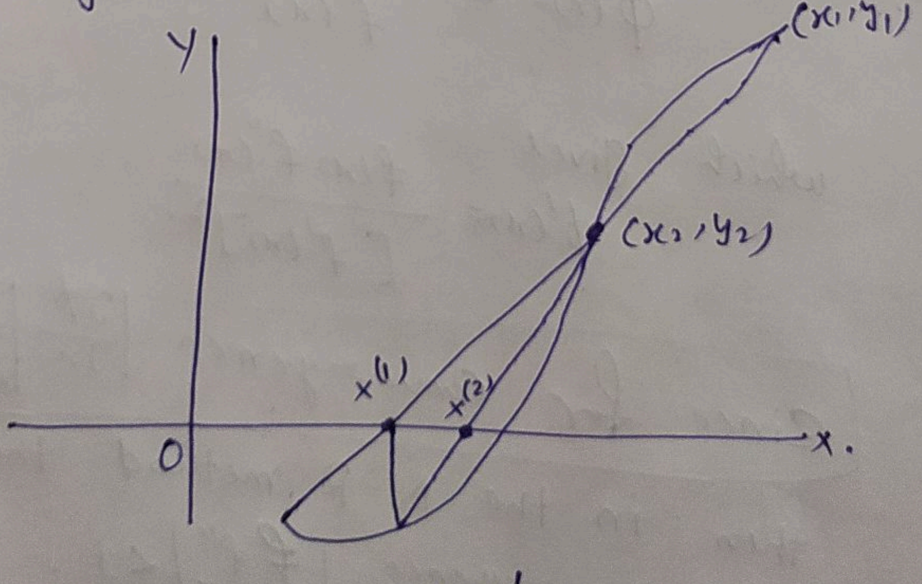
(1)

22 May 2021

Secant Method

In the Secent method also two values x_1 and x_2 are taken in the neighbourhood of the root but they are not ought to be on the opposite sides of the root like Regula – Falsi Method. That is, $f(x_1)$ and $f(x_2)$ may have same sign or opposite sign. Then a straight line (Secant). is drawn through (x_1, y_1) and (x_2, y_2) intersecting the x-axis at a point x. One of the points say (x_1, y_1) is discarded and again a line is drawn through (x_2, y_2) and (x, y). In actual computations (x_1, y_1) is replaced by (x_2, y_2) and the new point (x, y) replaces (x_2, y_2). The process is repeated until two successive value of x agree within desired accuracy.



Secant method.

(2)

Example Using Secant method find the positive root of $x^2 - 6e^{-x} = 0$ correct up to two decimal take the initial value $x_1 = 2.5$ and $x_2 = 2$.

Sol.

$$\boxed{x = \frac{x_1 y_2 - x_2 y_1}{y_2 - y_1}}$$

when $y_1 = f(x_1)$, $y_2 = f(x_2)$.

Iteration	x_1	$y_1 = f(x_1)$	x_2	$y_2 = f(x_2)$	x	$y = f(x)$
1	2.5	5.758	2	3.188	1.380	0.395
2	2	3.188	1.380	0.395	1.292	0.021
3	1.380	0.395	1.292	0.021	1.287	−0.0002.
4	1.292	0.021	1.287	−0.0002	1.287	

Ans. 1.287

Note:

$$\phi(x) = x - \frac{f(x)}{f'(x)}$$

which gives

$$\phi'(x) = \frac{f(x)\, f''(x)}{[f'(x)]^2}.$$

$$\boxed{\text{Since for Convergence } \left|\frac{\partial \phi}{\partial x}\right|_{x=\alpha} < 1.}$$

then in the N-R method the condition for convergence $\left|\frac{f f''}{f'^2}\right| < 1$.

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(3)

Convergence of Secant / Regula - Falsi method.

Let x_n denote the n^{th} iterate of the root of $f(x)=0$ or zero of the function $y=f(x)$. If α is the exact root, let $\boxed{\alpha = x_n + \epsilon_n}$ where ϵ_n is the error in x_n.

Let us suppose that x_{n-1} and x_n have been computed, then the next iterate x_{n+1} is given by the formula

$$x_{n+1} = \frac{x_{n-1} y_n - x_n y_{n-1}}{y_n - y_{n-1}}$$

$$= \frac{x_{n-1} f(x_n) - x_n f(x_{n-1})}{f(x_n) - f(x_{n-1})}$$

or $$\alpha - \epsilon_{n+1} = \frac{(\alpha-\epsilon_{n-1}) f(\alpha-\epsilon_n) - (\alpha-\epsilon_n) f(\alpha-\epsilon_{n-1})}{f(\alpha-\epsilon_n) - f(\alpha-\epsilon_{n-1})}$$

Using Taylor's Series

$$= \frac{(\alpha-\epsilon_{n-1})\{f(\alpha) - \epsilon_n f'(\alpha) + \frac{\epsilon_n^2}{2!} f''(\alpha) + \cdots\} - (\alpha-\epsilon_n)\{f(\alpha) - \epsilon_{n-1} f'(\alpha) + \frac{\epsilon_{n-1}^2}{2} f''(\alpha) + \cdots\}}{\{f(\alpha) - \epsilon_n f'(\alpha) + \frac{\epsilon_n^2}{2} f''(\alpha) + \cdots\} - \{f(\alpha) - \epsilon_{n-1} f'(\alpha) + \frac{\epsilon_{n-1}^2}{2} f''(\alpha) + \cdots\}}$$

$$= \frac{[-(\alpha-\epsilon_{n-1})\epsilon_n + (\alpha-\epsilon_n)\epsilon_{n-1}] f'(\alpha) + \frac{f''(\alpha)}{2}[(\alpha-\epsilon_{n-1})\epsilon_n^2 - (\alpha-\epsilon_n)\epsilon_{n-1}^2]}{(-\epsilon_n + \epsilon_{n-1}) f'(\alpha) + \frac{f''(\alpha)}{2}(\epsilon_n^2 - \epsilon_{n-1}^2)}$$

Since $f(\alpha)=0$

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(4)

$$= \frac{-\alpha(\epsilon_n-\epsilon_{n-1})f' + \frac{f''}{2}\left[\alpha(\epsilon_n^2-\epsilon_{n-1}^2) - \epsilon_{n-1}\cdot\epsilon_n(\epsilon_n-\epsilon_{n-1})\right]}{-(\epsilon_n-\epsilon_{n-1})f' + \frac{f''}{2}(\epsilon_n-\epsilon_{n-1})(\epsilon_n+\epsilon_{n-1})}.$$

where $f'' = f''(\alpha)$; $f' = f'(\alpha)$

$$= \frac{-\alpha f' + \frac{f''}{2}\left[\alpha(\epsilon_n+\epsilon_{n-1}) - \epsilon_{n-1}\cdot\epsilon_n\right]}{-f' + \frac{f''}{2}(\epsilon_n+\epsilon_{n-1})}$$

$$= \frac{\alpha - K\left[\alpha(\epsilon_n+\epsilon_{n-1}) - \epsilon_{n-1}\cdot\epsilon_n\right]}{1-K(\epsilon_n+\epsilon_{n-1})},$$

where $K = \frac{f''}{2f'}$

$$= \left[\alpha - K\{\alpha(\epsilon_n+\epsilon_{n-1}) - \epsilon_{n-1}\epsilon_n\}\right]\left[1-K(\epsilon_n+\epsilon_{n-1})\right]^{-1}$$

$$= \left[\alpha - K\{\alpha(\epsilon_n+\epsilon_{n-1}) - \epsilon_{n-1}\epsilon_n\}\right]\left[1+K(\epsilon_n+\epsilon_{n-1}) + K^2(\epsilon_n+\epsilon_{n-1})^2 + \cdots\right]$$

Leaving out third and higher order terms in ϵ_{n-1} and ϵ_n.

$$= \alpha + K\epsilon_{n-1}\epsilon_n$$

or $\boxed{\epsilon_{n+1} = -K\epsilon_n\epsilon_{n-1}}$ where $K = \frac{f''(\alpha)}{2f'(\alpha)}$