Amplitude Modulation :⇒

"Amplitude modulation may be defind as a process in which the maximum amplitude of high frequency carrier signal is change proportional to the instantaneous value (amplitude) of the modulating or baseband signal."

"It is a process in which we go change amplitude of high frequency carrier signal according to amplitude of message signal."

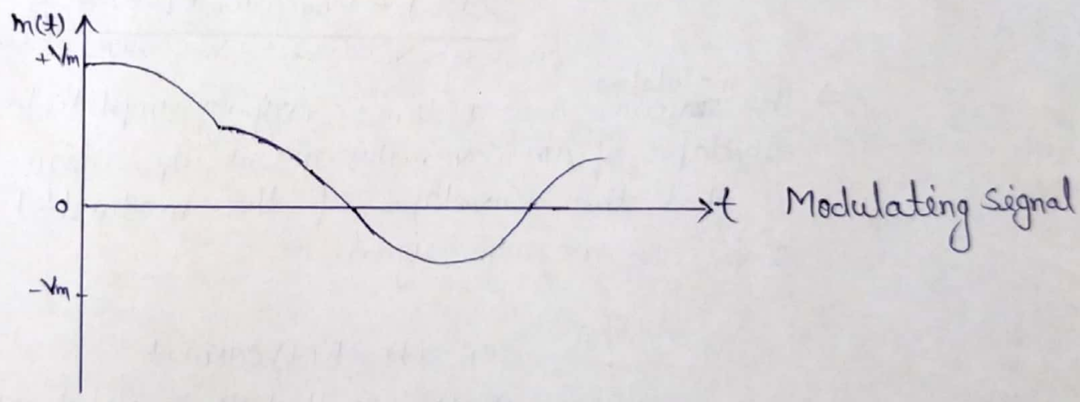
Let us consider a carrier signal $c(t)$ given as

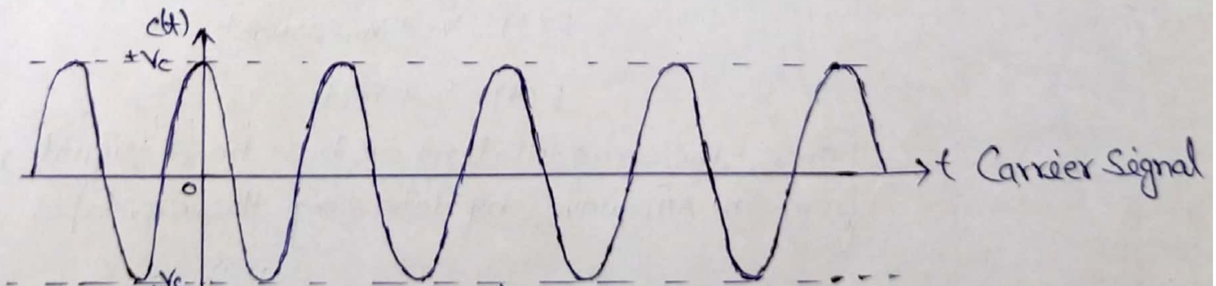
$$c(t) = V_c \cos \omega_c t \quad \text{- - - - - - - - -(i)}$$

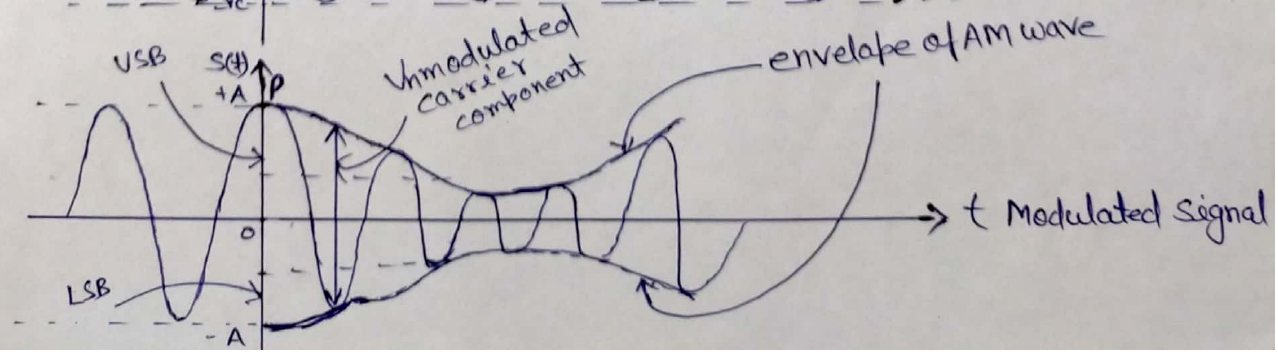
here V_c is the maximum amplitude of high frequency carrier signal and ω_c is the carrier frequency.

and also consider a message signal/modulating signal/base band signal given as

$$m(t) = V_m \cos \omega_m t \quad \text{- - - - - - - - (ii)}$$







⇒ Message signal modulates the amplitude of carrier signal thus generating a new wave i.e. amplitude modulated wave OR AM wave. Therefore the equation for amplitude modulated (AM) wave may be given as —

$$s(t) = A \cos\omega_c t \quad \text{------ (iii)}$$

* Now, Amplitude of AM wave :-

$$A = V_c + m(t)$$

$$A = V_c + V_m \cos\omega_m t$$

⇒ In the process of amplitude modulation, the frequency and phase of carrier wave remain constant, whereas the maximum amplitude varies according to the instantaneous value of the information signal. {⇒ at point 'P' the modulating signal is applied. after P point amplitude modulation occurs. this means amplitude V_c varies according to $m(t)$

⇒ i.e. the amplitude of AM wave is changing around V_c in according to the value of $m(t)$, hence the amplitude of AM wave

$$A = V_c + m(t)$$

$$\text{OR } A = V_c + V_m \cos\omega_m t$$

⇒ Now, the expression of AM wave is

$$s(t) = [V_c + V_m \cos\omega_m t] \cos\omega_c t$$

$$\text{OR } s(t) = V_c \cos\omega_c t + V_m \cos\omega_m t \cos\omega_c t$$

⇒ The modulating wave has a time-varying amplitude called as the envelope of AM wave. This means the unique property of AM wave is that the envelope of the modulated carrier has the same shape as message signal.

$$\text{OR } s(t) = E(t) \cos\omega_c t$$

where, $E(t)$ is called the envelop of AM wave. i.e.

$$E(t) = V_c + V_m \cos\omega_m t$$

$$E(t) = V_c + m(t)$$

⇒ hence, the modulating OR base band signal may be recovered from an AM wave by detecting the envelope.

Spectrum of AM wave OR Frequency-Domain Representation

The wave equation of AM wave will be

$$s(t) = [V_c + V_m \cos\omega_m t] \cos\omega_c t$$

OR
$$s(t) = V_c\left[1 + \frac{V_m}{V_c}\cos\omega_m t\right]\cos\omega_c t$$

where, we know that modulation index

$$\Rightarrow \quad m = \frac{V_m}{V_c}$$

Now, the AM wave equation can be expressed as

$$s(t) = V_c[1 + m\cos\omega_m t]\cos\omega_c t$$

OR
$$s(t) = V_c\cos\omega_c t + mV_c\cos\omega_m t\cos\omega_c t$$

$$\left\{\begin{array}{l}\text{we know that,} \\ \quad 2\cos A\cdot\cos B = \cos(A+B) + \cos(A-B)\end{array}\right\}$$

$$\Rightarrow \quad \cancel{\text{OR } s(t) = V_c\cos\omega_c t + \frac{mV_c}{2}[}$$

OR
$$s(t) = \underbrace{V_c\cos\omega_c t}_{\text{Carrier signal}} + \underbrace{\frac{mV_c}{2}\left[\cos(\omega_c+\omega_m)t\right]}_{\text{Upper Side Band (USB) signal}} + \underbrace{\frac{mV_c}{2}\left[\cos(\omega_c-\omega_m)t\right]}_{\text{Lower Side Band (LSB) signal}}$$

Carrier signal OR Unmodulated carrier signal

(generally keep $\omega_c > \omega_m$, therefor sidebands do not overlap each other)

⇒ Now, if we will have to find its spectrum, first we have to take the Fourier-Transform of AM wave.

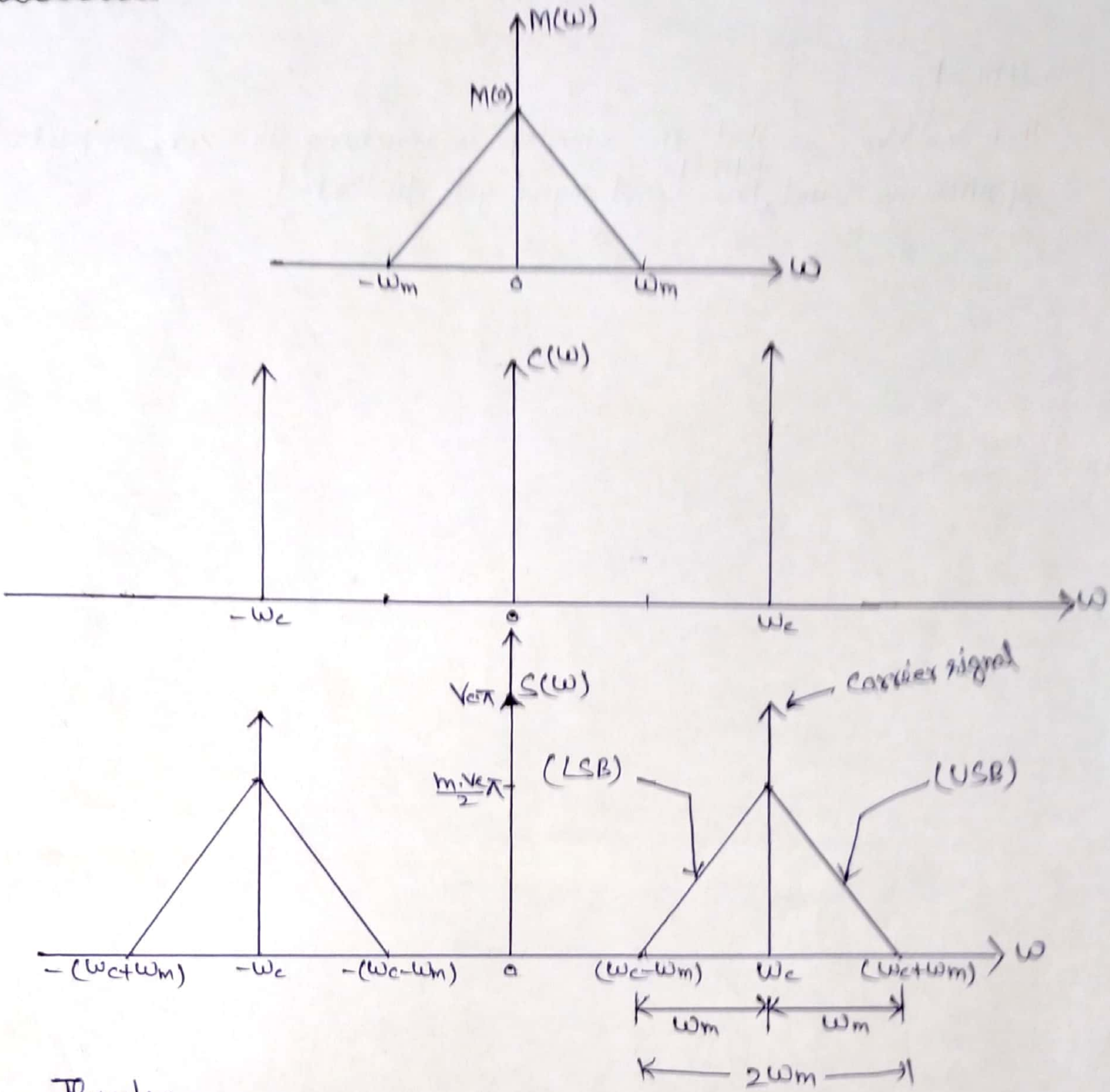
⇒ Let the modulating signal $m(t)$ be band-limited to the interval $-\omega_m \le \omega \le \omega_m$. then the F.T. of $\cos\omega_c t$ consist of two impulse at ω_c and $-\omega_c$ as

$$V_c\cos\omega_c t \xleftrightarrow{F.T.} V_c\pi[\delta(\omega-\omega_c) + \delta(\omega+\omega_c)]$$

Similarly,
$$\frac{mV_c}{2}[\cos(\omega_c+\omega_m)t] \xleftrightarrow{F.T.} \frac{mV_c}{2}\cdot\pi[\delta(\omega-(\omega_c+\omega_m)) + \delta(\omega+(\omega_c+\omega_m))]$$

and
$$\frac{mV_c}{2}[\cos(\omega_c-\omega_m)t] \xleftrightarrow{F.T.} \frac{mV_c}{2}\cdot\pi[\delta(\omega-(\omega_c-\omega_m)) + \delta(\omega+(\omega_c-\omega_m))]$$

Spectrums :⇒



Therefor,

The transmission bandwidth (B.W.) of AM signal is given as

$$B.W. = 2\omega_m \text{ rad/Hz}$$

OR

$$B.W. = 2 f_m \text{ Hz}$$

Modulation Index :⇒

⇒ In AM system the modulation index is defined as the measure of extent of amplitude variation about an unmodulated maximum carrier. Mathematically, $m_a = \frac{|m(t)|_{max}}{\text{maximum carrier amplitude}} = \frac{|m(t)|_{max}}{V_c}$

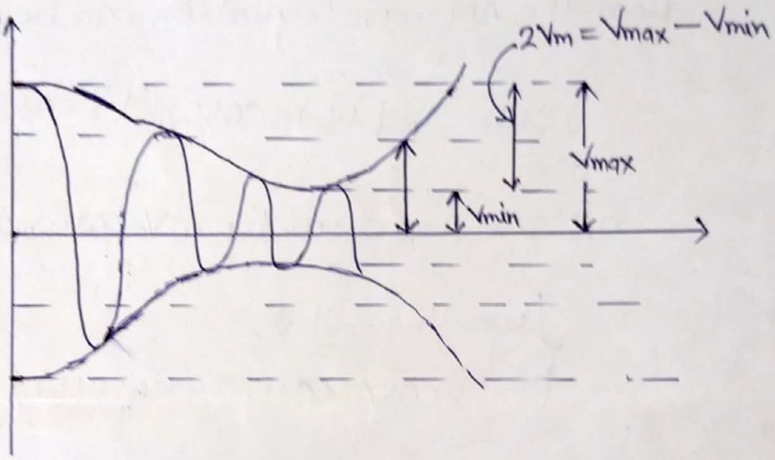
OR

⇒ Ratio of amplitude of message signal to amplitude of carrier signal.

Mathematically,

$$\boxed{m = \frac{V_m}{V_c}}$$

⇒



$(V_c > V_m)$

⇒ $2V_m = V_{max} - V_{min}$

$V_m = \frac{V_{max} - V_{min}}{2}$

⇒ $V_c = V_{max} - V_m$

$= V_{max} - \frac{V_{max} - V_{min}}{2}$

$= \frac{2V_{max} - V_{max} + V_{min}}{2}$

$= \frac{V_{max} + V_{min}}{2}$

⇒ Now,

$$\boxed{\text{Modulation index } (m) = \frac{V_{max} - V_{min}}{V_{max} + V_{min}}}$$

* Modulation index is also known as depth of modulation, degree of modulation or modulation factor. Also modulation index multiplied by 100 is known as percentage modulation.

Physical significance of modulation index :-

⇒ If $m < 1$,

then the maximum amplitude of baseband signal is less than maximum carrier amplitude i.e. $V_m < V_c$, so that the envelop is not reaching the zero amplitude axis of AM wave and the baseband signal may be fully recoverd from the envelope of AM wave.

⇒ If $m = 1$,

then there is 100% modulation. This is the basic condition of modulation.

⇒ If $m > 1$,

then $V_m > V_c$, So that the envelop is reaching the zero amplitude axis of AM wave and shape of base band signal gets distorted.