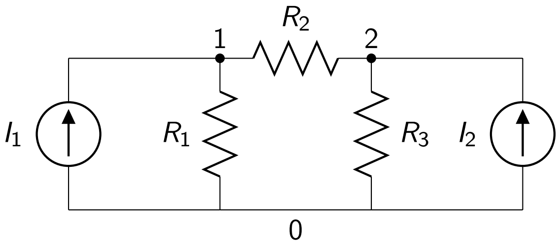
Nodal Analysis

- In nodal analysis, node voltages are found by solving a set of equations.
- Choosing node voltages rather than element voltages as variables reduces the number of equations.

It uses the following steps.

1. Select a node as the reference node. Assign voltages to other nodes with respect to the reference node.
2. Apply KCL at every node except reference node.
3. Solve the set of equations for node voltages.

Circuit with Current Sources



Let 0 be the reference node and the voltages of 1 and 2 be V_1 and V_2 respectively.
By applying KCL at node 1,

$$I_1 = \frac{V_1}{R_1} + \frac{V_1 - V_2}{R_2}$$

$$I_1 = V_1\left(\frac{1}{R_1} + \frac{1}{R_2}\right) - V_2\frac{1}{R_2} \quad (1)$$

By applying KCL at node 2,

$$I_2 = \frac{V_2}{R_3} + \frac{V_2 - V_1}{R_2}$$

$$I_2 = -V_1\frac{1}{R_2} + V_2\left(\frac{1}{R_2} + \frac{1}{R_3}\right) \tag{2}$$

Solve (1) and (2) for node voltages by any standard method.
By placing (1) and (2) in matrix form, we have

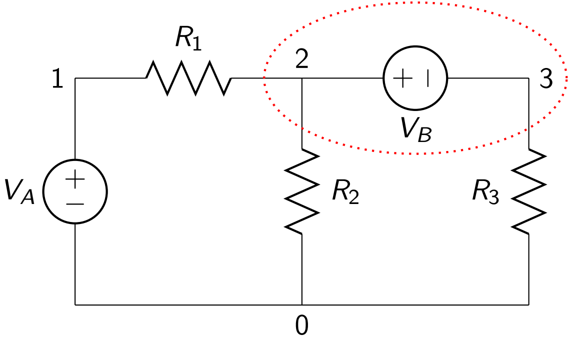
$$\begin{pmatrix} \left(\frac{1}{R_1} + \frac{1}{R_2}\right) & -\frac{1}{R_2} \\ -\frac{1}{R_2} & \left(\frac{1}{R_2} + \frac{1}{R_3}\right) \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \end{pmatrix} = \begin{pmatrix} I_1 \\ I_2 \end{pmatrix}$$

$$[G][V] = [I]$$

This can be solved for V_1 and V_2 by matrix inversion.

$$[V] = [G]^{-1}[I]$$

Circuit with Voltage Sources



There are three nodes. Let V_1, V_2 and V_3 be the node voltages. Since a voltage source V_A is connected between 1 and 0,

$$V_1 = V_A$$

To find V_2 and V_3, we need to apply KCL at node 2 and 3. As we do not know the current through the voltage source V_B, combine 2 and 3 and form a **super node**.

Applying KCL for super node,

$$\frac{V_2 - V_A}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3} = 0$$

$$V_2\left(\frac{1}{R_1} + \frac{1}{R_2}\right) + V_3\frac{1}{R_3} = \frac{V_A}{R_1} \tag{3}$$

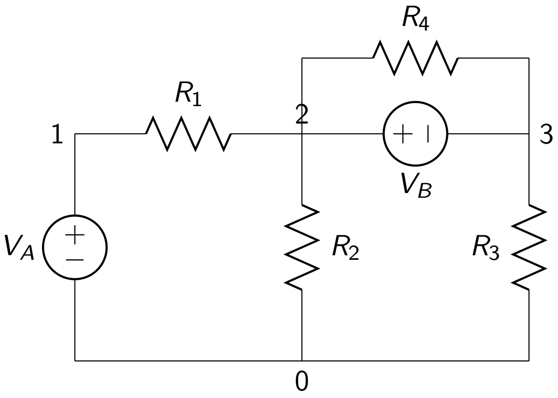
We also know that

$$V_2 - V_3 = V_B \tag{4}$$

Solve (3) and (4) for V_2 and V_3 by any standard method.

Whenever there is a voltage source (dependent or independent) between two non reference nodes, form a super node by combining them and apply KCL for the super node.

Let us consider the same circuit with R_4 between 2 and 3.

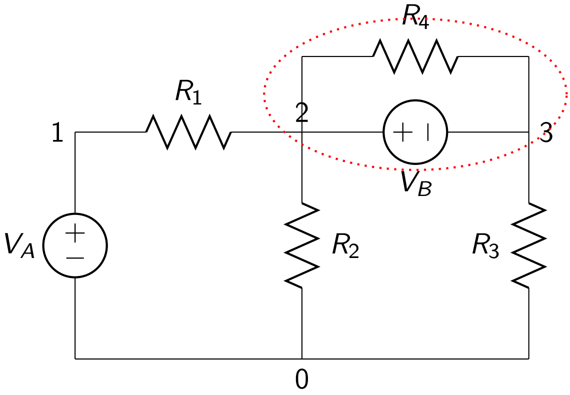


Applying KCL for super node,

$$\frac{V_2 - V_A}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3} + \frac{V_2 - V_3}{R_4} + \frac{V_3 - V_2}{R_4} = 0$$

The last two terms cancel each other.

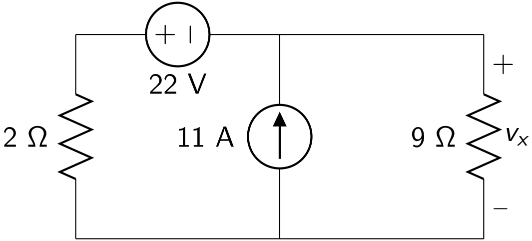
Since R_4 does not contribute anything, super node is formed by including it.



Super node method requires both KCL and KVL to solve for node voltages.

Test yourself

Find v_x using nodal analysis.

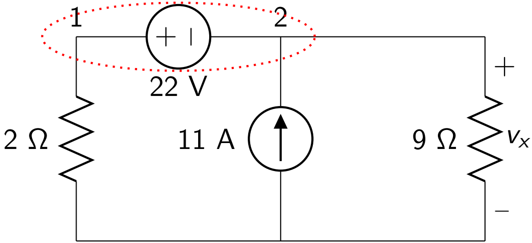


KCL at super node,

$$\frac{v_1}{2} + \frac{v_2}{9} = 11$$

and

$$v_1 - v_2 = 22$$



Solving this, $v_2 = 0$ V.
Since $v_x = v_2 = 0$ V.

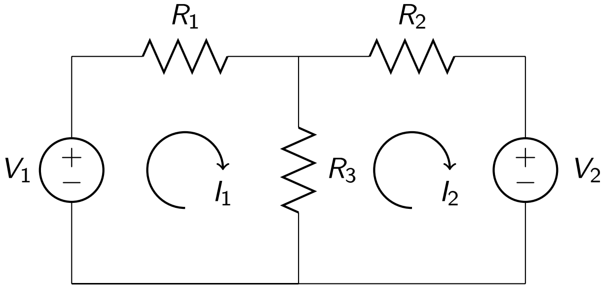
Mesh Analysis

- It is another way of analyzing circuits
- It uses KVL to find mesh currents.
- It is applicable only to *planar* circuit.

Mesh analysis uses the following steps

1. Assign mesh currents to all the meshes.
2. Apply KVL to each of the meshes.
3. Solve the resulting simultaneous equations for mesh currents.

Circuit with Voltage Sources



Applying KVL for mesh 1

$$-V_1 + I_1 R_1 + (I_1 - I_2)R_3) = 0$$

$$I_1(R_1 + R_3) - I_2 R_3 = V_1 \tag{5}$$

Applying KVL for mesh 2

$$(I_2 - I_1)R_3) + I_2 R_2 + V_2 = 0$$

$$-I_1 R_3 + I_2(R_2 + R_3) = -V_2 \tag{6}$$

Solve (5) and (6) for I_1 and I_2 by any standard method. They can also be solved by matrix inversion.

$$\begin{pmatrix} (R_1 + R_3) & -R_3 \\ -R_3 & R_2 + R_3 \end{pmatrix} \begin{pmatrix} I_1 \\ I_2 \end{pmatrix} = \begin{pmatrix} V_1 \\ -V_2 \end{pmatrix}$$

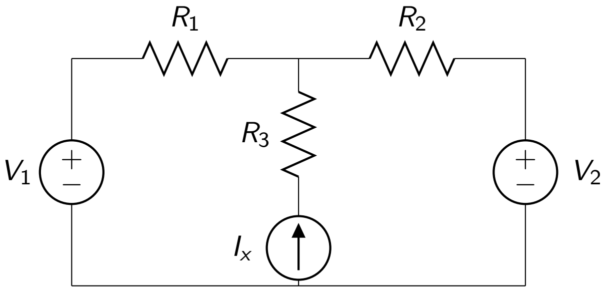
$$[R][I] = [V]$$

This can be solved for I_1 and I_2 by matrix inversion.

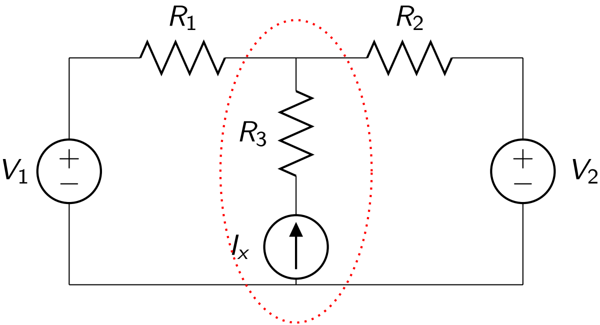
$$[I] = [R]^{-1}[V]$$

Notice that the matrix R is symmetric in this case.

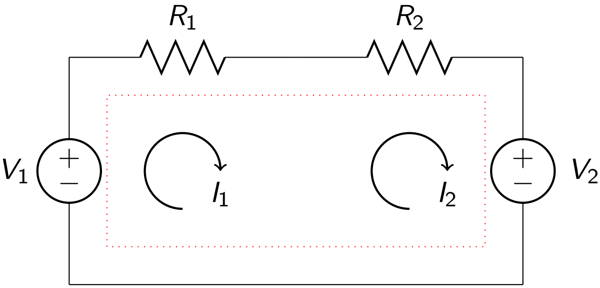
Circuit with Current Source



As we do not know the voltage across the current source, a **super mesh** is formed by excluding the current source and any elements connected in series.



R1
R2
R3
V1
V2
Ix



R1
R2
V1
V2
I1
I2

By applying KVL for super mesh,

$$-V_1 + I_1R_1 + I_2R_2 + V_2 = 0$$

$$I_1R_1 + I_2R_2 = V_1 - V_2 \tag{7}$$

We also know that

$$I_2 - I_1 = I_x \tag{8}$$

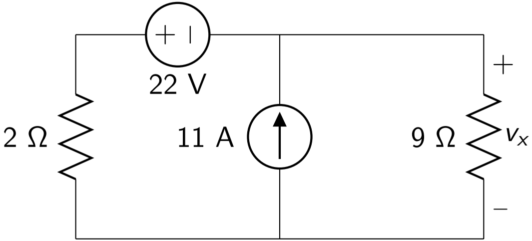
Solve (7) and (8) for mesh currents I_1 and I_2.

1. Super mesh requires the application of both KVL and KCL.
2. Super mesh has no current of its own.

Whenever there is a current source (dependent or independent) between two meshes, form a super mesh by excluding the current source and any elements connected in series.

Test yourself

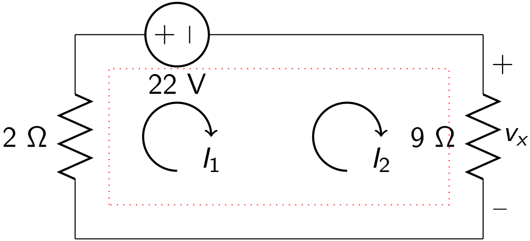
Find v_x using mesh analysis.



By writing KVL for super mesh,

$$2I_1 + 22 + 9I_2 = 0$$

and



$$I_1 - I_2 = -11$$

Solving this I_2 =0 A and $v_x = 0$ V.