Revised Simplex Method:

Standard form (I): II (For maximization problem).

Ex $\max z = 2x_1 + x_2$

subject to

$$3x_1 + 4x_2 \le 6$$
$$6x_1 + x_2 \le 3$$
$$x_1, x_2 \ge 0$$

Standard form

$\max z = 2x_1 + x_2 + 0s_1 + 0s_2$

subject to

$$3x_1 + 4x_2 + s_1 = 6$$
$$6x_1 + x_2 + s_2 = 3$$
$$x_1, x_2, s_1, s_2 \ge 0$$

Rewrite standard form as

$$z - 2x_1 - x_2 - 0s_1 - 0s_2 = 0$$
$$3x_1 + 4x_2 + s_1 + 0s_2 = 6$$
$$6x_1 + x_2 + 0s_1 + s_2 = 3$$
$$x_1, x_2, s_1, s_2 \ge 0$$

II

$$\begin{array}{ccccc} \beta_0^{(1)} & a_1^{(1)} & a_2^{(1)} & \beta_1^{(1)}(s_1) & \beta_2^{(1)}(s_2) \\ \end{array}$$
$$\left[\begin{array}{ccccc} 1 & -2 & -1 & 0 & 0 \\ 0 & 3 & 4 & 1 & 0 \\ 0 & 6 & 1 & 0 & 1 \end{array}\right] \begin{bmatrix} z \\ x_1 \\ x_2 \\ s_1 \\ s_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 6 \\ 3 \end{bmatrix}$$

z $\quad\underbrace{\quad}_{\text{columns of } A (a_j)}$ $\quad\underbrace{\quad}_{\beta}$.

$B = [\beta_0^{(1)}, \beta_1^{(1)}, \beta_2^{(1)}]$

↳ Initial Basis matrix.

Note: 1st column $\beta_0^{(1)}$ corresponding to z is not changed in problem

Addition Table

$a_1^{(1)}$	$a_2^{(1)}$
-2	-1
3	4
6	1

Iteration 1:

B.V.	B_1^{-1}			X_B	X_k	$\min X_B/X_k$, $x_k > 0$
	$\beta_0^{(1)}(a_3)$	$\beta_1(s_1)$	$\beta_2^{(1)}(s_2)$			
z	1	0	0	0	-2	–
s_1	0	1	0	6	3	2
s_2	0	0	1	3	6	1/2 →

Optimality Test: $[\Delta_1, \Delta_2] = -(1^{st} \text{ row of } B_1^{-1})[a_1^{(1)}\ a_2^{(1)}]$

$$= -(1\ 0\ 0)\begin{bmatrix} -2 \\ 3 \\ 6 \end{bmatrix}\begin{bmatrix} -1 \\ 4 \\ 1 \end{bmatrix} = [2, 1]$$

$\Delta_i \ge 0$ ∴ Not optimal

$\Delta_1 = 2\ (> \Delta_2)$ ∴ $\boxed{a_1 \uparrow \text{ i.e, } x_1 \text{ enters}}$.

$$X_1 = B_1^{-1} a_1^{(1)} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} -2 \\ 3 \\ 6 \end{bmatrix} = \begin{bmatrix} -2 \\ 3 \\ 6 \end{bmatrix}$$

∴ ~~X_k~~ = 1. k=1

~~$\min_{x_{i1}>0} \frac{x_B}{x_1}$~~ $\min_{x_{i4}>0} \frac{x_B}{x_1} = \left\{ -, \frac{6}{3}, \frac{3}{6} \right\} = \frac{1}{2} \; (= s_2 (\beta_2^{(1)}))$

$\Rightarrow \boxed{\beta_2^{(1)} (s_2) \text{ leaves}}$

Improved solution:

$$\begin{array}{c} \begin{array}{cccc} \beta_1^{(1)} & \beta_2^{(1)} & x_B & x_k \end{array} \\ \left[\begin{array}{ccc} 0 & 0 & 0 \\ 1 & 0 & 6 \\ 0 & 1 & 3 \end{array}\right] \begin{array}{c} -2 \\ 3 \\ \boxed{6} \end{array} \end{array} \longrightarrow \begin{array}{c} \begin{array}{cccc} \beta_1^{(1)} & \beta_2^{(1)} & x_B & x_k \end{array} \\ \left[\begin{array}{ccc} 0 & 1/3 & 1 \\ 1 & -1/2 & 9/2 \\ 0 & 1/6 & 1/2 \end{array}\right] \begin{array}{c} 0 \\ 0 \\ 1 \end{array} \end{array}$$

Iteration 2:

BV	B_1^{-1}			x_B	X_k	$\min_{x_{ik}>0} x_B/x_k$
	$\beta_0^{(1)}(z)$	$\beta_1^{(1)}(s_1)$	$\beta_2^{(1)}(a_1^{(1)})$			
z	1	0	1/3	1	-2/3	—
s_1	0	1	-1/2	9/2	7/2	9/7 →
x_1	0	0	1/6	1/2	1/6	3

Additional Table

$a_4^{(1)}$	$a_2^{(1)}$
0	-1
0	4
1	1

Optimality Test: $[\Delta_4, \Delta_2] = -(1 \;\; 0 \;\; 1/3) \begin{bmatrix} 0 & -1 \\ 0 & 4 \\ 1 & 1 \end{bmatrix} = [-1/3, 2/3]$

$\Delta_2 = 2/3 > 0 \; \therefore \boxed{a_2^{(1)} (x_2) \uparrow}$

$X_2 = B_1^{-1} a_2^{(1)} = \begin{bmatrix} 1 & 0 & 1/3 \\ 0 & 1 & -1/2 \\ 0 & 0 & 1/6 \end{bmatrix} \begin{bmatrix} -1 \\ 4 \\ 1 \end{bmatrix} = \begin{bmatrix} -2/3 \\ 7/2 \\ 1/6 \end{bmatrix}$ $\therefore X_k = X_2$

$\min_{x_{i2}>0} \frac{X_B}{X_2} = \left\{ -, \frac{9}{7}, 3 \right\} = \frac{9}{7} \; (= s_1) \Rightarrow \boxed{s_1 (\beta_1^{(1)}) \text{ leaves}}$

Improved solution

$$\begin{array}{c} \begin{array}{cccc} \beta_1^{(1)} & \beta_2^{(1)} & x_B & x_k \end{array} \\ \left[\begin{array}{ccc} 0 & 1/3 & 1 \\ 1 & -1/2 & 9/2 \\ 0 & 1/6 & 1/2 \end{array}\right] \begin{array}{c} -2/3 \\ \boxed{7/2} \\ 1/6 \end{array} \end{array} \longrightarrow \begin{array}{c} \begin{array}{cccc} \beta_1^{(1)} & \beta_2^{(1)} & x_B & x_k \end{array} \\ \left[\begin{array}{ccc} 4/21 & 5/21 & 13/7 \\ 2/7 & -1/7 & 9/7 \\ -1/21 & 4/21 & 2/7 \end{array}\right] \begin{array}{c} 0 \\ 1 \\ 0 \end{array} \end{array}$$

Iteration 3

BV	$\beta_0^{(1)}$	$\beta_1^{(1)}$	$\beta_2^{(1)}$	X_B	X_k	$\min\limits_{X_{ik}>0} X_B/X_{ik}$
	B^{-1}					
z	1	$4/21$	$5/21$	$13/7$		
x_2	0	$2/7$	$-1/7$	$9/7$		
x_1	0	$-1/21$	$4/21$	$2/7$		

Additional Table

(s_2) $a_4^{(1)}$	(s_1) $a_3^{(1)}$
0	0
0	1
1	0

Optimality Test: $[\Delta_4, \Delta_5] = -(1\ \frac{4}{21}\ \frac{5}{21})\begin{bmatrix}0&0\\0&1\\1&0\end{bmatrix} = [-\frac{5}{21}, -\frac{4}{21}]$

$\because$ All $\Delta_j \le 0$ $\therefore$ optimality is reached.

Optimal soln: $x_1 = \frac{2}{7}$ $x_2 = \frac{9}{7}$ $z_{max} = \frac{13}{7}$ — Ans

Ex 2 $\max z = 5x_1 + 3x_2$

subject to:

$3x_1 + 5x_2 \le 15$

$5x_1 + 2x_2 \le 10$

$x_1, x_2 \ge 0$

Standard LPP

$\max z = 5x_1 + 3x_2 + 0s_1 + 0s_2$

s.t.

$3x_1 + 5x_2 + s_1 = 15$

$5x_1 + 2x_2 + s_2 = 10$

$s_1, s_2, x_1, x_2 \ge 0$

Rewriting it as system of eqn

$z - 5x_1 - 3x_2 - 0s_1 - 0s_2 = 0$

$3x_1 + 5x_2 + s_1 + 0s_2 = 15$

$5x_1 + 2x_2 + 0s_1 + s_2 = 10$

$x_1, x_2, s_1, s_2 \ge 0$

$$\left[\begin{array}{ccccc} \beta_0^{(1)} & a_1^{(1)} & a_2^{(1)} & \beta_1^{(1)} & \beta_2^{(1)} \\ 1 & -5 & -3 & 0 & 0 \\ 0 & 3 & 5 & 1 & 0 \\ 0 & 5 & 2 & 0 & 1 \\ z & \multicolumn{2}{c}{\underbrace{\qquad}_{a}} & \multicolumn{2}{c}{\underbrace{\qquad}_{\beta}} \end{array}\right] \begin{bmatrix} z \\ x_1 \\ x_2 \\ s_1 \\ s_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 15 \\ 10 \end{bmatrix} \longleftarrow$$

$B = [\beta_0^{(1)}, \beta_1^{(1)}, \beta_2^{(1)}]$

Iteration 1

BV	$\beta_0^{(1)}(z)$	$\beta_1^{(1)}(s_1)$	$\beta_2^{(1)}(s_2)$	X_B	X_k	$\min\limits_{X_{ik}>0} \frac{X_B}{X_k}$
	B_1^{-1}					
z	1	0	0	0	-5	—
s_1	0	1	0	15	3	5
s_2	0	0	1	10	5	2 →

Additional Table

$a_1^{(1)}$	$a_2^{(1)}$
-5	-3
3	5
5	2

Optimality test: $[\Delta_1, \Delta_2] = -(1\ 0\ 0)\begin{bmatrix}-5\\3\\5\end{bmatrix}\begin{bmatrix}-3\\5\\2\end{bmatrix} = [5, 3]$

$\Delta_1 = 5\ (>\Delta_2) \Rightarrow \boxed{a_1^{(1)}(x_1)\uparrow}$

$$X_1 = B_1^{-1} a_1^{(1)} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -5 \\ 3 \\ 5 \end{bmatrix} = \begin{bmatrix} -5 \\ 3 \\ 5 \end{bmatrix}. \qquad X_k = X_1$$

$$\min_{i,\, x_{i1}} \frac{X_B}{X_1} = \left\{ -, \frac{15}{3}, \frac{5}{2} \right\} = 2.5 \; (= s_2 (= \beta_2^{(1)}))$$

$$\Rightarrow \boxed{s_2 (\beta_2^{(1)}) \text{ - leaves}}$$

Improved solution

$$\begin{array}{cccc} \beta_1^{(1)} & \beta_2^{(1)} & X_B & X_k \\ \left[\begin{matrix} 0 \\ 1 \\ 0 \end{matrix}\right. & \begin{matrix} 0 \\ 0 \\ 1 \end{matrix} & \left.\begin{matrix} 0 \\ 15 \\ 10 \end{matrix}\right] & \begin{matrix} -5 \\ 3 \\ \boxed{5} \end{matrix} \end{array} \longrightarrow \begin{array}{cccc} \beta_1^{(1)} & \beta_2^{(1)} & X_B & X_k \\ \left[\begin{matrix} 0 \\ 1 \\ 0 \end{matrix}\right. & \begin{matrix} 1 \\ -3/5 \\ 1/5 \end{matrix} & \left.\begin{matrix} 10 \\ 9 \\ 2 \end{matrix}\right] & \begin{matrix} 0 \\ 0 \\ 1 \end{matrix} \end{array}$$

Iteration 2:

BV	B_1^{-1}			X_B	X_k	$\min \frac{X_B}{X_k}$, x_{k2}
	$\beta_0^{(1)}$	$\beta_1^{(1)}$	$\beta_2^{(1)}$			
z	1	0	1	10	-1	—
s_1	0	1	$-3/5$	9	$+19/5$	$45/19$ →
x_1	0	0	$1/5$	2	$2/5$	5

Addition Table

$a_4^{(1)}$	$a_2^{(1)}$
0	-3
0	5
1	2

Optimality Test

$$[\Delta_4, \Delta_2] = -(1 \;\; 0 \;\; 1) \begin{bmatrix} 0 & -3 \\ 0 & 5 \\ 1 & 2 \end{bmatrix} = [-1, 1]$$

$$\Delta_2 = 1 > 0 \quad \therefore \boxed{a_2^{(1)} (x_2) \uparrow}$$

$$X_2 = B_1^{-1} a_2^{(1)} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -3/5 \\ 0 & 0 & 1/5 \end{bmatrix} \begin{bmatrix} -3 \\ 5 \\ 2 \end{bmatrix} = \begin{bmatrix} -1 \\ 19/5 \\ 2/5 \end{bmatrix} \qquad X_k = X_2$$

$$\min_{i,\, x_{i2}} \frac{X_B}{X_2} = \left\{ -1, \frac{45}{19}, [illegible] \right\} = \frac{45}{19} \; (= s_1) \qquad \therefore \boxed{s_1 \text{ leaves } (= \beta_1^{(1)})}$$

Improved soln

$$\begin{array}{cccc} \beta_1^{(1)} & \beta_2^{(1)} & X_B & X_k \\ \left[\begin{matrix} 0 \\ 1 \\ 0 \end{matrix}\right. & \begin{matrix} 1 \\ -3/5 \\ 1/5 \end{matrix} & \left.\begin{matrix} 10 \\ 9 \\ 2 \end{matrix}\right] & \begin{matrix} -1 \\ \boxed{19/5} \\ 2/5 \end{matrix} \end{array} \longrightarrow \begin{array}{cccc} \beta_1^{(1)} & \beta_2^{(1)} & X_B & X_k \\ \left[\begin{matrix} 5/19 \\ 5/19 \\ -2/19 \end{matrix}\right. & \begin{matrix} 16/19 \\ -3/19 \\ 5/19 \end{matrix} & \left.\begin{matrix} 235/19 \\ 45/19 \\ 20/19 \end{matrix}\right] & \begin{matrix} 0 \\ 1 \\ 0 \end{matrix} \end{array}$$

Iteration 3

BV	$\beta_0^{(1)}(z)$	$\beta_1^{(1)}(a_5^{(1)})$	$\beta_2^{(1)}(a_1^{(1)})$	X_B	X_K	$\min_{X_K>0} \frac{X_B}{X_K}$
		B_1^{-1}				
z	1	5/19	16/19	235/19		
x_2	0	5/19	−3/19	45/19		
x_1	0	−2/19	5/19	20/19		

Additional table

$a_4^{(1)}$ (s_1)	$a_5^{(1)}$ (s_2)
0	0
0	1
1	0

Optimality Test: $[\Delta_4, \Delta_3] = -(1\ \frac{5}{19}\ \frac{16}{19})\begin{bmatrix}0&0\\0&1\\1&0\end{bmatrix} = \left[\frac{-16}{19}, \frac{-15}{19}\right]$

$\because$ All $\Delta_j \le 0$, optimality is reached

Optimal soln: $x_1 = \frac{20}{19}$, $x_2 = \frac{45}{19}$ $z_{max} = \frac{235}{19}$

Ex3 $\max z = 6x_1 - 2x_2 + 3x_3$

subject to

$2x_1 - x_2 + 2x_3 \le 2$

$x_1 + 4x_3 \le 4$

$x_1, x_2, x_3 \ge 0$

$z - 6x_1 + 2x_2 - 3x_3 - 0s_1 - 0s_2 = 0$

$2x_1 - x_2 + 2x_3 + 1s_1 + 0s_2 = 2$ $x_1, x_2, s_1, s_2, x_3 \ge 0$

$x_1 + 0x_2 + 4x_3 + 0s_1 + s_2 = 4$

Iteration 1

BV	$\beta_0^{(1)}$	$\beta_1^{(1)}$	$\beta_2^{(1)}$	X_B	X_K	$\min_{X_K>0} \frac{X_B}{X_K}$
		B_1^{-1}				
z	1	0	0	0	−6	—
s_1	0	1	0	2	2	1 →
s_2	0	0	1	4	1	4

Addition table

$a_1^{(1)}$	$a_2^{(1)}$	$a_3^{(1)}$
−6	2	−3
2	−1	2
1	0	4

optimality test: $[\Delta_1, \Delta_2, \Delta_3] = -(1\ 0\ 0)\begin{bmatrix}-6&2&-3\\2&-1&2\\1&0&4\end{bmatrix} = [6, -2, 3]$

$\therefore \Delta_1 = 6\ (> \Delta_j)$ $\therefore$ $\boxed{a_1^{(1)}(x_1)\uparrow}$

$X_1 = B_1^{-1} a_1^{(1)} = \begin{bmatrix}-6\\2\\1\end{bmatrix}$ $X_K = X_1$

Improved Solution:

$$\begin{array}{cccc} \beta_1^{(1)} & \beta_2^{(1)} & X_B & X_k \\ \end{array}$$

$$\left[\begin{array}{ccc} 0 & 0 & 0 \\ 1 & 0 & 2 \\ 0 & 1 & 4 \end{array}\right] \begin{array}{c} -6 \\ \boxed{2} \\ 1 \end{array} \longrightarrow \left[\begin{array}{ccc} 3 & 0 & 6 \\ 1/2 & 0 & 1 \\ -1/2 & 1 & 3 \end{array}\right] \begin{array}{c} 0 \\ 1 \\ 0 \end{array}$$

$$\begin{array}{cccc} \beta_1^{(1)} & \beta_2^{(1)} & X_B & X_1 \\ \end{array}$$

Iteration 2

BV	B_1^{-1}			X_B	X_k	$\min_{x_{ik}>0} \frac{X_B}{X_1}$
	$\beta_0^{(1)}$	$\beta_1^{(1)}$	$\beta_2^{(1)}$			
z	1	3	0	6	-1	—
x_1	0	$1/2$	0	1	$-1/2$	—
S_2	0	$-1/2$	1	3	$1/2$	6 →

Additional table

$\overset{s_1}{a_4^{(1)}}$	$a_2^{(1)}$	$a_3^{(1)}$
0	2	-3
1	-1	2
0	0	3

optimality Test: $[\Delta_4, \Delta_2, \Delta_3] = -(-1, 3\ 0)\begin{bmatrix} 0 & 2 & -3 \\ 1 & -1 & 2 \\ 0 & 0 & 3 \end{bmatrix}$

$= [0\ \ 1\ -3]$

$\Delta_2 = 1 > 0$ $\boxed{\therefore a_2^{(1)} (x_2) \uparrow}$

$X_2 = B_1^{-1} a_2^{(1)} = \begin{bmatrix} 1 & 3 & 0 \\ 0 & 1/2 & 0 \\ 0 & -1/2 & 1 \end{bmatrix} \begin{bmatrix} +2 \\ -1 \\ 0 \end{bmatrix} = \begin{bmatrix} -1 \\ -1/2 \\ 1/2 \end{bmatrix}$ $X_k = X_2$

$\min_{i, x_{i2}} \frac{X_B}{X_2} = \left\{ -\ -, 6 \right\} = 6 = \boxed{S_2 (\beta_2^{(1)}) \text{ leaves}}$

Improved solution

$$\begin{array}{cccc} \beta_1^{(1)} & \beta_2^{(1)} & X_B & X_k \\ \end{array}$$

$$\left[\begin{array}{ccc} 3 & 0 & 6 \\ 1/2 & 0 & 1 \\ -1/2 & 1 & 3 \end{array}\right] \begin{array}{c} -1 \\ -1/2 \\ 1/2 \end{array} \longrightarrow \left[\begin{array}{ccc} 2 & 2 & 12 \\ 0 & 1 & 4 \\ -1 & 2 & 6 \end{array}\right] \begin{array}{c} 0 \\ 0 \\ 1 \end{array}$$

$$\begin{array}{cccc} \beta_1^{(1)} & \beta_2^{(1)} & X_B & X_k \\ \end{array}$$

Iteration 3 :

	B_1^{-1}					
BV	$\beta_0^{(1)}$	$\beta_1^{(1)}$	$\beta_2^{(1)}$	x_B	X_k	min $\frac{x_B}{x_k}$, $x_k > 0$
z	1	2	2	12		
x_1	0	0	1	4		
x_2	0	-1	2	6		

Addition Table

$a_4^{(1)}$	$a_5^{(1)}$	$a_3^{(1)}$
0	0	-3
1	0	2
0	1	4

Optimality Test: $[\Delta_4, \Delta_5, \Delta_3] = -(1\ 2\ 2)\begin{bmatrix} 0 & 0 & -3 \\ 1 & 0 & 2 \\ 0 & 1 & 4 \end{bmatrix} = [-2, -2, -9]$

$\because$ All $\Delta_j \le 0$ $\therefore$ optimality is reached

$\boxed{x_1 = 4,\ x_2 = 6 \quad z_{max} = 12}$ → optimal solution.