Graphical Method: LPP involving only two decision variables can be effectively solved by graphical method - i.e., finding values of decision variables by using graphs.

Procedure: (1) Consider each inequality constraint as equation

(2) Plot the graph of lines in step 1.

[Put $x = 0$ → this gives y-value, say y_1 $(0, y_1)$]
[Put $y = 0$ → " " x - " , say x_1 $(x_1, 0)$]
[Join these points to get the graph of line]

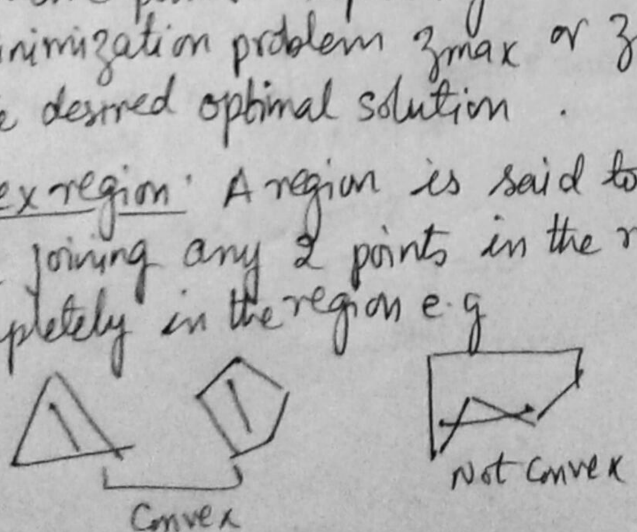
(3) If inequality is '≤', then region below the line ~~represents~~ is considered and if '≥' then the region above the line is considered.

(4) The region common to all the constraints, in the first quadrant & is the required feasible region

(5) Determine the corner points of feasible region. These are called extreme points.

(6) Find value of objective function (z) at these extreme points. Depending on maximization or minimization problem z_{max} or z_{min} will be the desired optimal solution.

Convex region: A region is said to be convex if line joining any 2 points in the region, lie completely in the region e.g



Note: (1) Feasible region (if bounded) constitutes a convex set.
(2) Within feasible region, feasible solution corresponds to extreme points

Ex1 The ABC company has been producer of picture tubes for TV sets and certain circuits for radio. It has built a new plant that can operate 48 hours/week. Production of an AM radio will require 2hrs and production of an AM-FM radio will require 3hrs. Each AM radio will contribute Rs 40 to the profits while an AM-FM radio will contribute Rs 80 to profits. The marketing department, after extensive research has determined that a maximum of 15 AM and 10 AM-FM radios can be sold each week.
(1) Formulate LPP to determine optimum production that maximizes the profits
(2) Find the solution graphically.

Solution 1: max $z = 40x + 80y$

subject to

$2x + 3y \le 48$. (hour constraint)

$x \le 15$ (AM radio)

$y \le 10$. (AM-FM radio.

$x, y \geq 0$.

where x: No. of AM radios units
y: " " AM-FM " units

step 1: $2x + 3y = 48$

$x = 15,\ y = 10$.

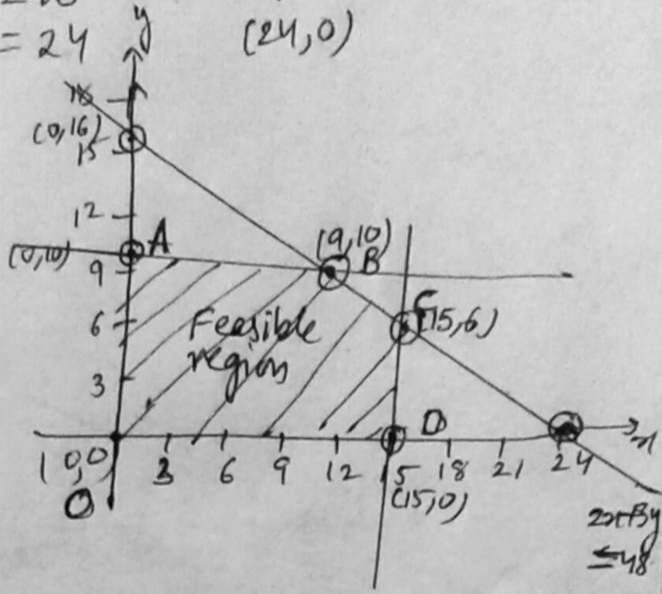
$x \geq 0,\ y \geq 0$

P.T.O.

step 2: $x=0 \Rightarrow y=16$ $(0,16)$

$y=0 \Rightarrow x=24$ $(24,0)$

step 3, 4, 5: (a) → Extreme points



$z_A = 40(0)+80(10)$
$= 800$

$z_B = 40(9)+80(10)$
$= 1160$

$z_C = 40(15)+80(6)$
$= 1080$

$z_D = 40(15)+80(0)$
$= 600$

∴ Optimal soln occurs at extreme point B.

i.e, $x=9$ $y=10$ & $z_{max}=1160$

If 9 units of AM & 10 units of AM-FM are sold then the company will have a maximum profit of Rs 1160 (subject to given constraints).

Ex 2 minimize $z = 4x+3y$

subject to

$200x+100y \geq 4000$
$x+2y \geq 50$
$40x+40y \geq 1400$
$x, y \geq 0$.

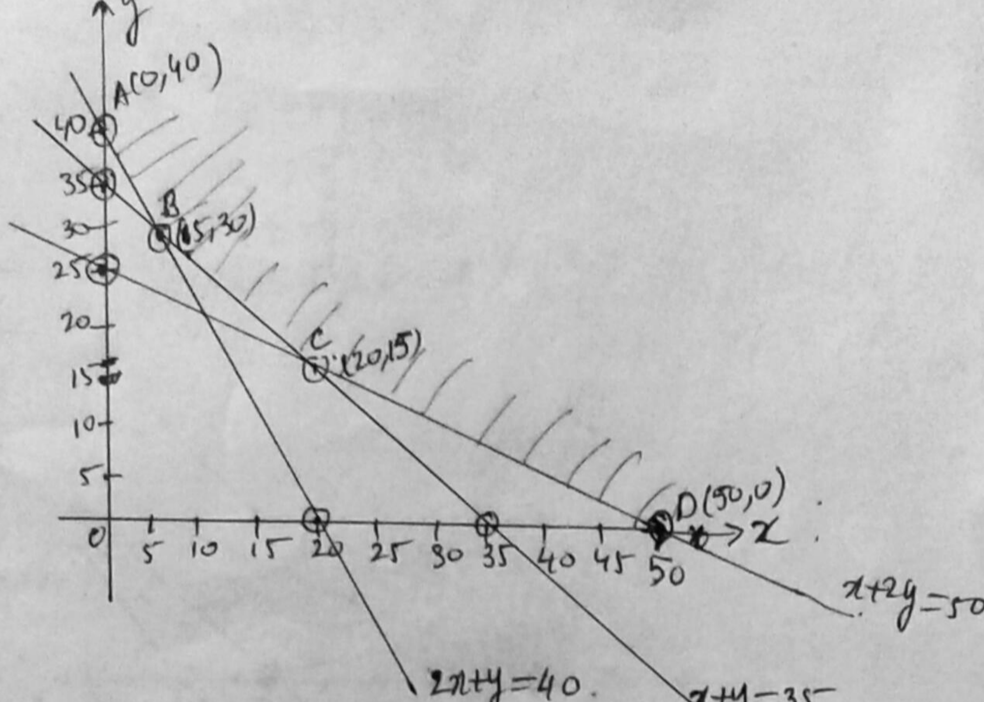
→ Solve this LPP graphically

Soln 2: The above LPP can be written as

$\min z = 4x+3y$

subject to

$2x+y \geq 40$
$x+2y \geq 50$
$x+y \geq 35$
$x, y \geq 0$.



$\left.\begin{array}{r} 2x+y=40 \\ x+y=35 \end{array}\right] \Rightarrow x=5, y=30.$

$\left.\begin{array}{r} x+2y=50 \\ x+y=35 \end{array}\right] \Rightarrow x=20, y=15$

Shaded region is the feasible region. Now, compute value of z at the ~~boundary~~ points of intersection of lines in & on shaded region.

$z_A = 4(0)+3(40)=120$ $z_B = 4(5)+3(30)=110$

$z_C = 4(20)+3(15)=125$ $z_D = 4(50)+3(0)=200$.

Since minimum value of z is at B, the optimal solution to the problem is $x=5$ $y=30$ and $z_{min}=110$.

Ex3 Solve the LPP given below graphically and find the optimal solution:

$\max z = 300x + 400y$

subject to

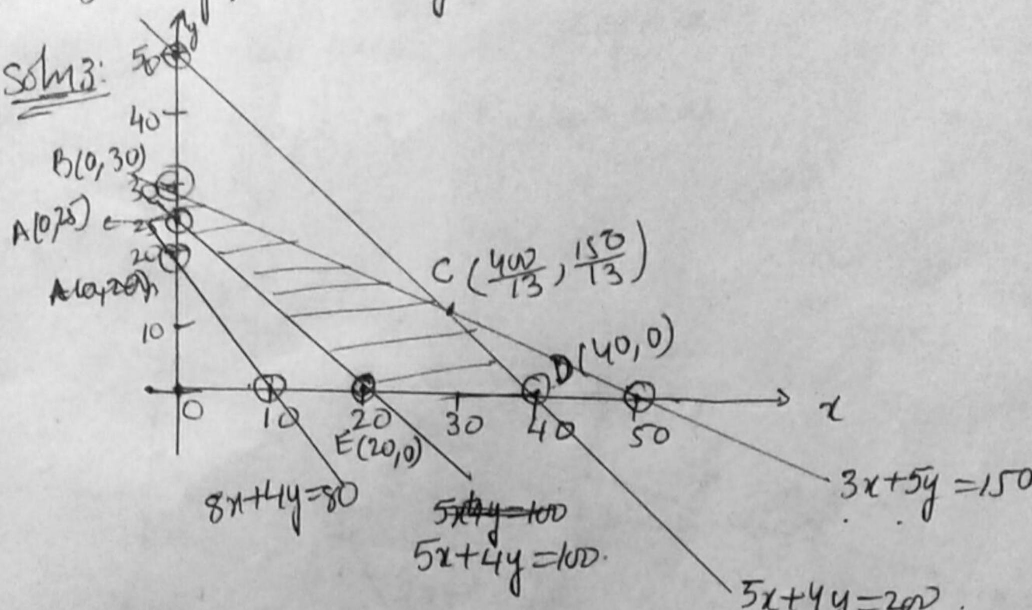
$5x + 4y \leq 200$

$3x + 5y \leq 150$

$5x + 4y \geq 100$

$8x + 4y \geq 80$ ~~8x+4y ≥ 80~~ , $x, y \geq 0$.

Soln 3:



$\therefore z_A = 10{,}000$

$z_B = 12{,}000$

$z_C = \frac{1{,}80{,}000}{13} \simeq 13846.15$

$z_D = 12{,}000$

$z_E = 6000$

$\therefore$ Maximum occurs at C. So the optimal soln is $x = \frac{400}{13}$, $y = \frac{150}{13}$

and $z_{max} = \frac{180000}{13}$.

Now, consider example 2 which we have done. In this we ~~repla~~ take maximize $z = 4x + 3y$ i.e, we maximize objective function instead of minimizing which we have already done. The constraint conditions remain the same. Lets see what is the resultant optimal solution now.

Ex 4: max $z = 4x + 3y$

subject to $200x + 100y \geq 4000$

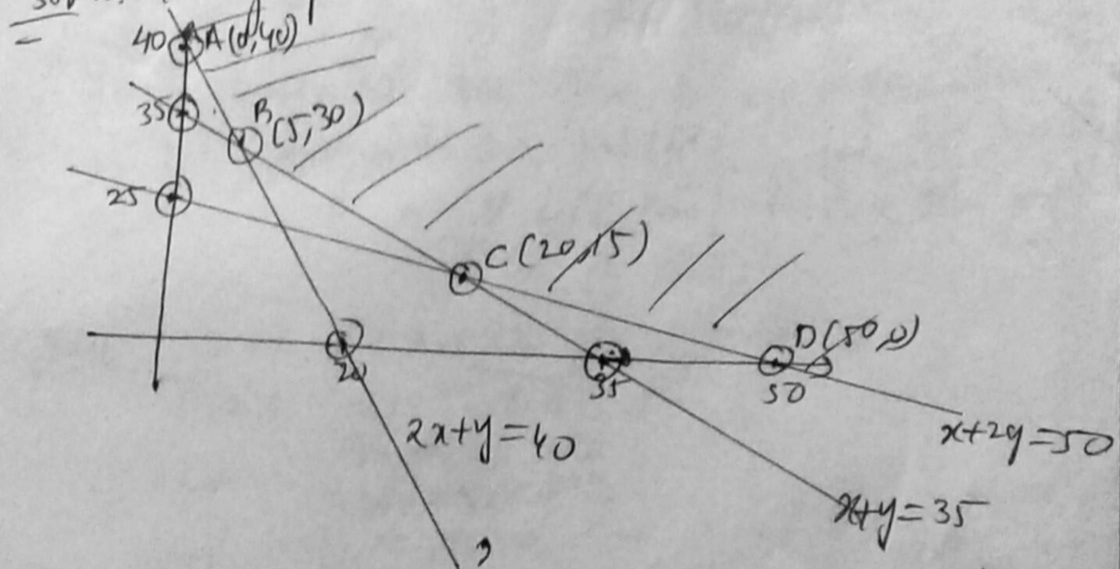
$x + 2y \geq 50$

$40x + 40y \geq 1400$

$x, y \geq 0$

Solve the given LPP graphically.

Soln: The graph as we saw was.



If you see the feasible region, the points A, B, C, D marked will have ~~z~~ z value less than any other point in shaded ~~reg~~ region. Hence, **unbounded solution** i.e, as you go on increasing either x or y the value of z will go on increasing.

Ex 5: Solve the LPP given below graphically

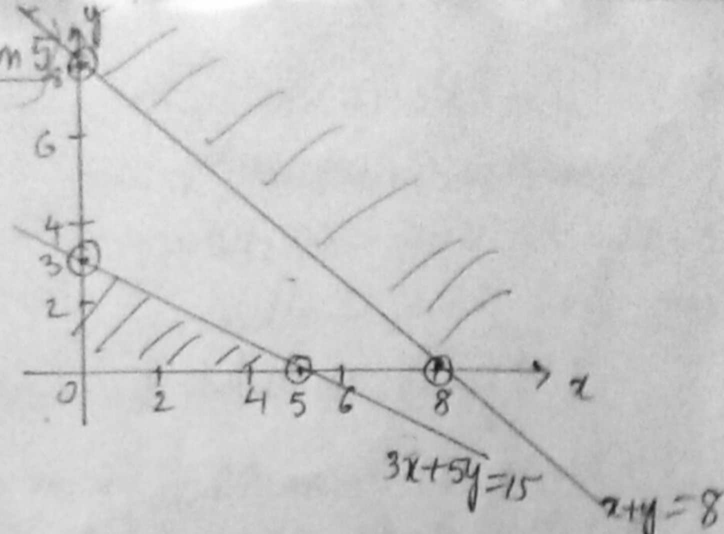
min $z = 3x + 2y$

subject to $x + y \geq 8$

$3x + 5y \leq 15$

$x \geq 0, y \geq 0$

Solution 5:



Here as you can see there is no region where all the constraints are satisfied. Hence, no feasible solution exists or LPP has infeasible solution

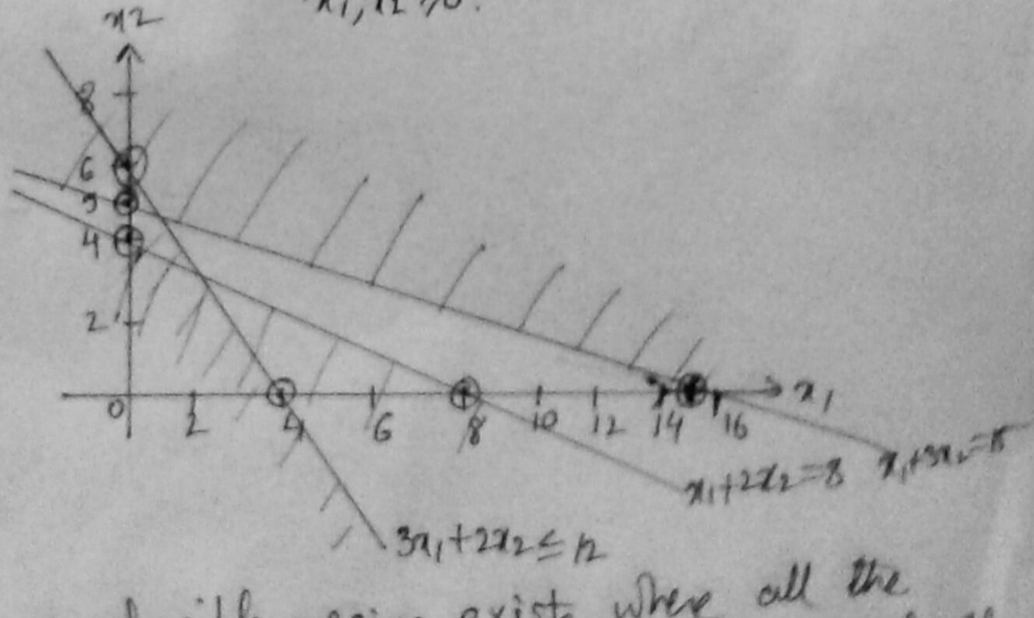
Ex 6: max $z = x_1 + x_2$

s.t. $x_1 + 2x_2 \leq 8$

$3x_1 + 2x_2 \leq 12$

$x_1 + 3x_2 \geq 15.$

$x_1, x_2 \geq 0.$



No feasible region exists where all the three constraints are satisfied. Hence Infeasible solution.