Revised Simplex Method (Standard Form II)

Ex Solve LPP by revised simplex method

$\min z = x_1 + 2x_2$

subject to

$2x_1 + 5x_2 \geq 6$

$x_1 + x_2 \geq 2$

$x_1, x_2 \geq 0$

Conversion into maximization prob

$\max z' = -x_1 - 2x_2$ where $z' = -z$

subject to

$2x_1 + 5x_2 + s_1 + A_1 = 6$

$x_1 + x_2 - s_2 + A_2 = 2$

$x_1, x_2, s_1, s_2, A_1, A_2 \geq 0$

Now add all the constraint eqns; we have

$3x_1 + 6x_2 - s_1 - s_2 + A_1 + A_2 = 8$

multiply it by (-1),

$-3x_1 - 6x_2 + s_1 + s_2 + A_3 = -8$ where $A_3 = -(A_1 + A_2)$

Hence, we have standard II form is

$z' + x_1 + 2x_2 = 0$

$-3x_1 - 6x_2 + s_1 + s_2 + A_3 = -8$ (Note)

$2x_1 + 5x_2 - s_1 + A_1 = 6$

$x_1 + x_2 - s_2 + A_2 = 2$

$x_1, x_2, s_1, s_2, A_1, A_2 \geq 0$ z', A_3 are unrestricted

System of constraint equations is

Columns: $e_1^{(2)}$ (z'), $a_1^{(2)}$, $a_2^{(2)}$, $a_3^{(2)}$, $a_4^{(2)}$, $e_2^{(2)}$ (A_3), $\beta_1^{(2)}$ (A_1), $\beta_2^{(2)}$ (A_2)

$$\begin{bmatrix} 1 & 1 & 2 & 0 & 0 & 0 & 0 & 0 \\ 0 & -3 & -6 & 1 & 1 & 1 & 0 & 0 \\ 0 & 2 & 5 & -1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & -1 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} z' \\ x_1 \\ x_2 \\ s_1 \\ s_2 \\ A_3 \\ A_1 \\ A_2 \end{bmatrix} = \begin{bmatrix} 0 \\ -8 \\ 6 \\ 2 \end{bmatrix}$$

Basis Matrix

Columns: $e_1^{(2)}$, $e_2^{(2)}$, $\beta_1^{(2)}$, $\beta_2^{(2)}$

$$B_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_1 = \begin{bmatrix} 1 & 2 & 0 & 0 \\ -3 & -6 & 1 & 1 \\ 2 & 1 & -1 & 0 \\ 1 & 5 & 0 & -1 \end{bmatrix}$$

Note 1. (1) First 2 columns and associated variables z' and A_3 never change.

(2) In Phase I we shall max A_3 (not z'). This phase will end once max $A_3 = 0$.

Phase I Iteration 1

BV	$e_1^{(2)}$	$e_2^{(2)}$	$\beta_1^{(2)}$	$\beta_2^{(2)}$	X_B	X_k	min $\frac{X_B}{X_k}$, $X_k>0$
z'	1	0	0	0	0	2	—
A_3	0	1	0	0	-8	-6	—
A_1	0	0	1	0	6	5	6/5 →
A_2	0	0	0	1	2	1	2

(The columns $e_1^{(2)}, e_2^{(2)}, \beta_1^{(2)}, \beta_2^{(2)}$ form B_2^{-1}.)

$a_1^{(2)}$	$a_2^{(2)}$	$a_3^{(2)}$	$a_4^{(2)}$
1	2	0	0
-3	-6	1	1
2	5	-1	0
1	1	0	-1

max $A_3 = -8$

Optimality Test:

$$\Delta_j = -(2^{nd} \text{ row of } B_2^{-1})(a_1^{(2)}\ a_2^{(2)}\ a_3^{(2)}\ a_4^{(2)})$$

$$= -(0\ 1\ 0\ 0)\begin{bmatrix} 1 & 2 & 0 & 0 \\ -3 & -6 & 1 & 1 \\ 2 & 5 & -1 & 0 \\ 1 & 1 & 0 & -1 \end{bmatrix} = [+3\ \ +6\ \ -1\ \ -1]$$

$\Delta_2 = 6$ is max. positive $\therefore$ $a_2^{(2)}$ $(x_2)\uparrow$ Enters

$X_2 = B_2^{-1} a_2^{(2)} = \begin{bmatrix} 2 \\ -6 \\ 5 \\ 1 \end{bmatrix}$ $X_k = X_2$. From table A_1 leaves

Improved solution

$$\begin{array}{cccc} \beta_1^{(1)} & \beta_2^{(1)} & X_B & X_k \end{array}$$

$$\left[\begin{array}{ccc|c} 0 & 0 & 0 & 2 \\ 0 & 0 & -8 & -6 \\ 1 & 0 & 6 & \boxed{5} \\ 0 & 1 & 2 & 1 \end{array}\right] \longrightarrow \left[\begin{array}{ccc|c} -2/5 & 0 & -12/5 & 0 \\ 6/5 & 0 & -4/5 & 0 \\ 1/5 & 0 & 6/5 & 1 \\ -1/5 & 1 & 4/5 & 0 \end{array}\right]$$

Iteration 2:

BV	$e_1^{(2)}$	$e_2^{(2)}$	$\beta_1^{(2)}$	$\beta_2^{(2)}$	X_B	X_{1k}	min $\frac{X_B}{X_{ik}}$, $X_{ik}>0$
z'	1	0	$-2/5$	0	$-12/5$	$1/5$	—
A_3	0	1	$6/5$	0	$-4/5$	$-3/5$	—
x_2	0	0	$1/5$	0	$6/5$	$2/5$	3
A_2	0	0	$-1/5$	1	$4/5$	$3/5$	$4/3$ →

(B_2^{-1} = columns $e_1^{(2)}, e_2^{(2)}, \beta_1^{(2)}, \beta_2^{(2)}$)

$a_1^{(2)}$	A_1	$a_3^{(2)}$	$a_4^{(2)}$
1	0	0	0
-3	0	1	1
2	1	-1	0
1	0	0	-1

$\max A_3 = -4/5$

Optimality Test $\Delta_j = -\left(0 \;\; 1 \;\; \frac{6}{5} \;\; 0\right)\begin{bmatrix} 1 & 0 & 0 & 0 \\ -3 & 0 & 1 & 1 \\ 2 & 1 & -1 & 0 \\ 1 & 0 & 0 & -1 \end{bmatrix} = \left[\frac{3}{5} \;\; \frac{-6}{5} \;\; \frac{1}{5} \;\; -1\right]$

$\Delta_1 = \frac{3}{5}$ is max +ive. $\therefore$ $a_1^{(2)}$ $(x_1)\uparrow$ Enters

$$X_1 = B_2^{-1}a_1^{(2)} = \begin{bmatrix} 1 & 0 & -2/5 & 0 \\ 0 & 1 & 6/5 & 0 \\ 0 & 0 & 1/5 & 0 \\ 0 & 0 & -1/5 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ -3 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 1/5 \\ -3/5 \\ 2/5 \\ 3/5 \end{bmatrix} \qquad X_k = X_1$$

From table: A_2 leaves

Improved solution

$$\left[\begin{array}{ccc|c} -2/5 & 0 & -12/5 & 1/5 \\ 6/5 & 0 & -4/5 & -3/5 \\ 1/5 & 0 & 6/5 & 2/5 \\ -1/5 & 1 & 4/5 & \boxed{3/5} \end{array}\right] \longrightarrow \left[\begin{array}{ccc|c} -1/3 & -1/3 & -8/3 & 0 \\ 1 & 1 & 0 & 0 \\ 1/3 & -2/3 & 2/3 & 0 \\ -1/3 & 5/3 & 4/3 & 1 \end{array}\right]$$

Iteration 3

BV	e_1	e_2	β_1	β_2	X_B	X_{1k}	min $\frac{X_B}{X_k}$, $X_{ik}>0$
z'	1	0	$-1/3$	$-1/3$	$-8/3$		
A_3	0	1	1	1	$\boxed{0}$		
x_2	0	0	$1/3$	$-2/3$	$2/3$		
x_1	0	0	$-1/3$	$5/3$	$4/3$		

(B^{-1} = columns $e_1, e_2, \beta_1, \beta_2$)

A_2	A_1	$a_3^{(2)}$	$a_4^{(2)}$
0	0	0	0
0	0	1	1
0	1	-1	0
1	0	0	-1

$\max A_3 = 0$ $\therefore$ phase I ends. In phase II remove A_1, A_2 (artificial variables) column from additional table

Phase II:

BV	$e_1^{(2)}$	$e_2^{(2)}$	$\beta_1^{(2)}$	$\beta_2^{(2)}$	X_B	X_k	min $\frac{X_B}{X_k}$
z'	1	0	$-1/3$	$-1/3$	$-8/3$		
A_3	0	1	1	1	1		
x_2	0	0	$1/3$	$-2/3$	$+2/3$		
x_1	0	0	$-1/3$	$5/3$	$4/3$		

(B_2^{-1} spans columns $e_1^{(2)}$ to $\beta_2^{(2)}$)

$a_3^{(2)}$	$a_4^{(2)}$
0	0
1	1
-1	0
0	-1

Optimality test

$\Delta_j = -(1^{st}$ row of $B_2^{-1})\,[a_3^{(2)}\ a_4^{(2)}]$ (Note)

$$= -(1\ \ 0\ \ -1/3\ \ -1/3)\begin{bmatrix}0 & 0\\ 1 & 1\\ -1 & 0\\ 0 & -1\end{bmatrix} = [-1/3,\ -1/3]$$

$\because$ All $\Delta_j \le 0$ $\therefore$ optimal solution is obtained.

$x_1 = 4/3 \quad x_2 = 2/3 \quad z'_{max} = -8/3 \quad \therefore$ $\boxed{z_{min} = 8/3}$.

Ex Using revised simplex method solve

~~max $z = x_1 + 2x_2 + 3x_3 - x_4$~~

~~subject to~~

~~$x_1 + 2x_2 + x_3 = 15$~~

~~$2x_1 + x_2 + 5x_3 = 20$~~

~~$x_1 + 2x_2 + x_3 + x_4 = 10$~~

~~$x_1, x_2, x_3, x_4 \ge 0$.~~

(min $z = 2x_1 + x_2$

subject to

$3x_1 + x_2 \le 3$

$4x_1 + 3x_2 \ge 6$

$x_1 + 2x_2 \le 3$

$x_1, x_2 \ge 0$.

(solution: $x_1 = \frac{3}{5}$ $x_2 = \frac{6}{5}$ $z_{min} = 12/5$)