Power Meaurement

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Power Measurement in DC circuit

Electric power (P) consumed by a load (R) supplied from a dc power supply (V_S) is the product of the voltage across the load (V_R) and the current flowing through the load (I_R):

$$P = V_R \times I_R \tag{7.1}$$

Thus, power measurement in a dc circuit can be carried out using a voltmeter (V) and an ammeter (A) using any one of the arrangements shown in Figure 7.1.

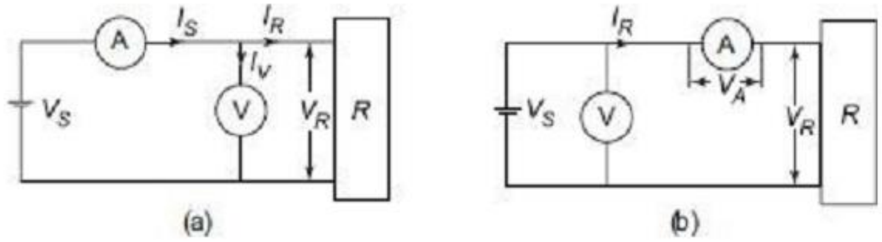


Figure 7.1 *Two arrangements for power measurement in dc circuits*

Power Measurement in DC circuit

One thing should be kept in mind while using any of the two measuring arrangements shown in Figure 7.1; that both the voltmeter and the ammeter requires power for their own operations. In the arrangement of Figure 7.1(a), the voltmeter is connected between the load and the ammeter. The ammeter thus, in this case measures the current flowing into the voltmeter, in addition to the current flowing into the load.

$$\text{Current through the voltmeter} = I_V = V_R/R_V \tag{7.2}$$

where, R_V is the internal resistance of the voltmeter.

$$\begin{aligned}\text{Power consumed by the load} &= V_R \times I_R = V_R \times (I_S - I_V)\\ &= V_R \times I_S - V_R \times I_V\\ &= V_R \times I_S - V_R^2/R_V \qquad (7.3)\\ &= \text{Power indicated by instruments} - \text{Power loss in voltmeter}\end{aligned}$$

Power Measurement in DC circuit

Thus, Power indicated = Power consumed + Power loss in voltmeter

In the arrangement of Figure 7.1(b), the voltmeter measures the voltage drop across the ammeter in addition to that dropping across the load.

$$\text{Voltage drop across ammeter} = V_A = I_R \times R_A \tag{7.4}$$

where, R_A is the internal resistance of the ammeter.

$$\begin{aligned}\text{Power consumed by the load } &= V_R \times I_R = (V_S - V_A) \times I_R \\ &= V_S \times I_R - V_A \times I_R \\ &= V_S \times I_R - I_R^2 \times R_A \qquad (7.5) \\ &= \text{power indicated by instruments} - \text{Power loss in ammeter}\end{aligned}$$

Thus, Power indicated = Power consumed + Power loss in Ammeter

Thus, both arrangements indicate the additional power absorbed by the instruments in addition to indicating the true power consumed by the load only. The corresponding measurement errors are generally referred to as insertion errors.

Example

Two incandescent lamps with 80 Ω and 120 Ω resistances are connected in series with a 200 V dc source. Find the errors in measurement of power in the 80 Ω lamp using a voltmeter with internal resistance of 100 kΩ and an ammeter with internal resistance of 0.1 mΩ, when (a) the voltmeter is connected nearer to the lamp than the ammeter, and (b) when the ammeter is connected nearer to the lamp than the voltmeter

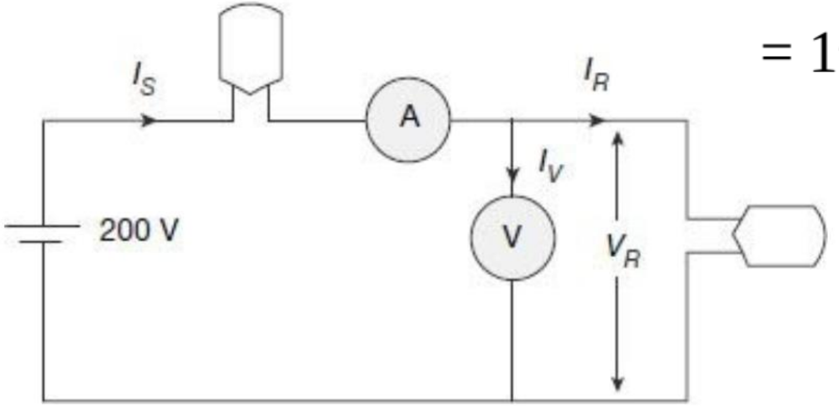
Solution

Solution Assuming both the instruments to be ideal, i.e., the voltmeter with infinite internal impedance and ammeter with zero internal impedance, the current through the series circuit should be

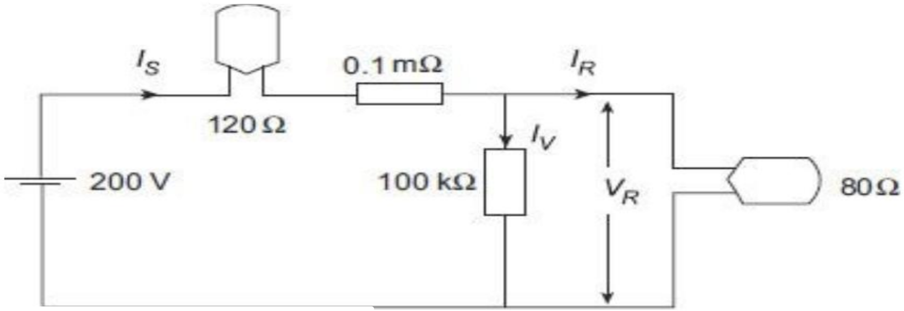
$$= 200/(80 + 120) = 1 \text{ A}$$

Hence, true power consumed by the 80 Ω lamp would have been

$$= 1^2 \text{ X } 80 = 80\ \Omega$$



However, considering the internal resistance of the ammeter and volt equivalent circuit will look like



Supply current (ammeter reading)

$$I_S = \text{Supply voltage/Equivalent resistance of the circuit}$$

$$= \frac{\text{Supply voltage}}{\text{(Series of Lamp 1 and ammeter)} + \text{(Parallel of Lamp 2 and voltmeter)}}$$

$$= 200 \Big/ \left((120 + 0.1 \times 10^{-3}) + {(100 \times 10^3 \times 80)} \Big/ {(100 \times 10^3 + 80)} \right)$$

$$= 1.0003 \text{ A}$$

Actual current through the 80 Ω lamp is

$$I_R = 1.0003 \times \frac{100 \times 10^3}{100 \times 10^3 + 80} \text{ A}$$

$$= 0.9995 \text{ A}$$

Voltage across the 80 Ω lamp (voltmeter reading) is

$$V_R = I_R \times 80$$

$$= 79.962 \text{ V}$$

Thus, actual power consumed by the 80 Ω lamp is

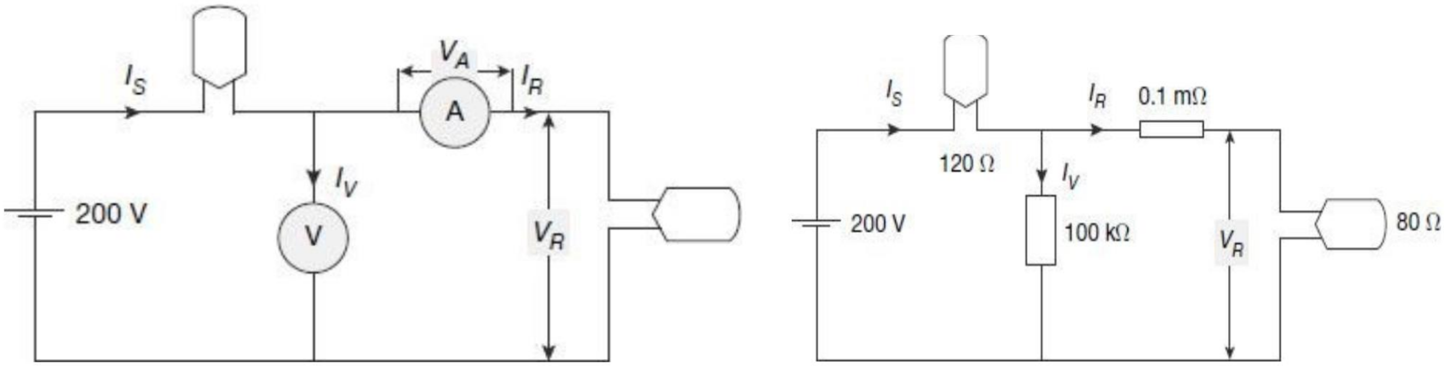
$$V_R \times I_R = 79.962 \times 0.9995 = 79.922 \text{ W}$$

Power consumption as indicated by the two meters

$$= \text{Voltmeter reading} \times \text{Ammeter reading}$$

$$= 79.962 \times 1.0003 = 79.986 \text{ W}$$

(b) In this case, the actual circuit and its equivalent will look like Figure 7.4.



Supply current

$$I_S = \text{Supply voltage/Equivalent resistance of the circuit}$$

$$= \frac{\text{Supply voltage}}{\text{Series of Lamp 1 and [Parallel of voltmeter and (Series of ammeter and Lamp 2)]}}$$

$$= 200\Bigg/\left(120 + \frac{[100\times10^3 \times (80+0.1\times10^{-3})]}{100\times10^3 + (80+0.1\times10^{-3})}\right)$$

$$= 1.003 \text{ A}$$

Current through the 80 Ω lamp (ammeter reading) is

$$I_R = 1.0003\times\frac{100\times10^3}{100\times10^3 + (80+0.1\times10^{-3})}$$

$$= 0.9995 \text{ A}$$

Voltage across the 80 Ω lamp

$$= I_R \times 80 = 79.96 \text{ V}$$

Voltmeter reading

$$= I_V \times 100 \times 10^3$$

$$= (I_S - I_R) \times 100 \times 10^3 = 80 \text{ V}$$

Thus, actual power consumed by the 80 Ω lamp is

$$V_R \times I_R = 79.96 \times 0.9995 = 79.92 \text{ W}$$

Power consumption as indicated by the two meters

$$= \text{Voltmeter reading} \times \text{Ammeter reading}$$

$$= 80 \times 0.9995 = 79.96 \text{ W}$$

Thus, we can have the following analysis:

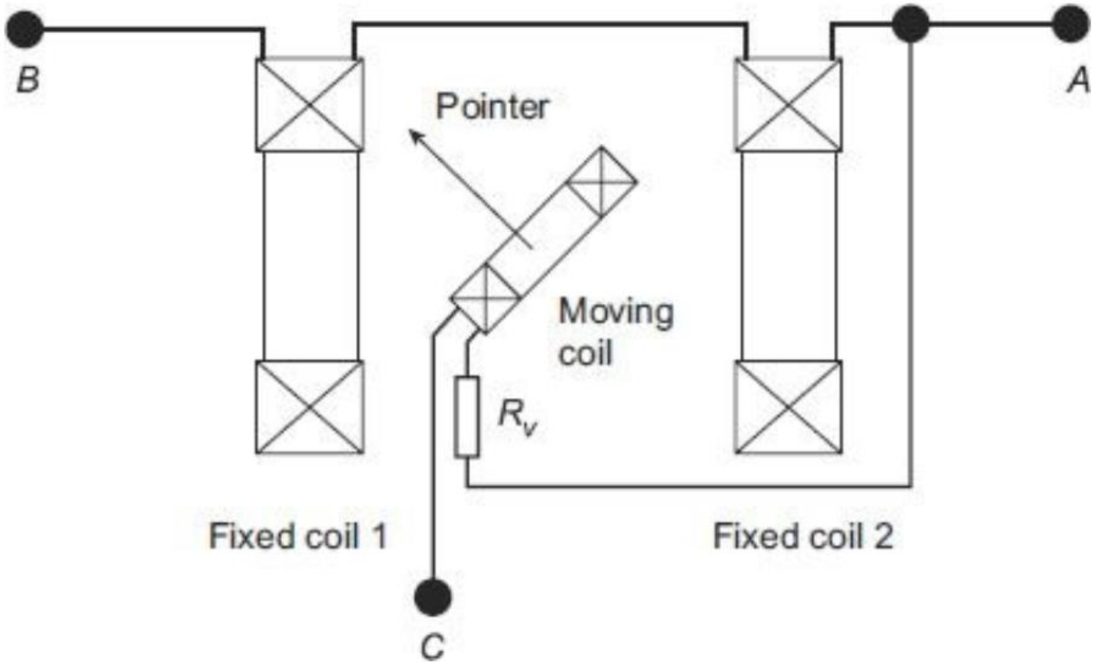
Case	*Power Consumption by 80 W lamp (W)*			*% Error from ideal*
	Ideal Power	*Actual Power*	*Meter Indication*	
a	80	79.922	79.986	0.0175
b	80	79.92	79.96	0.05

Dynamometer

- Power in dc circuits can also be measured by wattmeter. Wattmeter can give direct indication of power and there is no need to multiply two readings as in the case

  when ammeter and voltmeter is used.
- The type of wattmeter most commonly used for such power measurement is the *dynamometer*. It is built by
  - (1) two fixed coils, connected in series and positioned coaxially with space between them, and
  - (2) a moving coil, placed between the fixed coils and fitted with a pointer. Such a construction for a dynamometer-type wattmeter is shown in Figure

Dynamometer



Dynamometer

- It can be shown that the torque produced in the dynamometer is proportional to the product of the current flowing through the fixed coils times that through the moving coil.
- The fixed coils, generally referred to as current coils, carry the load current while the moving coil, generally referred to as voltage coil, carries a current that is proportional, via the multiplier resistor *RV* , to the voltage across the load resistor *R*.
- As a consequence, the deflection of the moving coil is proportional to the power consumed by the load.

Power measurement in ac circuit

In alternating current circuits, the instantaneous power varies continuously as the voltage and current varies while going through a cycle. In such a case, the power at any instant is given by

$$p(t) = v(t) \times i(t) \tag{7.6}$$

where, $p(t)$, $v(t)$, and $i(t)$ are values of instantaneous power, voltage, and current respectively.

Thus, if both voltage and current can be assumed to be sinusoidal, with the current lagging the voltage by phase-angle φ, then

$$v(t) = V_m \sin \omega t$$

and $$i(t) = I_m \sin (\omega t - \varphi)$$

where, V_m and I_m are peak values of voltage and current respectively, and w is the angular frequency.

Power measurement in ac circuit

The instantaneous power $p$is therefore given by

$$p(t) = V_m I_m \sin \omega t \sin(\omega t - \varphi) \tag{7.7}$$

or,

$$p(t) = \frac{V_m I_m}{2}[\cos\varphi - \cos(2\omega t - \varphi)]$$

Average value of power over a complete cycle in such a case will be

$$\begin{aligned}
P &= \frac{1}{2T}\int_0^{2T} p(t)\,dt = \frac{1}{2T}\int_0^{2T} \frac{V_m I_m}{2}[\cos\varphi - \cos(2\omega t - \varphi)]\,dt \\
&= \frac{V_m I_m}{2T}\int_0^{2T}\left[\cos\varphi - \cos\left(\frac{4\pi}{T}t - \varphi\right)\right]dt \\
&= \frac{V_m I_m}{2T}\left[\cos\varphi\, t\Big|_0^T - \frac{T}{4\pi}\sin\left(\frac{4\pi}{T}t - \varphi\right)\Big|_0^T\right] \\
&= \frac{V_m I_m}{4T}[\cos\varphi\, T - 0] \\
&= \frac{V_m I_m}{2}\cos\varphi \\
&= \frac{V_m}{\sqrt{2}}\frac{I_m}{\sqrt{2}}\cos\varphi \\
&= V\, I\cos\varphi
\end{aligned} \tag{7.8}$$

where, $V$and I are rms values of voltage and current respectively and cos j is power factor of the load.

Power measurement in ac circuit

Involvement of the power-factor term cos *j* in the expression for power in ac circuit indicates that ac power cannot be measured simply by connecting a pair of ammeter and voltmeter. A wattmeter, with in-built facility for taking in to account the power factor, can only be used for measurement of power in ac circuits.

Electrodynamometer type wattmeter

Schematic diagram displaying the basic constructional features of a electrodynamometer-type wattmeter is shown in Figure 7.9.

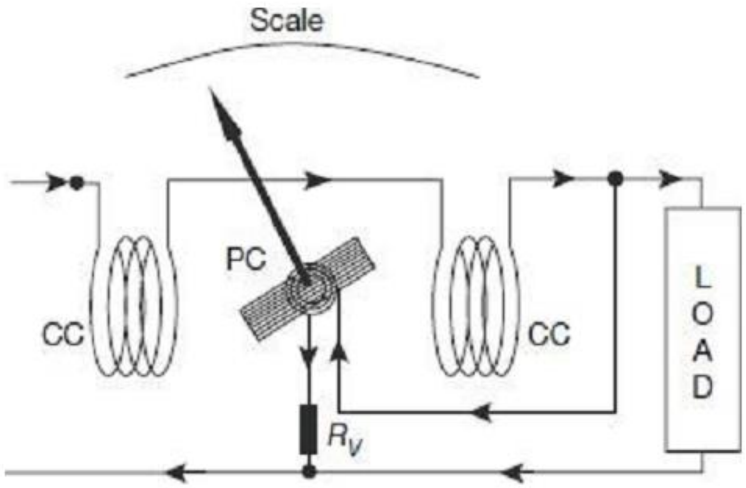


Figure 7.9 *Schematic of electrodynamometer-type wattmeter*

Electrodynamometer type wattmeter

Fixed coils

- Such an instrument has two coils connected in different ways to the same circuit of which power is to be measured. The *fixed coils* or the *field coils* are connected in series with the load so as to carry the same current as the load.
- The fixed coils are hence, termed as the *Current Coils (CC)* of the wattmeter. The main magnetic field is produced by these fixed coils. This coil is divided in two sections so as to provide more uniform magnetic field near the centre and to allow placement of the instrument moving shaft.
- Fixed coils are usually wound with thick wires for carrying the main load current through them.
- Windings of the fixed coil is normally made of stranded conductors running together but, insulated from each other.
- Such stranding of the fixed coils also reduces Eddy-current loss in the conductors.
- Fixed coils are mounted rigidly with the coil supporting structures to prevent any small movement whatsoever and resulting field distortions. Mounting supports are made of ceramic, and not metal, so as not to disturb the magnetic field distribution.

Electrodynamometer type wattmeter

Moving coil system

- The **moving coil** that is connected across the load carries a current proportional to the voltage. Since the moving coil carries a current proportional to the voltage, it is called the *voltage coil* or the *pressure coil* or simply *PC* of the wattmeter.
- A high value **non-inductive resistance** is connected in series with the voltage coil to restrict the current through it to a small value.
- The moving coil, made of fine wires, is wound either as a self-sustaining air-cored coil,or else wound on a nonmetallic former. A metallic former, otherwise would induce Eddycurrents in them under influence of the alternating field.

Electrodynamometer type wattmeter

Movement and Restoring System

- The moving, or voltage coil along with the pointer is mounted on an aluminum spindle in case jewel bearings are used to support the spindle.
- For higher sensitivity requirements, the moving coil may be suspended from a torsion head by a metallic suspension which serves as a lead to the coil.
- In other constructions, the coil may be suspended by a silk fibre together with a spiral spring which gives the required torsion.
- The phosphor-bronze springs are also used to lead current into and out of the moving coil.
- In any case, the torsion head with suspension, or the spring, also serves the purpose of providing the restoring torque to bring the pointer back to its initial position once measurement is over.

Electrodynamometer type wattmeter

Damping system

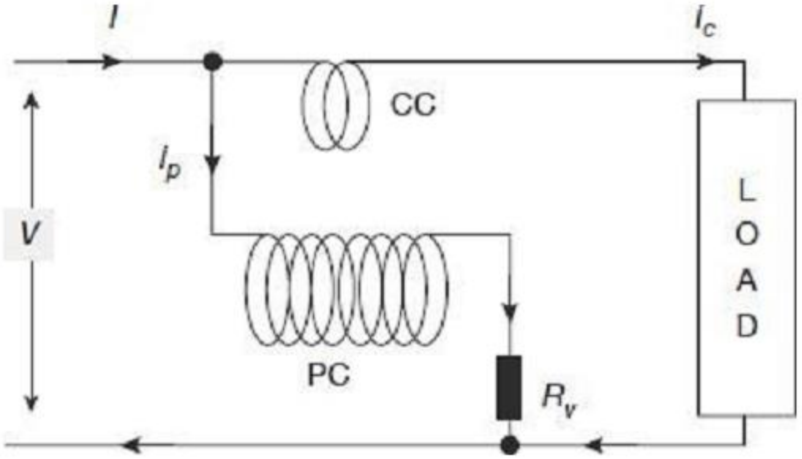
- Damping in such instruments may be provided by small aluminum vanes attached at the bottom of the spindle.
- These vanes are made to move inside enclosed air chambers, thereby creating the damping torque.
- In other cases, the moving coil itself can be stitched on a thin sheet of mica, which acts as the damping vane while movements.

Shielding System

- It is essential to shield the electrodynamometer-type instruments from effects of external magnetic fields. Enclosures of such instruments are thus made of alloys with high permeability to restrict penetration of external stray magnetic fields into the instrument.

Electrodynamometer type wattmeter-Working

The schematic operational circuit of an electrodynamometer-type wattmeter being used for measurement of power in a circuit is shown in Fig



Electrodynamometer type wattmeter- Working

V = voltage to be measured (rms)

I = current to be measured (rms)

i_P = voltage (pressure) coil instantaneous current

i_C = current coil instantaneous current

R_V = external resistance connected with pressure coil

R_P = resistance of pressure coil circuit (PC resistance + R_V)

M = mutual inductance between current coil and pressure coil

θ = angle of deflection of the moving system

ω = angular frequency of supply in radians per second

Electrodynamometer type wattmeter- Working

φ = phase-angle lag of current I with respect to voltage V

the instantaneous torque of the electrodynamometer wattmeter shown in Figure is given by

$$T_i = i_P i_C \frac{dM}{d\theta} \qquad (7.9)$$

Instantaneous value of voltage across the pressure-coil circuit is

$$v_P = \sqrt{2} \times V \sin \omega t$$

If the pressure coil resistance can be assumed to be very high, the whole pressure coil can be assumed to be behaving like a resistance only. The current i_P in the pressure coil thus, can be assumed to in phase with the voltage v_P, and its instantaneous value is

$$i_P = {v_P}/{R_P} = \sqrt{2} \times {V}/{R_P} \sin \omega t = \sqrt{2} \times I_P \sin \omega t$$

where $I_P = V/R_P$ is the rms value of current in pressure coil.

Electrodynamometer type wattmeter- Working

Assuming that the pressure-coil resistance is sufficiently high to prevent branching out of any portion of the supply current towards the pressure coil, the current coil current can be written as

$$i_C = \sqrt{2} \times I \sin(\omega t - \varphi)$$

Thus, instantaneous torque from (7.9) can be written as

$$
\begin{aligned}
T_i &= \sqrt{2} \times I_p \sin \omega t \times \sqrt{2} \times I \sin(\omega t - \varphi) \frac{dM}{d\theta} \\
&= 2 I_p I \sin \omega t \sin(\omega t - \varphi) \frac{dM}{d\theta} \\
&= I_p I \{\cos\varphi - \cos(2\omega t - \varphi)\} \frac{dM}{d\theta} \qquad (7.10)
\end{aligned}
$$

Electrodynamometer type wattmeter-Working

Presence of the term containing 2*w t*, indicates the instantaneous torque as shown in (7.10) varies at twice the frequency of voltage and current.

Average deflecting torque over a complete cycle is

$$T_d = \frac{1}{T}\int_0^T T_i \, d\omega t = \frac{1}{2\pi}\int_0^{2\pi} I_p I \{\cos\varphi - \cos(2\omega t - \varphi)\} \frac{dM}{d\theta} d\omega t$$

$$= \frac{I_p I}{2\pi}\left[\omega t \cos\varphi\right]_0^{2\pi} \frac{dM}{d\theta}$$

$$= I_p I \cos\varphi \frac{dM}{d\theta} \tag{7.11}$$

$$= \frac{V}{R_p} I \cos\varphi \frac{dM}{d\theta}$$

$$= \frac{VI\cos\varphi}{R_p}\frac{dM}{d\theta} \tag{7.12}$$

Electrodynamometer type wattmeter-Working

With a spring constant K, the controlling torque provided by the spring for a final steady-state deflection of θ is given by

$$T_C = K\theta$$

Under steady-state condition, the average deflecting torque will be balanced by the controlling torque provided by the spring. Thus, at balanced condition $T_C = T_d$

$$T_C = T_d$$

$$K\theta = \frac{VI\cos\varphi}{R_P}\frac{dM}{d\theta}$$

$$\theta = \frac{VI\cos\varphi}{KR_P}\frac{dM}{d\theta}$$

$$\theta = \left(K_1\frac{dM}{d\theta}\right)P \qquad (7.13)$$

where, P is the power to be measured and $K_1 = 1/KR_P$ is a constant.

Electrodynamometer type wattmeter- Working

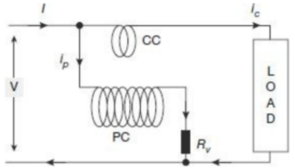
Steady-state deflection θ is thus found to be an indication of the power P to be measured.

Example

An electrodynamometer-type wattmeter has a current coil with a resistance of 0.1 Ω and a pressure coil with resistance of 6.5 kΩ. Calculate the percentage errors while the meter is connected as (i) current coil to the load side, and (ii) pressure coil to the load side. The load is specified as (a) 12 A at 250 V with unity power factor, and (b) 12 A at 25 V with 0.4 lagging power factor.

Solution

(a) Load specified as 12 A at 250 V with unity power factor



(i) Current coil (CC) on load side

True power = *VI* cos *j*

= 250 X 12 X 1

= 3000 Ω

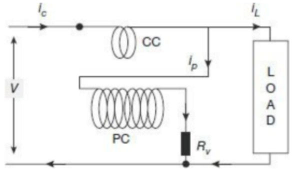
Power lost in CC = I^2 X r_C (where r_C is the resistance of CC)

= 12^2 X 0.1

= 14.4 Ω

The wattmeter will thus read total power = 3000 + 14.4 = 3014.4 Ω

Hence, error in measurement $= \frac{14.4}{3000} \times 100\% = 0.48\%$



(ii) Pressure coil (CC) on load side

True power = *VI* cos *j*

= 250 X 12 X 1

= 3000 Ω

Power lost in PC = V^2/R_P (where R_P is the resistance of PC)

= 250^2/6500

= 9.6 W

The wattmeter will thus read total power = 3000 + 9.6 = 3009.6 Ω

Hence, error in measurement $= \frac{9.6}{3000} \times 100\% = 0.32\%$

Solution

(b) Load specified as 12 A at 250 V with 0.4 power factor

(i) Current coil (CC) on load side

True power = $VI \cos j$

$= 250 \text{ X } 12 \text{ X } 0.4$

$= 1200\ \Omega$

Power lost in CC = $I^2 \text{ X } r_C$ (where r_C is the resistance of CC)

$= 12^2 \text{ X } 0.1$

$= 14.4$ W

The wattmeter will thus read total power = 1200 + 14.4 = 1214.4 Ω

Hence, error in measurement $= \frac{14.4}{1200} \times 100\% = 1.2\%$

(ii) Pressure coil (CC) on load side

True power = 1200 Ω

Power lost in PC = V^2/R_P (where R_P is the resistance of PC)

$= 250^2/6500$

$= 9.6$ W

The wattmeter will thus read total power = 1200 + 9.6 = 1209.6 Ω

Hence, error in measurement $= \frac{9.6}{1200} \times 100\% = 0.8\%$

An electrodynamometer-type wattmeter is used for power

Example

measurement of a load at 100 V and 9 A at a power factor of 0.1 lagging. The pressure coil circuit has a resistance of 3000 Ω and inductance of 30 mH. Calculate the percentage error in wattmeter reading when the pressure coil is connected (a) on the load side, and (b) on the supply side. The current coil has a resistance of 0.1 Ω and negligible inductance. Assume 50 Hz supply frequency.