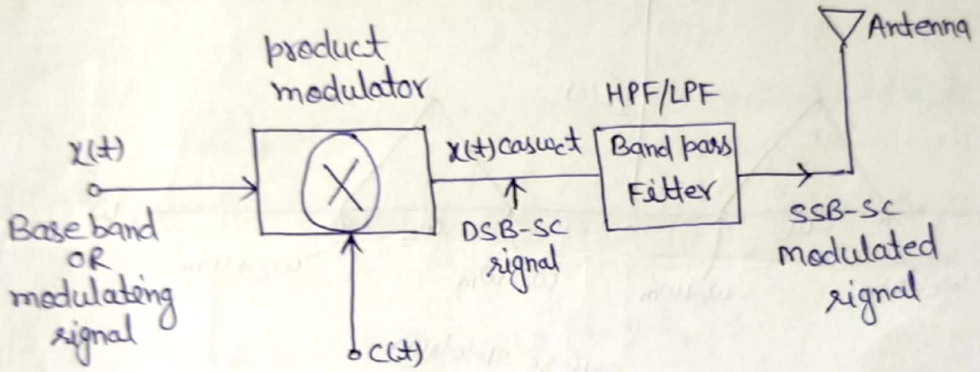
Single sideband Suppressed-Carrier (SSB-SC) Modulation :⇒

In AM and DSB-SC modulation are wasteful of bandwidth since they both need a transmission bandwidth equal to twice the message signal bandwidth.

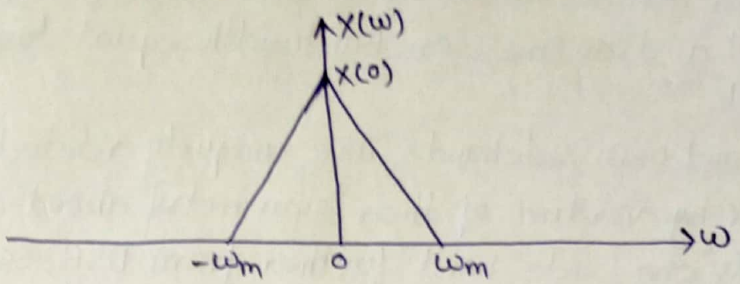
The LSB and USB sidebands are uniquely related i.e. real images of each other by virtue of their symmetry about the carrier frequency. If we suppress one side band further from DSB-SC signal then there is no effect on message signal, and the remaining signal is called SSB-SC modulated signal and this type of technique is known as SSB-SC modulation technique.

Thus, SSB-SC system reduces the transmission bandwidth by half.

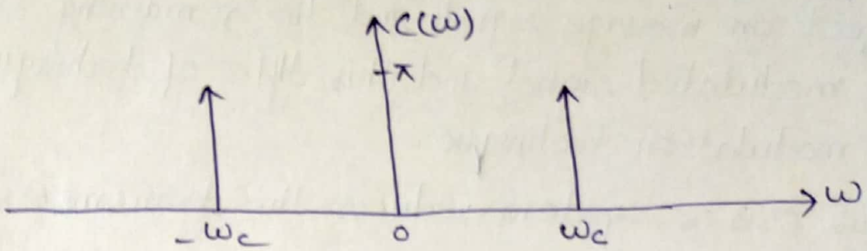


The concept of single sideband modulation with the help of different frequency spectrums.

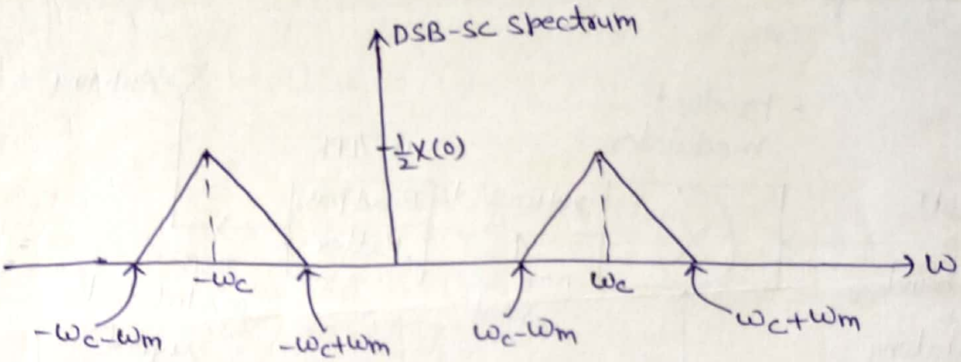
X(t)



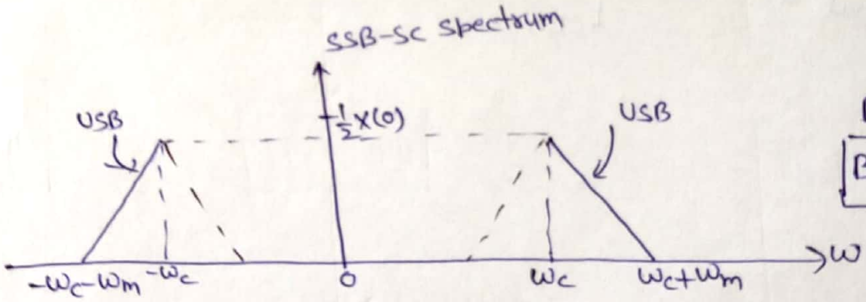
c(t)



$S_{DSB-SC}(t)$



$S_{SSB-SC}(t)$/HPF

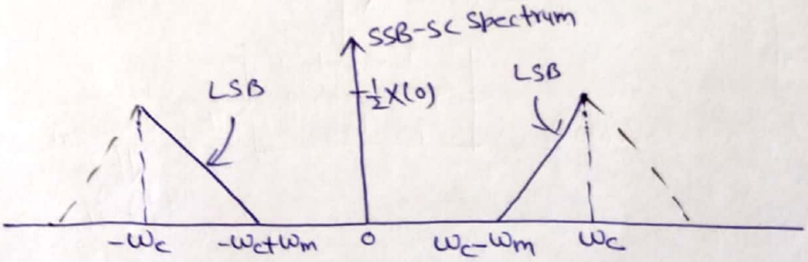


$B = \omega_c + \omega_m - \omega_c$

$\boxed{B = \omega_m}$

50% of DSB-SC

$S_{SSB-SC}(t)$/LPF

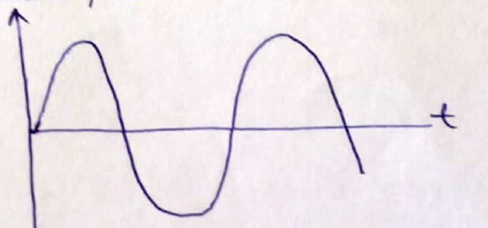


$B = \omega_c - (\omega_c - \omega_m)$

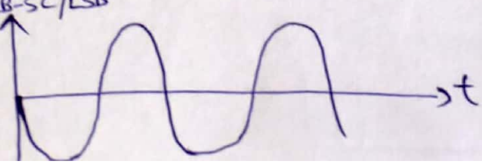
$\boxed{B = \omega_m}$

50% DSB-SC

S_{SSB-SC}/USB



S_{SSB-SC}/LSB



Frequency-Domain Representation of SSB-SC Wave :⇒

Let us consider a single-tone modulating signal

$$m(t) = V_m \cos \omega_m t = V_m \cos 2\pi f_m t$$

and also consider a carrier signal

$$c(t) = V_c \cos \omega_c t = V_c \cos 2\pi f_c t$$

Now, the expression for AM wave/signal

$$S(t) = V_c \cos \omega_c t + m(t) V_c \cos \omega_c t = V_c \cos 2\pi f_c t + m(t) V_c \cos 2\pi f_c t$$

$$S(t) = \underbrace{V_c \cos \omega_c t}_{c(t)} + \underbrace{\frac{mV_c}{2}\left[\cos(\omega_c+\omega_m)t\right]}_{USB} + \underbrace{\frac{mV_c}{2}\left[\cos(\omega_c-\omega_m)t\right]}_{LSB}$$

and, we get the expression for DSB-SC signal

$$S(t)_{DSB\text{-}SC} = m(t) V_c \cos 2\pi f_c t = \underbrace{\frac{V_c V_m}{2} \cos 2\pi (f_c+f_m)t}_{USB} + \underbrace{\frac{V_c V_m}{2} \cos 2\pi (f_c-f_m)t}_{LSB}$$

Now the similarly, the expression for SSB-SC signal

$$S(t)_{SSB\text{-}SC} = \frac{V_c V_m}{2} \cos 2\pi (f_c \pm f_m) t$$

$$\left[\cos 2\pi (A \pm B) = \cos 2\pi (A)t \cdot \cos 2\pi (B)t \mp \sin 2\pi (A)t \cdot \sin 2\pi (B)t\right]$$

$$S(t)_{SSB\text{-}SC} = \frac{V_c V_m}{2} \cos 2\pi f_c t \cdot \cos 2\pi f_m t \mp \frac{V_c V_m}{2} \sin 2\pi f_c t \cdot \sin 2\pi f_m t$$

$$S(t)_{SSB\text{-}SC} = \frac{V_c}{2} m(t) \cos 2\pi f_c t \mp \frac{V_c}{2} \hat{m}(t) \sin 2\pi f_c t$$ ✓

↑ Hilbert transform

Hilbert Transform:-

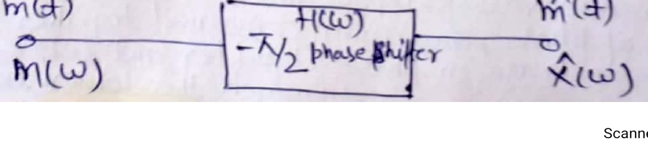
The function $\hat{m}(t)$ is a signal obtained by shifting phase of every component present in $m(t)$ by $(-\pi/2)$

This means that $\hat{m}(t)$ is the Hilbert transform of $x(t)$ i.e.

$$\hat{m}(t) = \frac{1}{\pi} m(t) \otimes \frac{1}{t}$$

$$\text{OR} \quad \hat{m}(t) = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{x(\tau)}{t-\tau} d\tau$$

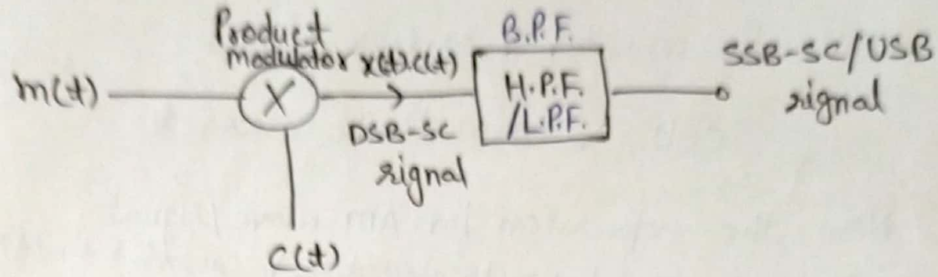
* It is also known as $-\pi/2$ phase shifter.



Generation of SSB-SC Signal:⇒

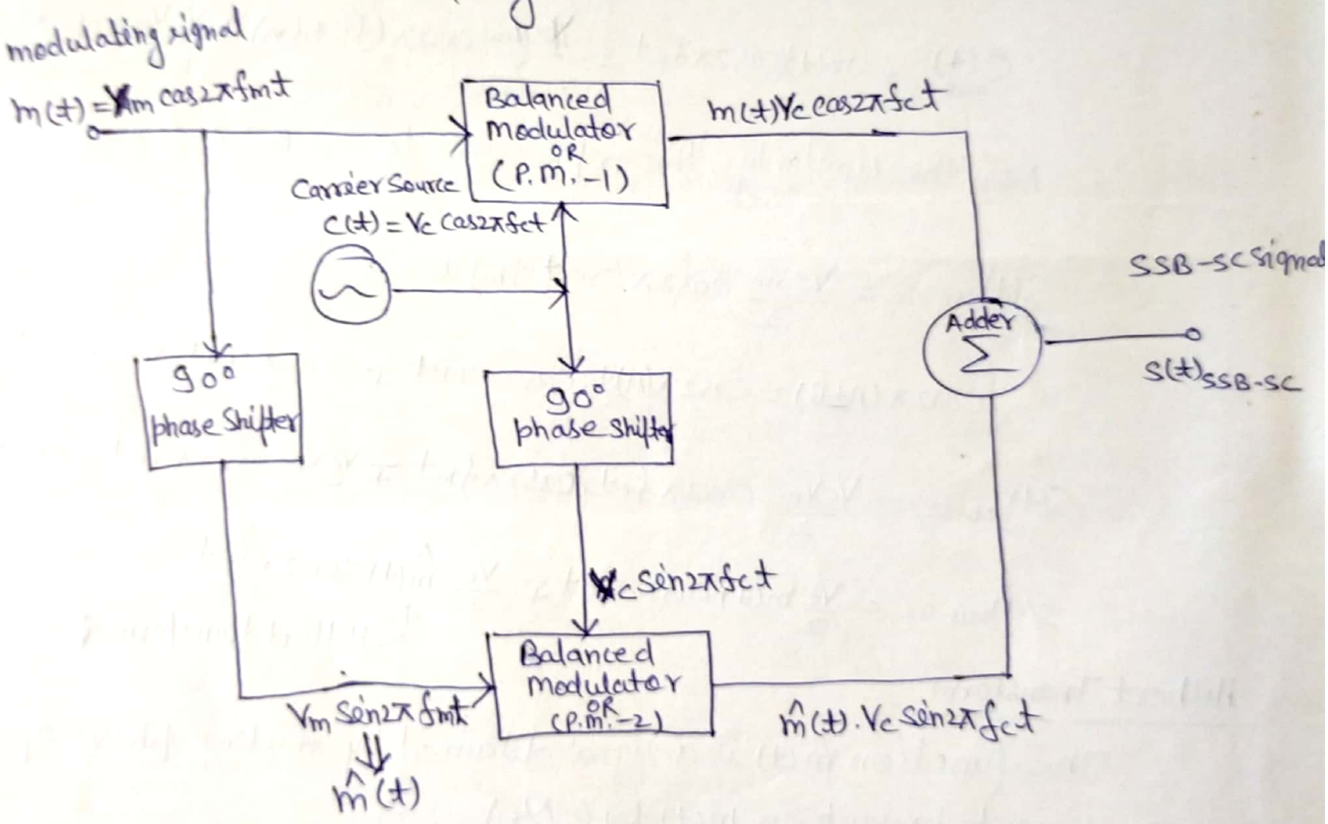
SSB-SC signals may be generated by two methods

(i) Frequency discrimination Method OR Filter Method



(ii) Phase Discrimination Method OR phase-shift method.

This method makes use of two balanced modulators and two phase-shifting networks.



$$S(t)_{SSB-SC} = m(t)V_c \cos 2\pi f_c t + \hat{m}(t) V_c \sin 2\pi f_c t$$

The modulators M_1, receives the carrier voltage shifted by 90° and the modulating voltage, whereas another balanced modulator M_2, receives the modulating voltage shifted by 90° and the carrier voltage.

Both modulators produces an output consisting only of sidebands. Both the upper sideband leads the input carrier voltage by 90°. one of the lower sideband leads the reference voltage by 90° and the other lags it by 90°. The two lower sidebands are thus out of phase, and when combined together in the adder, they cancel each other. The upper sidebands are in phase at adder and hence they add producing SSB in which the lower sideband has been cancelled.

Mathematical Analysis :⇒

Let the expression for carrier be $\sin \omega_c t$ and that for modulating signal $\sin \omega_m t$.

Now, the balanced modulator M_1 will receive $\sin \omega_m t$ and $\sin(\omega_c t + 90)$ whereas M_2 will receive $\sin(\omega_m t + 90^\circ)$ and $\sin \omega_c t$.

We know that the output of balance modulator M_1 will contain sum and difference frequencies.

Hence,

$$v_1 = \cos[(\omega_c t + 90) - \omega_m t] - \cos[(\omega_c t + 90^\circ) + \omega_m t]$$

$$\text{or, } v_1 = \underset{[LSB]}{\cos(\omega_c t - \omega_m t + 90^\circ)} - \underset{[USB]}{\cos(\omega_c t + \omega_m t + 90^\circ)}$$

Similarly, the output of balance modulator M_2, will contain

$$v_2 = \cos[\omega_c t - (\omega_m t + 90^\circ)] - \cos[\omega_c t + (\omega_m t + 90^\circ)$$

$$\text{or, } v_2 = \cos(\omega_c t - \omega_m t - 90^\circ) - \cos(\omega_c t + \omega_m t + 90^\circ)$$

Therefore, the output of the adder will be

$$v_0 = v_1 + v_2 = 2\cos(\omega_c t + \omega_m t + 90^\circ)$$

which is the expression for SSB-SC.