① Lecture - 4

Example Find the root of the equation $\cos x = xe^x$ using the Regula-Falsi method correct to four decimal places.

Sol: Let $f(x) = \cos x - xe^x = 0$

So that

$$f(0) = 1$$

$$f(1) = \cos 1 - e = -2.17798$$

i.e. the root lies between 0 and 1.

$\therefore$ taking $x_0 = 0$, $x_1 = 1$, $f(x_0) = 1$
and $f(x_1) = -2.17798$ in the,
we get

$$x_2 = x_0 - \frac{x_1 - x_0}{f(x_1) - f(x_0)} f(x_0) \quad \text{— (i)}$$

$$= 0 + \frac{1}{3.17798} \times 1$$

$$= 0.31467$$

Now $f(0.31467) = 0.51987.$

i.e. the root lies between 0.31467 and 1.

$\therefore$ taking $x_0 = 0.31467$, $x_1 = 1$, $f(x_0) = 0.51987$,
$f(x_1) = -2.17798$ in (i) we get

$$x_3 = 0.31467 + \frac{0.68533}{2.69785} \times 0.51987$$

$$= 0.44673.$$

Now

$$f(0.44673) = 0.20356$$

i.e. the root lies between 0.44673 and 1.

$\therefore$ taking $x_0 = 0.44673$,

$x_1 = 1$,

$f(x_0) = 0.20356$,

$f(x_1) = -2.17798$ in (i), we get.

$$x_4 = 0.44673 + \frac{0.55327}{2.38154} \times 0.20356$$

$$= 0.49402$$

Repeating this process, the successive approximate are

$x_5 = 0.50995$,

$x_6 = 0.51520$

$x_7 = 0.51692$

$x_8 = 0.51748$

$x_9 = 0.51767$

$x_{10} = 0.51775$ etc.

Hence the root is 0.5177 correct to 4 decimal places.

(3) Find a real root of the equation $x \log_{10} x = 1.2$ by regula-falsi method correct to four decimal places.

Sol. Let $f(x) = x \log_{10} x - 1.2$

So that $f(1) = -ve$, $f(2) = -ve$ and $f(3) = +ve$

$\therefore$ a root lies between 2 and 3.

Taking $x_0 = 2$ and $x_1 = 3$, $f(x_0) = -0.59794$ and $f(x_1) = 0.23136$, in the method of false position, we get

$$x_2 = x_0 - \frac{x_1 - x_0}{f(x_1) - f(x_0)} f(x_0) \quad \text{—(i)}$$

$= 2.72102$

Now $f(x_2) = f(2.72102) = -0.01709$

i.e. the root lies between 2.72102 and 3

$\therefore$ taking $x_0 = 2.72102$
$x_1 = 3$, $f(x_0) = -0.01709$
and $f(x_1) = 0.23136$ in (i), we get.

$$x_3 = 2.72102 + \frac{0.27898}{0.23136 + 0.01709} \times 0.01709$$

$= 2.74021$

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Repeating this process, the successive approxi-mation are

$$x_4 = 2.74024,$$

$$x_5 = 2.74063 \text{ etc.}$$

Hence the root is 2.7406 correct to 4 decimal places.

Example Use the method of false position to find the fourth root of 32 correct to three decimal places.

Solution Let $x = (32)^{1/4}$

So that $x^4 - 32 = 0$

Take $f(x) = x^4 - 32$, Then $f(2) = -16$ and $f(3) = 49$, i.e. a root lies between 2 and 3.

$\therefore$ taking $x_0 = 2$, $x_1 = 3$, $f(x_0) = -16$, $f(x_1) = 49$ in the method of False position, we get

$$x_2 = x_0 - \frac{x_1 - x_0}{f(x_1) - f(x_0)} f(x_0) \quad \text{— (i)}$$

$$= 2 + \frac{16}{65} = 2.2462$$

Now, $f(x_2) = f(2.2462) = -6.5438$.

i.e. the root lies between 2.2462 and 3.

$\therefore$ taking $x_0 = 2.2462$,

$x_1 = 3$,

$f(x_0) = -6.5438$,

$f(x_1) = 49$.

in (i) we get

$$x_3 = 2.2462 - \frac{3 - 2.2462}{49 + 6.5438}(-6.5438)$$

$$= 2.335.$$

Now $f(x_3) = f(2.335) = -2.2732$ i.e.

the root lies between 2.335 and 3.

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$\therefore$ taking $x_0 = 2.335$, $x_1 = 3$, $f(x_0) = -2.2732$ and $f(x_1) = 49$ in (i) we obtain

$$x_4 = 2.335 - \frac{3 - 2.335}{49 + 2.2732}(-2.2732)$$

$$= 2.3645$$

Repeating this process, the successive approximation are $x_5 = 2.3770$, $x_6 = 2.3779$ etc [Since $x_5 = x_6$ upto 3 decimal places], we take $(32)^{1/4} = 2.378$.

Newton - Raphson (N-R) Method:

Bisection method, R-F Method and Secant, we needed two values in the neighbourhood of the root. ~~while in~~

In the N-R Method only one value is required.

Geometrically:

Let us suppose that x_0 is the initial estimate for the root α of $f(x) = 0$. If we draw a tanget at point (x_0, y_0) of the curve $y = f(x)$, then.

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$$y - y_0 = \left(\frac{dy}{dx}\right)_{x=x_0} (x - x_0) \quad \text{—— (A)}$$

The point where the tangent (A) cuts the x-axis ($y=0$), say $x = x_1$, is the next estimate of the root. i.e

$$\boxed{x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}, \quad y_0 = f(x_0).}$$

This process may be repeated to get the next estimate x_2 using x_1 and so on. Let us suppose that we have computed nth estimate x_n, the next estimate may be computed

$$\boxed{x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}, \quad n = 0, 1, ---}$$

—— (B)

This iterative formula (B) is known as N-R formula,

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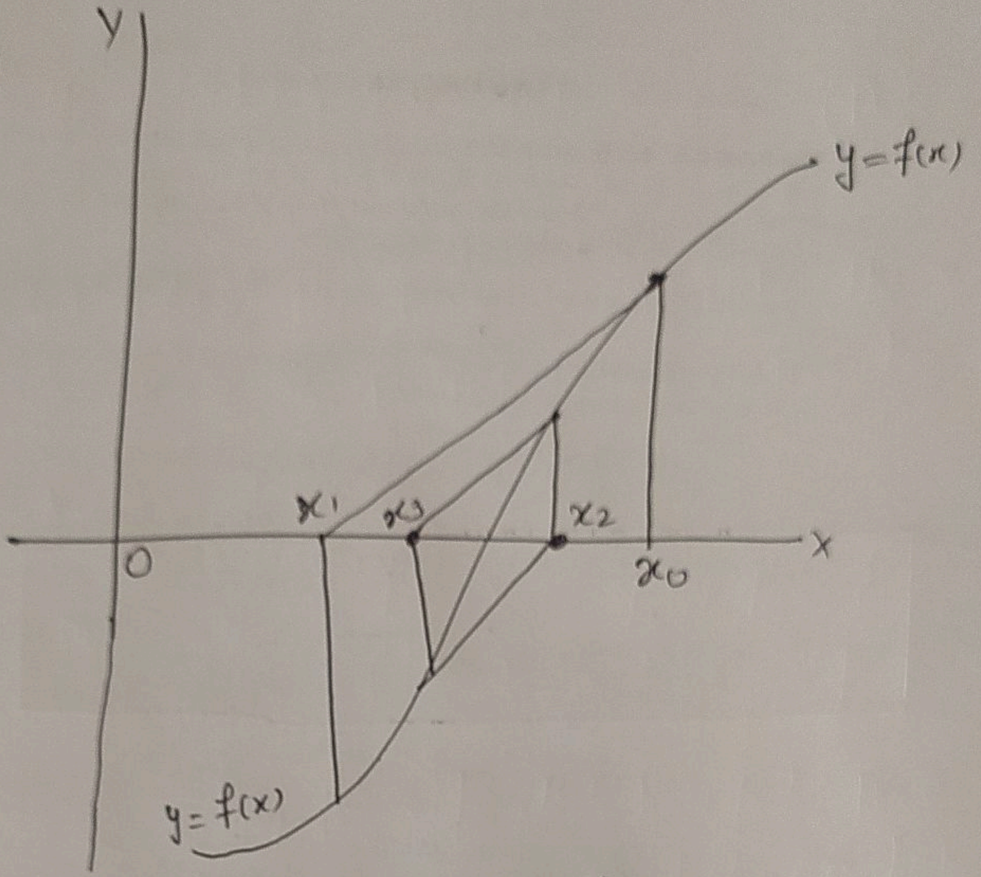

N-R Method.

Analytically:

Let x_0 be an approximate root of $f(x)=0$ and let $x_1 = x_0+h$ be the correct root so that $f(x_1)=0$. Expaning $f(x_0+h)$ by Taylor series.

$$f(x_0)+h f'(x_0)+\frac{h^2}{2!}f''(x_0)+\cdots=0$$

Neglecting the second and higher order derivative we have

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$$f(x_0) + h f'(x_0) = 0$$

which gives $h = -\dfrac{f(x_0)}{f'(x_0)}$.

A better approximation than x_0 is therefore given by x_1, where

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

Successive approximation are given by $x_2, x_3, \ldots, x_{n+1}$, where

$$\boxed{x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}}$$

which is the Newton - Raphson method.