect Method

It is also called Armstrong method. This method is given by the scientist Armstrong. In this method we obtain NBFM signals and then converted into WBFM signals using frequency multiplier. The value of frequency multiplier n depends on the maximum frequency deviation required in the WBFM signals.

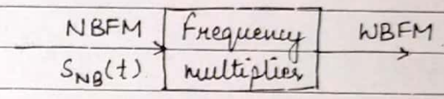


$$n = \frac{\text{maximum modulation index } (m_f)}{m_f\ (\text{NBFM})} = \frac{\delta_{max}}{\delta_{NBFM}}$$

Maximum allowed frequency deviation at the input of frequency multiplier:

$$\delta_{in} = \frac{\delta_{max}}{n}$$

Q. Consider the frequency multiplier shown in fig. and the NBFM signal $S_{NB}(t) = A\cos(\omega_c t + m_f \sin\omega_m t)$ with $m_f < 0.5$ and $f_c = 200\ \text{kHz}$. Let the range of f_m be from 50 Hz to 15 kHz and maximum frequency deviation at the output of frequency multiplier is 75 kHz. Determine required frequency multiplication factor n and the allowed maximum frequency deviation at the input of frequency multiplier.



Soln: $S_{NB}(t) = A\cos(\omega_c t + m_f \sin\omega_m t)$

$m_f < 0.5$

$f_c = 200\ \text{kHz}$

$50 < f_m < 15\ \text{kHz}$

$n =$

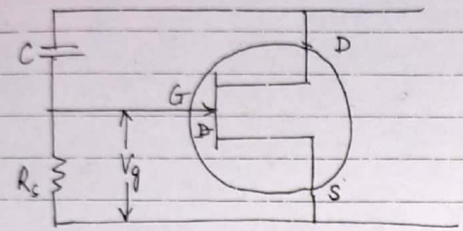
$$(m_f)_{max} = \frac{\delta}{f_m} = \frac{75\ \text{kHz}}{50} = 1500$$

$$(m_f)_{min} = \frac{\delta}{f_m} = \frac{75\ \text{kHz}}{15\ \text{kHz}} = 5$$

$$n = \frac{1500}{0.5} = 3000$$

$$\delta_{in} = \frac{\delta_{max}}{n} = \frac{75\ \text{kHz}}{3000} = 25\ \text{Hz}$$

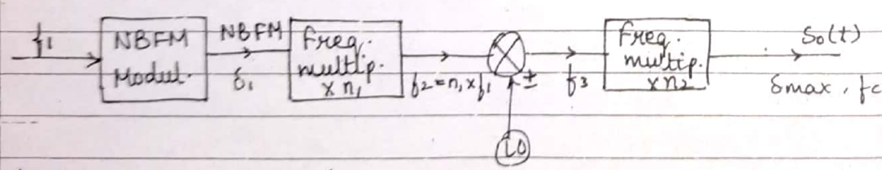
2. Reactance Modulator



The concept of reactance modulator is based on the fact that when we change the reactance or capacitance of a tuned circuit then FM is generated.
There are a no. of devices in which reactance is varied by applying an external voltage. For reactance modulator purpose we prefer 3-terminal devices such as FET and BJT. A reactance modulator circuit using the FET is shown in the fig. The transconductance of FET is varied when external voltage is applied. The reactance depends on the transconductance. The equivalent capacitance of circuit is given by:

$$C_{eq} = g_m RC$$

Q.



A block diagram of Armstrong transmitter is shown in the fig. Cal. maximum frequency deviation δ_{max} at the output of FM transmitter and carrier frequency f_c if $f_1 = 200$ kHz, $f_{LO} = 10.8$ MHz, $\delta_1 = 25$ Hz, $n_1 = 64$ and $n_2 = 48$.

Soln.

$$n = \frac{m_f(WBFM)}{m_f(NBFM)} = \frac{\delta(WBFM)}{\delta'(NBFM)}$$

$$\delta_{max} = n \times \delta_1$$
$$= n_1 \times n_2 \times \delta_1$$

$$f_3 = f_2 \pm f_{LO} = n_1 \times f_1 \pm f_{LO} = 64 \times 200 \times 10^3 \pm 10.8 \times 10^6$$
$$= 23.6 \text{ MHz} \quad (+f_{LO})$$

$$f_c = n_2 \times f_3$$
$$= 48 \times 23.6 \text{ MHz}$$
$$= 1.13 \times 10^9 \text{ Hz}$$
$$= 1.13 \text{ GHz}$$

$$f_3 = 64 \times 200 \times 10^3 - 10.8 \times 10^6 \quad (-f_{LO})$$
$$= 2 \text{ MHz}$$

$$f_c = n_2 \times f_3$$
$$= 48 \times 2 \text{ MHz}$$
$$= 96 \text{ MHz}$$

Q. An angle modulated signal is given by:
$$S_0(t) = 100 \cos(2\pi f_c t + 4 \sin 2000 \pi t)$$
where $f_c = 10$ MHz. Determine a) avg. transmitted power.
b) Peak frequency deviation and peak phase deviation.

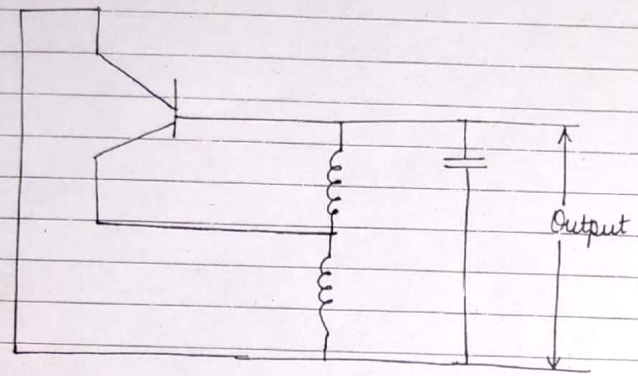
Soln. $S_0(t) = 100 \cos 2\pi f_c t + 4 \sin 2000 \pi t$

Peak phase deviation $= |\phi(t)|_{max}$

Peak frequency deviation $= \frac{1}{2\pi} \left| \frac{d}{dt} \phi(t) \right|_{max}$

Direct Method

i) Voltage Controlled Mett Oscillator Method



Frequency of oscillation can be controlled by applying an external voltage.

$$f_i(t) = \frac{1}{2\pi\sqrt{(L_1+L_2)\,C_i(t)}}$$

$C_i(t)$ = Instantaneous capacitance

$C_i(t) = C_0 + \Delta C \cos 2\pi f_m t$

C_0 = Initial capacitance

= unmodulated capacitance

$$f_i(t) = \frac{1}{2\pi\sqrt{(L_1+L_2)\cdot(C_0+\Delta C\cos 2\pi f_m t)}}$$

$$= \frac{1}{2\pi\sqrt{(L_1+L_2)\,C_0\left(1+\frac{\Delta C}{C_0}\cos 2\pi f_m t\right)}}$$

$$f_i(t) = \frac{1}{2\pi\sqrt{(L_1+L_2)\,C_0}\sqrt{1+\frac{\Delta C}{C_0}\cos 2\pi f_m t}}$$

f_0 = unmodulated frequency of oscillator $= \frac{1}{2\pi\sqrt{(L_1+L_2)C_0}}$

$$f_i(t) = f_0 \frac{1}{\sqrt{1+\frac{\Delta C}{C_0}\cos 2\pi f_m t}}$$

$$= f_0\left[1+\frac{\Delta C}{C_0}\cos 2\pi f_m t\right]^{-1/2}$$

$$= f_0\left[1-\frac{\Delta C}{2C_0}\cos 2\pi f_m t\right]$$

Let us assume : $\frac{\Delta C}{2C_0} = -\frac{\Delta f}{f_0}$

$$f_i(t) = f_0\left[1+\frac{\Delta f}{f_0}\cos 2\pi f_m t\right]$$

$$f_i(t) = f_0 + \Delta f \cos 2\pi f_m t$$

$$\Delta f = \delta$$

$$f_i(t) = f_0 + \delta \cos 2\pi f_m t$$

Hence, Hartley Oscillator can be used as a FM generator.

$\phi(t) = 4 \sin 2000\pi t$

$\frac{d}{dt}\phi(t) = \frac{d}{dt}(4 \sin 2000\pi t) = 4 \times 2000\pi \cos 2000\pi$

$\phi'(t) = 8000\pi \cos 2000\pi t$.

Peak phase deviation $= |\phi(t)|_{max}$

$= 4$ rad

Peak frequency deviation $= \frac{1}{2\pi} \times |\phi'(t)|_{max}$

$= \frac{1}{2\pi} \times 8000\pi$

$= 4000$ Hz

$= 4$kHz

Q. Consider the following angle modulated signal

$S_\phi(t) = 10\cos[2\times 10^6 \pi t + 10\sin(10^3\pi t)] + 5\sin(2\times 10^3)\pi t]$.

Determine maximum phase deviation.

Soln. Maximum phase deviation $= |\phi(t)|_{max}$

$S_\phi(t) = A_c \cos(\omega_c t + \phi(t))$

$\phi(t) = 10 \sin(10^3\pi t) + 5\sin(2\times 10^3)\pi t$

$\frac{d\phi(t)}{dt} = 10\cos(10^3\pi t)\cdot 10^3\pi + 5\cos(2\times 10^3\pi t) \times 2\times 10^3\pi$

$0 = 10\times 10^3\pi \cos(500\times 2\pi t) + 10\times 10^3 \pi \cos(10^3 \times 2\pi t)$

$0 = 10^4\pi\,(\cos(500\times 2\pi t) + \cos(10^3 \times 2\pi t))$

$0 = \cos(2\pi t \times 500) + \cos(2\pi t \times 1000)$

$[-\cos\theta = \cos(\pi - \theta)]$

$t = \frac{1}{3000}$,

$\phi(t)|_{max} = 10 \sin 10^3\pi \times \frac{1}{3000} + 5 \sin \frac{2\pi \times 10^3}{3000}$

$= 10\sin(\pi/3) + 5\sin(2\pi/3)$

$= \frac{15\sqrt{3}}{2}$ Rad/sec.

Q. Consider the following NBFM signal:

$S(t) = A_c \cos 2\pi f_c t - m_f A_c \sin(2\pi f_c t)\sin(2\pi f_m t)$

The NBFM signal is demodulated by using the envelope detector. Determine envelope of this signal and calculate the ratio of maximum to minimum value of envelope.

Soln. $S(t) = A_c \cos(2\pi f_c t) - m_f A_c \sin(2\pi f_c t)\sin(2\pi f_m t)$

$e(t) = \sqrt{A_c^2 + (A_c m_f \sin(2\pi f_m t))^2}$

$= \sqrt{A_c^2 + A_c^2 m_f^2 \sin^2(2\pi f_m t)} = \sqrt{A_c^2(1 + m_f^2 \sin^2 2\pi f_m t)}$

$= A_c \sqrt{1 + m_f^2 \sin^2 2\pi f_m t}$

$e(t)$ is max. if $\sin 2\pi f_m t = 1$

$e(t)_{max} = A_c\sqrt{1+m_f^2}$

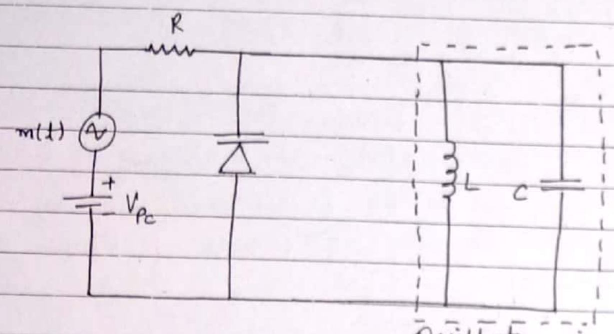
$e(t)$ is min. if $\sin 2\pi f_m t = 0$

$e(t)_{min} = A_c$

$$\therefore \frac{e(t)_{max}}{e(t)_{min}} = \frac{A_c\sqrt{1+m_f^2}}{A_c} = \sqrt{1+m_f^2}$$

Varactor diode F.M. Modulation

It is variable capacitance diode. The junction capacitance can be varied according to external applied voltage.



Oscillator circuit to generate carrier.

$$C_d \propto \frac{1}{\sqrt{V_{ext}}}$$

$$C_d = k\, V_{ext}^{-1/2}$$

$$V_{ext} = V_{Pc} + m(t)$$

$$f = \frac{1}{2\pi\sqrt{LC}}$$

$$f_i = \frac{1}{2\pi\sqrt{L(C_o + C_d)}}$$

$$= \frac{1}{2\pi\sqrt{L(C_o + k\,V_{ext}^{-1/2})}}$$

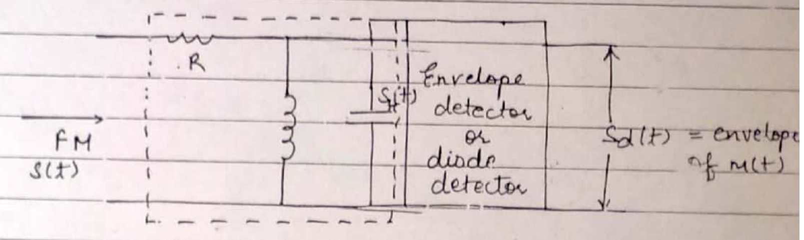
Imp

Demodulation of F.M.

1. Frequency Discrimination method
2. Phase Discrimination method

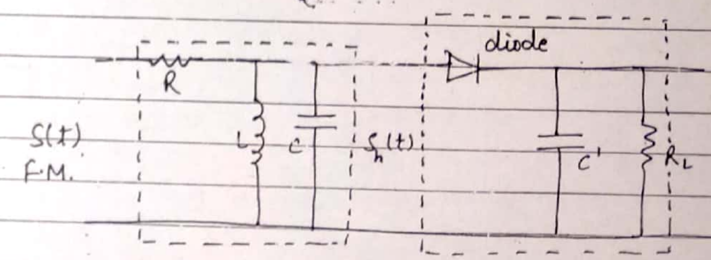
Frequency Discrimation Method

It is also known as slope detector method. Envelope detector is followed by a tuned circuit.

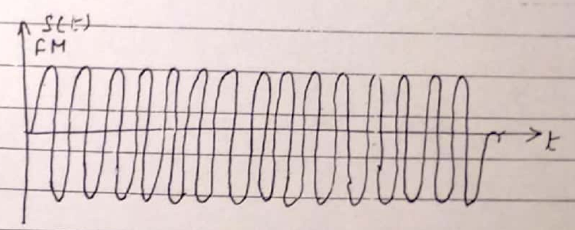


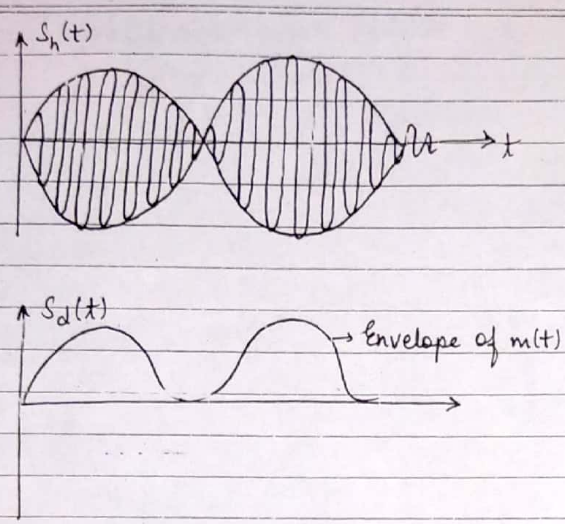
Tuned circuit (tuned above or below f_c)

⇓



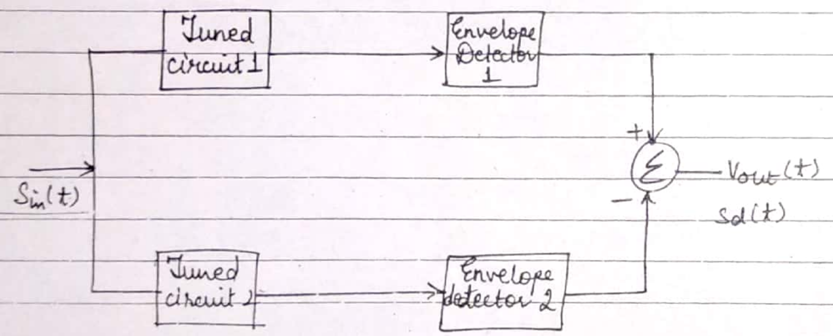
Tuned circuit Diode detector



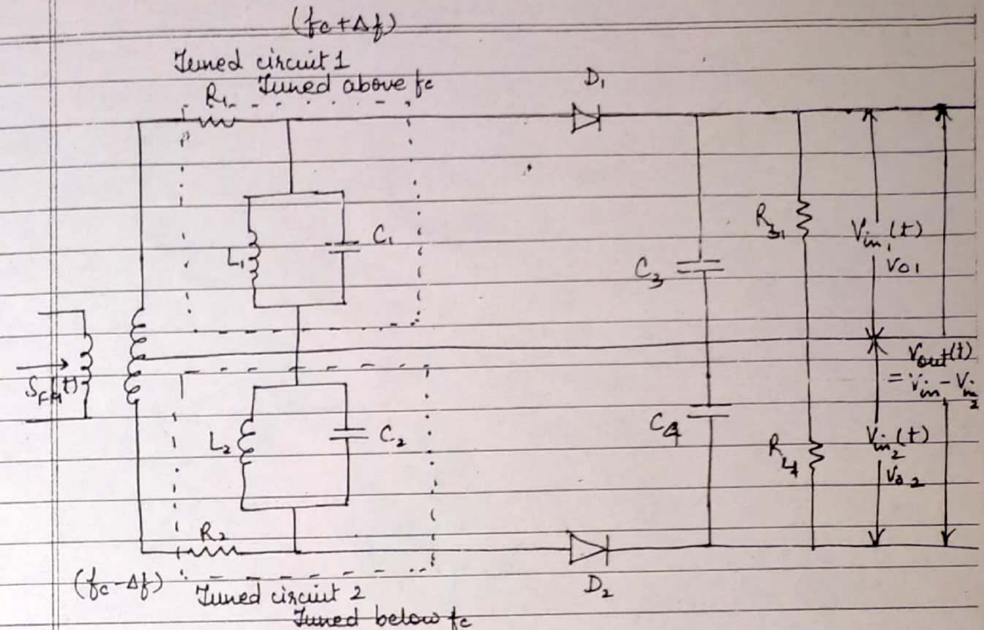


Voltage is varied by variation of frequency. Then the amplitude is varied and finally m(t) is varied.

Balanced Slope Detector



Block Diagram



Assignment -

Q. When FM signals are demodulated by using the slope detector there is ~~no~~ occurance of interferance. Discuss the above statement -

Q. Calculate bandwidth of F.M. signals for following two conditions -

i) If the baseband frequency is doubled.

ii) The baseband frequency is decreased by the factor 1/2.

Assume that the F.M. is commercial F.M. and frequency of baseband signal is 430 kHz.

Noise

i) Thermal noise / Johnson noise / white noise - It occurs due to the presence of passive resistance. The p.d.f. of thermal noise is uniform p.d.f.

$P_n \propto T\delta f$

$P_n = KT\delta f$

P_n = Thermal noise power
T = Absolute temperature (in K)
δf = Operating bandwidth of system
k = Boltzman constant $= 1.38 \times 10^{-23}$ J/K

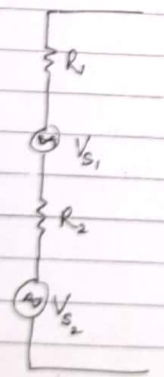
$$P_n = \frac{V^2}{R_L} = \frac{(V_n/2)^2}{R_L}$$

$$P_n = \frac{V_n^2}{4R_L}$$

$$V_n^2 = 4R_L P_n$$

$$V_n = \sqrt{4R_L P_n} = \sqrt{4R_L KT\delta f}$$

If the devices or resistors are connected in series:



$$V_n = \sqrt{4KT\delta f(R_1+R_2)}$$

If the devices or resistors are connected in paralle

$$V_n = \sqrt{4kT\delta f R_{eq}}$$

$$R_{eq} = \frac{R_1 R_2}{R_1+R_2}$$

Noise Figure / Noise Factor :

$$F = \frac{\text{Signal to noise ratio at input}}{\text{Signal to noise ratio at output}}$$

$$F = \frac{(S/N)\,\text{input}}{(S/N)\,\text{output}}$$

$$\boxed{F = 1 + \frac{R_{eq}}{R_a}}$$

R_{eq} = Noise equivalent resistance
R_a = Antenna Resistance

Noise Equivalent Temperature

$$T_{eq} = T_o(F-1)$$

F = Noise Figure (in Ratio)
T_o = Absolute temp = 290 K

$$\boxed{F\,(\text{in dB}) = 10\log_{10} F}$$

Q. A receiver is connected to an antenna whose resistance is 50 Ω have a noise equivalent resistance 30 Ω. Cal. noise figure in dB and noise equivalent temperature.

Soln. $R_a = 50\Omega$

$R_{eq} = 30\Omega$

$$F = 1 + \frac{R_{eq}}{R_a}$$

$$= 1 + \frac{30}{50} = 1.6$$

$$F(\text{in dB}) = 10\log_{10} F$$

$$= 10\log_{10} 1.6$$

$$= 2.04$$

$$T_{eq} = T_0(F-1)$$

$$= 290(2.04-1)$$

$$= 290(1.04)$$

$$= 301.9\ K$$

<u>Balanced Slope Detector</u> (continued from the diagram).

Three cases are possible:

<u>Case 1</u>: $(f_c - \Delta f) < f_{in} < f_c$

Input of diode D_1 is less than input of diode D_2.

V_{01} is less than V_{02}

$$V_0 = V_{01} - V_{02}$$

$$V_0 = \text{negative}$$

<u>Case 2</u>: $f_{in} = f_c$

Voltage of diode D_1 and D_2 are equal.

$$V_{01} = V_{02}$$

$$V_0 = 0.$$

<u>Case 3</u>: $f_c < f_{in} < f_c + \Delta f$

$$V_{01} > V_{02}$$

$$V_{out} = \text{Positive}$$

<u>Overall Noise Figure : Multistage Amplifier</u>

$$F = F_1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1 G_2} + \dots$$

$F_1, F_2, F_3 \dots$ = Noise figures of Individual stages.

$G_1, G_2, G_3 \dots$ = Gain of each stage

<u>Overall Noise Temperature : Multistages</u>

$$T_{eq} = T_{eq_1} + \frac{T_{eq_2}}{G_1} + \frac{T_{eq_3}}{G_1 G_2} + \dots$$

Q. Two resistance sources R_1 and R_2 at absolute temp. T_1 and T_2 are connected in series and form white noise. Determine noise equivalent temperature

Soln.

$$\boxed{\begin{matrix} R_1 \\ T_1 \end{matrix}} \qquad \boxed{\begin{matrix} R_2 \\ T_2 \end{matrix}}$$

$$\forall\ P_n = \frac{V_n^2}{4R_L} = \frac{V_{n_1}^2 + V_{n_2}^2}{4R_L}$$

$$P_n = \frac{4R_1 kT_1 \delta f + 4R_2 kT_2 \delta f}{4R_L}$$

$$P_n = \frac{4k\delta f\,(R_1T_1 + R_2T_2)}{4R_L}$$

$$P_n = kT_{eq}\cdot\delta f$$

$$T_{eq} = \frac{P_n}{k\delta f}$$

$$= \frac{4k\delta f\,(R_1T_1 + R_2T_2)/4R_L}{k\delta f}$$

$$= \frac{R_1T_1 + R_2T_2}{R_L} \qquad R_1 = R_1T_1 + R_2T_2$$

$$= \frac{R_1T_1 + R_2T_2}{R_1 + R_2}$$

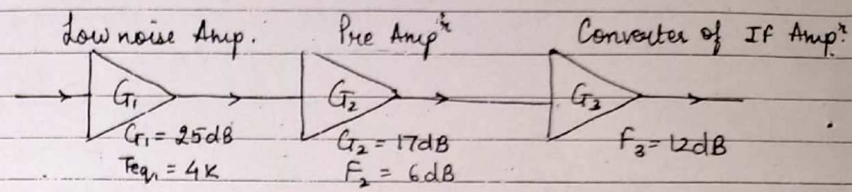
Q. Two resistance sources R_1 and R_2 at absolute temp T_1 and T_2 are connected in parallel and form white noise. Determine noise equivalent temperature.

Soln. $P_n = \dfrac{V_n^2}{4R_L} = \dfrac{V_{n_1}^2 + V_{n_2}^2}{4R_L}$

$$P_n = \frac{4R_1kT_1\delta f + 4R_2T_2k\delta f}{4R_L}$$

$$\therefore T_{eq} = \frac{(R_1T_1 + R_2T_2)(R_1+R_2)}{R_1R_2} \qquad R_L = \frac{R_1R_2}{R_1+R_2}$$

Q. Find out the noise equivalent temperature of the following low noise receiving system.



Assume the room temp to be 17°c.

Soln. $T_{eq} = T_{eq_1} + \dfrac{T_{eq_2}}{G_1} + \dfrac{T_{eq_3}}{G_1G_2}$

$$= 4 + \frac{T_{eq_2}}{25} + \frac{T_{eq_3}}{25\times 17}$$

$$T_{eq_2} = (F_2 - 1)T_0 = \cancel{5\times}(17 + 290)(F_2 - 1)$$

$$T_{eq_3} = (F_3 - 1)T_0 = \cancel{11\times}(17 + 290)\,(F_3 - 1)$$

$F_2 = 10\log_{10} F_2$

$6 = 10\log_{10} F_2$

$\therefore F_2 = 10^{0.6} = 3.98$

$G_1 = 10^{2.5} = 316.23$

$G_2 = 10^{1.7} = 50.11$

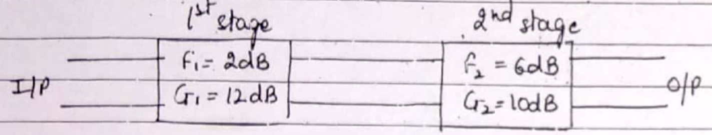
$F_3 = 10^{1.2} = 15.85$

$$T_{eq} = 4 + \frac{307\times(3.98-1)}{316.23} + \frac{307\times(15.85-1)}{316.23\times 50.11}$$

$$= 4 + 2.89 + 0.29$$

$$= 7.18\ K$$

Q. A cascade two stage amplifier shown in the fig. The 1^{st} stage has a noise figure 2dB and power gain 12dB. The 2^{nd} stage has noise figure 6dB and power gain 10dB. Determine overall noise figure in dB.



Soln. $F = F_1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1 G_2} + \ldots$

$$F = F_1 + \frac{F_2 - 1}{G_1}$$

$$F_1 = 10^{0.2} = 1.58$$

$$F_2 = 10^{0.6} = 3.98$$

$$G_1 = 10^{1.2} = 15.8$$

$$G_2 = 10^{1} = 10.$$

$$F = 1.58 + \frac{2.98}{15.8} = 1.76$$

$$F(dB) = 10 \log_{10} 1.76$$
$$= 2.45\ dB$$

Q. Determine overall noise figure of 3 stage cascaded amplifier. Each stage having power gain 10dB and noise figure 6dB.

Soln. $F_1 = F_2 = F_3 = 10^{0.6} = 3.98$

$G_1 = G_2 = G_3 = 10^{1} = 10.$

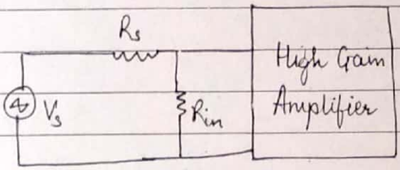
$$F = F_1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1 G_2}$$

$$F = 3.98 + \frac{2.98}{10} + \frac{2.98}{39.8}$$

$$= 4.35$$

$$F(dB) = 10 \log_{10} 4.35$$
$$= 6.34\ dB$$

Q. A source of internal resistance $50\,\Omega$ is connected to high gain amplifier. If the bandwidth of amplifier is 40 kHz and output (S/N) ratio 36 dB, find out source emf at the temp $17^{\circ}C$. Ignore the internal noise of amplifier. $k = 1.38 \times 10^{-23}$ J/K.



Soln. $R_s = R_{in} = 50\,\Omega$

$\delta f = 40\,kHz$

$T = 17^{\circ}C$

$k = 1.38 \times 10^{-23}$ J/K

S/N ratio at output $= 36\,dB = 10^{3.6} = 3981.07$

$$\text{S/N power at output} = \frac{\text{S/N power at input}}{\text{Noise power}}$$

$$P_n = \frac{V_n^2}{4R_L}$$

$$V_n^2 = 4 R_L k \delta T f$$

$$\therefore P_n = kT\delta f = 1.38 \times 10^{-23} \times 307 \times 40 \times 10^{3} = 1.67 \times 10^{-16}$$