3.7 Design Specifications of a Control System

A second order control system is required to satisfy three main specifications, namely, peak overshoot to a step input (M_P), settling time (t_s) and steadystate accuracy. Peak overshoot is indicative of damping (δ) in the system and for a given damping settling time indicates the undamped natural frequency of the system. The steadystate accuracy is specified by the steadystate error and error can be made to lie within given limits by choosing an appropriate error constant K_p, K_V or K_a depending on the type of the system. If any other specifications like rise time or delay time are also specified, they must be specified consistent with the other specifications. Most control systems are designed to be underdamped with a damping factor lying between 0.3 and 0.7. Let us examine the limitations in choosing the parameters of a type one, second order system to satisfy all the design specifications.

The expressions for δ, t_s and e_{ss} are given by,

$$\delta = \frac{1}{2\sqrt{K_v \tau}} \qquad .....(3.69)$$

$$t_s = \frac{4}{\delta \omega_n} \qquad .....(3.70)$$

$$e_{ss} = \frac{1}{K_v} \qquad .....(3.71)$$

In a second order system, the only variables are K_v and τ. Even if both of them are variable, we can satisfy only two out of the three specifications namely, δ, t_s and e_{ss}. Generally, we are given a system for which a suitable controller has to be designed. This means that the system time constant is fixed and the only variable available is the system gain K_v. By using a proportional controller, the gain can be adjusted to suit the requirement of the steadystate accuracy. If K_v is adjusted for an allowable limit on steadystate error, this value of K_v is usually large enough to make the system damping considerably less, as given by eqn. (3.69). Thus the transient behaviours of the system is not satisfactory. Hence suitable compensation schemes must be designed so that the dynamic response improves. Some control schemes used in industry are discussed in the next section.

3.7.1 Proportional Derivative Error Control (PD control)

A general block diagram of a system with a controller is given in Fig. 3.22.

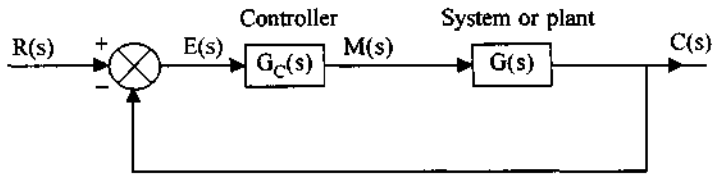

Fig. 3.22 General block diagram of a system with controller and unity feed back

For a second order, Type 1 system,

$$G(s) = \frac{K'_v}{s(\tau s+1)}$$

By choosing different configurations for the controller transfer function G_C (s) we get different control schemes. The input to the controller is termed as error signal or most appropriately, actuating signal. The output of the controller is called as the manipulating variable, m (t) and is the signal given as input to the system or plant. Thus, we have,

$$m(t) = K_P\left(e(t) + K_D\frac{de(t)}{dt}\right)$$

and $$M(s) = K_P\,(1 + K_D\,s)\,E\,(s)$$

The open loop transfer function with PD controller is given by,

$$G_0\,(s) = G_C\,(s)\,G\,(s)$$

$$= \frac{K_P(1+K_Ds)K'_v}{s(\tau s+1)}$$

The closed loop transfer function of the system is given by,

$$T(s) = \frac{\frac{K_P.K'_v}{\tau}(K_Ds+1)}{s^2+s\left(\frac{1+K_PK_DK'_v}{\tau}\right)+\frac{K_PK'_v}{\tau}}$$

If we define $$K_v = K_P\,K'_v,$$

$$T(s) = \frac{\frac{K_v}{\tau}(K_D\,s+1)}{s^2+s\left(\frac{1+K_vK_D}{\tau}\right)+\frac{K_v}{\tau}}$$

The damping and natural frequency of the system are given by,

$$\delta' = \frac{1+K_vK_D}{2\sqrt{K_v\tau}} = \delta + \frac{K_D}{2}\sqrt{\frac{K_v}{\tau}} \qquad .....(3.72)$$

$$\omega_n' = \sqrt{\frac{K_v}{\tau}} = \omega_n \qquad .....(3.73)$$

By a suitable choice of the proportional controller gain K_P (Amplifier gain), the steadystate error requirements can be met. As seen earlier, such a choice of K_P usually results in a low value of damping and hence this can be increased to a suitable value by of a proper choice of K_D, the gain of the derivative term in the controller, as given by the eqn. (3.72). It can be observed from eqn. (3.73) that the natural frequency is not altered for a given choice of K_P. Hence the settling time is automatically reduced since ω_n is fixed and δ for the compensated system has increased.

It may also be observed that, adding a derivative term in the controller introduces a zero in the forward path transfer function and we have seen that the effect of this is to increase the damping in the system.

3.7.2 Proportional Integral Controller (PI Control)

If the amplifier in the forward path is redesigned to include an integrator so that the output of the controller is given by,

$$m(t) = K_P\left(e(t) + K_1\int_0^t e(t)dt\right) \qquad .....(3.74)$$

or

$$M(s) = K_P\left(1 + \frac{K_1}{s}\right)E(s)$$

We have,

$$G_0(s) = G_C(s)\,G(s)$$

$$= \frac{K_P(s + K_1)K_v'}{s^2(\tau s + 1)} = \frac{K_v(T_1 s + 1)}{s^2(\tau s + 1)} \qquad .....(3.75)$$

Where $T_1 = \dfrac{1}{K_1}$

and $K_v = \dfrac{K_P K_v'}{K_1}$

and

$$T(s) = \frac{K_v(T_1 s + 1)}{\tau s^3 + s^2 + K_v T_1 s + K_v} \qquad .....(3.76)$$

From eqn. (3.75) we observe that the type of the system is changed from Type 1 to type 2 and hence the steadystate error for a unit velocity input is reduced to zero. Hence an integral controller is usually preferred wherever the steadystate accuracy is important. But the dynamics of the system can not be easily obtained, as the system order is increased from two to three, because of the introduction of integral control.

Moreover, the stability of the sytem (as will be discussed in chapter 4) may be affected adversely if the system order is increased. Since the system is of third order, it is usually designed to have two complex poles nearer to the imaginary axis and one real pole, as for away from origin as desirable. The response due to the complex poles dominate the overall response and hence the damping factor of these poles will have to be properly chosen to get a satisfactory transient response.

3.7.3 Proportional, Integral and Derivative Controller (PID Control)

An integral control eliminates steadystate error due to a velocity input, but its effect on dynamic response is difficult to predict as the system order increases to three. We have seen in sector 3.7.1 that a derivative term in the forward path improves the damping in the system. Hence a suitable combination of integral and derivative controls results in a proportional, integral and derivate control, usually called PID control. The transfer function of the PID controller is given by,

$$G_C(s) = K_P\left(1 + K_D s + \frac{K_I}{s}\right)$$

The overall forward path transfer function is given by,

$$G_o(s) = \frac{K_P K_v'\left(1 + K_D s + \frac{K_I}{s}\right)}{s(\tau s + 1)}$$

and the overall transfer function is given by,

$$T(s) = \frac{K_P K_v' \ (K_D s^2 + s + K_I)}{\tau s^3 + s^2(1 + K_p K_D) + sK_P K_v' \ s + K_P K_I}$$

Proper choice of K_P, K_D and K_I results in satisfactory transient and steadystate responses. The process of choosing proper K_P, K_D, at K_I for a given system is known as *tuning of a PID controller*.

3.7.4 Derivative Output Control

So far, we have discussed controllers in the forward path for which the input is the error. Some times control is provided by taking a signal proportional to the rate at which the output is changing and feeding back to the amplifier in the forward path. A typical block diagram fo such a system, employing rate feedback, as it is often known, is given in Fig. 3.23.

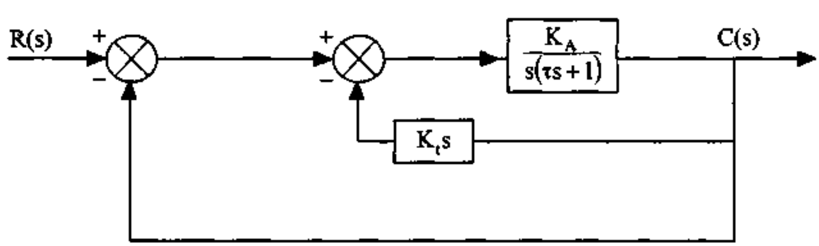

Fig. 3.23 (a) Block diagram of a system employing rate feedback

The inner loop provides the desired rate feedback as its output is proportional to $\frac{dc(t)}{dt}$. Simplifying the inner loop, we have,

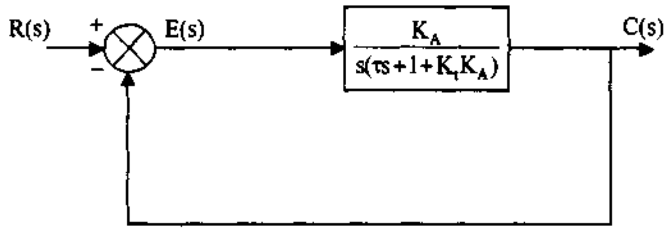

Fig. 3.23 (b) Simplified block diagram of Fig. 3.23 (a)

The forward path transfer function is given by,

$$G(s) = \frac{\frac{K_A}{1+K_t K_A}}{s\left(\frac{\tau}{1+K_t K_A}s+1\right)} = \frac{K'_v}{s(\tau' s+1)} \qquad .....(3.77)$$

Where,

$$K'_v = \frac{K_A}{1+K_t K_A} \qquad .....(3.78)$$

$$\tau' = \frac{\tau}{1+K_t K_A} \qquad .....(3.79)$$

Thus the new damping factor is given by,

$$\delta' = \frac{1}{2\sqrt{K_v'\tau'}} = \frac{1}{2\sqrt{\frac{K_A}{1+K_t K_A}\cdot\frac{\tau}{1+K_t K_A}}} \qquad .....(3.80)$$

$$= \frac{1+K_t K_A}{2\sqrt{K_A \tau}}$$

$$= (1 + K_t K_A)\,\delta \qquad .....(3.81)$$

$$\omega_n' = \sqrt{\frac{K_v'}{\tau'}} = \sqrt{\frac{K_A}{\tau}} = \omega_n \qquad .....(3.82)$$

The product, $K'_v\,\delta'$ is given by,

$$K'_v\,\delta' = \frac{K_A}{2\sqrt{K_A \tau}} = \frac{1}{2}\sqrt{\frac{K_A}{\tau}}$$

or

$$K_A = 4\,(K'_v\,\delta')^2\,\tau \qquad .....(3.83)$$

If the values of K'_v and δ' are specified, the amplifier gain (K_A) can be adjusted to get a suitable value using eqn. (3.82). For this vlaue of K_A, the rate feedback constant K_t is given by, using eqn. 3.78,

$$K_t = \left(\frac{1}{K'_v} - \frac{1}{K_A}\right) \qquad .....(3.84)$$

If rate feedback is not present,

$$K_v = K_A \qquad .....(3.85)$$

and
$$\delta = \frac{1}{2\sqrt{K_A \tau}}$$

With rate feedback, if same velocity error constant is specified, comparing eqn. (3.78) with eqn. (3.85), we see that the amplifier gain, K_A has to be more. Thus ω_n' given by eqn. (3.82) will be more. Hence the derivative output compensation increases both damping factor and natural frequency, thereby reducing the settilng time.

3.5 Example

Consider the position control system shown in Fig. 3.24 (a). Draw the block diagram of the system. The particulars of the system are the following.

Total Moment of Inertia referred to motor shaft, $J = 4 \times 10^{-3}$ kgm^2.

Total friction coefficient referred to motor shaft, $f = 2 \times 10^{-3}$ Nwm|rad|sec

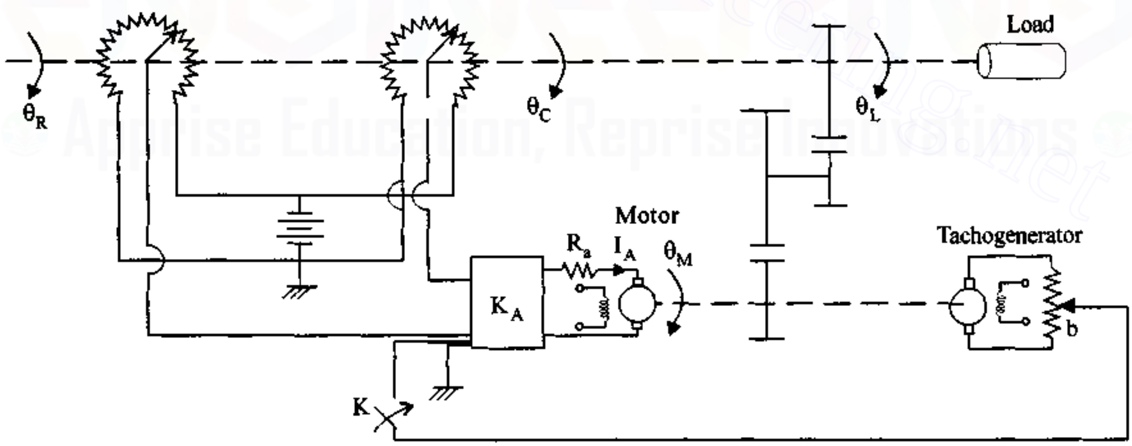

Fig. 3.24 (a) Schematic of a position control system.

Motor to load Gear ratio, $n = \dfrac{\theta_L}{\theta_M} = \dfrac{1}{50}$

Load to potentiometer gear ratio, $\dfrac{\dot{\theta}_L}{\dot{\theta}_C} = 1$

Motor torque constant,	$K_T = 2$ Nw-m | amp
Tachogenerator constant,	$K_t = 0.2$V | rad | sec
Sensitivity of error detector,	$K_P = 0.5$V | rad.
Amplifier gain,	K_A Amps/V. (variable)

(a) With switch K open, obtain the vlaue of K_A for a steadystate error of 0.02 for unit ramp input. Calculate the values of damping factor, natural frequency, peak overshoot and settling time.

(b) With switch K open, the amplifier is modified to include a derivative term, so that the armature current i_a (t) is given by

$$i_a(t) = K_A\left(e(t) + K_D\frac{de(t)}{dt}\right)$$

Find the vlaues of K_A and K_D to give a steadystate error within 0.02 for a unit ramp input and damping factor of 0.6. Find the natural frequency and settling time in this case.

(c) With switch K closed and with proportional control only, find the portion of tachogenerator voltage to be fedback, b, to get a peak overshort of 20%. Steadystate error should be less than 0.02 for a unit ramp input. Find the settling time and natural frequency.

Solution : The block diagram of the system is given in Fig. 3.24 (b).

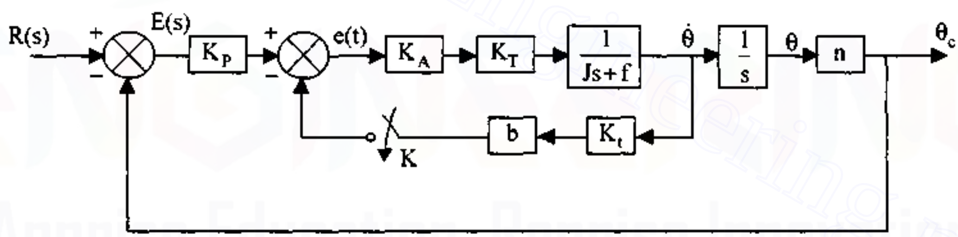

Fig. 3.24 (b) Block diagram of the position control system of Fig. 3.24 (a).

With switch K open,

$$G(s) = K_P K_A K_T \frac{n}{s(Js+f)}$$

$$= \frac{0.5\times 2K_A}{50}\cdot\frac{1}{s(4\times 10^{-3}s + 2\times 10^{-3})}$$

$$= \frac{K_A}{50\times 2\times 10^{-3}s\,(2s+1)} = \frac{10K_A}{s(2s+1)}$$

$$= \frac{K_v}{s(2s+1)}$$

Now $\qquad e_{ss} = 0.02$

$$e_{ss} = \frac{1}{K_v} = \frac{1}{10K_A}$$

$$K_A = \frac{1}{10 \times 0.02} = 5$$

$$K_v = 10\,K_A = 50$$

damping factor, $\qquad \delta = \dfrac{1}{2\sqrt{K_v \tau}} = \dfrac{1}{2\sqrt{50 \times 2}} = \dfrac{1}{20}\ 0.05$

Natural frequency, $\qquad \omega_n = \sqrt{\dfrac{K_v}{\tau}} = \sqrt{\dfrac{50}{2}}$

$$= 5.0 \text{ rad/sec.}$$

Peak overshoot, $\qquad M_P = e^{-\frac{\pi\delta}{\sqrt{1-\delta^2}}} = 85.45\%$

Settling time, $\qquad t_s = \dfrac{4}{\delta\omega_n} = \dfrac{4}{0.05 \times 5} \times 5 = 16 \text{ sec}$

Thus, it is seen that, using proportional control only (Adjusting the amplifier gain K_A) the steadystate error is satisfied, but the damping is poor, resulting in highly oscillatory system. The settling time is also very high.

(b) With the amplifier modified to include a derivative term,

$$i_a(t) = K_A \left[e(t) + K_D \frac{de(t)}{dt} \right]$$

The forward path transfer function becomes,

$$G(s) = \frac{K_P K_A (1 + K_D s) K_T n}{s(Js + f)} = \frac{0.5 \times K_A (1 + K_D s) 2}{2 \times 10^{-3} \times 50 s(2s+1)} = \frac{10 K_A (1 + K_D s)}{s(2s+1)}$$

To satisfy steadystate error requirements, K_A is again chosen as 5. The damping factor is given by,

$$\delta = \frac{1 + K_v K_D}{2\sqrt{K_v \tau}}$$

$$0.6 = \frac{1 + 50 K_D}{2\sqrt{50 \times 2}}$$

From which we get,

$$K_D = 0.22$$

$$\omega_n = \sqrt{\frac{K_v}{\tau}} = \sqrt{\frac{50}{2}} = 5 \text{ rad/sec}$$

$$t_s = \frac{4}{0.6 \times 5} = 1.333 \text{ sec}$$

The derivative control increases the damping and also reduces the setlling time. The natural frequency is unaltered.

(c) With switch K closed and with only proportional control, we have,

$$G(s) = \frac{K_P.n}{s}\left[\frac{K_A K_T}{Js + f + K_A K_T K_t b}\right]$$

$$= \frac{0.5}{50s}\left[\frac{2K_A}{4\times10^{-3}s + 2\times10^{-3} + 2\times0.2\times K_A b}\right]$$

The closed loop transfer function is given by,

$$T(s) = \frac{K_A}{0.2s^2 + (0.1 + 20K_A b)s + K_A}$$

The steadystate error, $e_{ss} = 0.02$

$$\therefore \qquad K_v = \frac{1}{0.02} = 50$$

The peak over shoot,

$$M_p = e^{-\frac{\pi\delta}{\sqrt{1-\delta^2}}} = 0.2$$

$$\therefore \qquad \delta = 0.456$$

From the expression for G (s), we have,

$$K_v = \underset{s \to 0}{\text{Lt}} \; s\,G(s)$$

$$K_v = \frac{K_A}{50(2\times10^{-3} + 0.4K_A b)} = \frac{K_A}{0.1 + 20K_A b}$$

From the expression for T (s), we have

$$\omega_n = \sqrt{\frac{K_A}{0.2}}$$

$$\delta = \frac{0.1 + 20K_A b}{0.2} \cdot \frac{1}{2\sqrt{\frac{K_A}{0.2}}}$$

Taking the product of K_v and δ, we have

$$K_v\delta = \frac{K_A}{0.1+20K_Ab} \times \frac{0.1+20K_Ab}{2\sqrt{0.2K_A}}$$

$$= \frac{K_A}{2\sqrt{0.2K_A}}$$

But $\qquad K_v = 50 \text{ and } \delta = 0.456$

$$(50 \times 0.456)^2 = \frac{K_A^2}{4\times 0.2K_A}$$

$\therefore \qquad K_A = 4 \times 0.2\,(50 \times 0.456)^2 = 415.872$

We notice that the value of K_A is much larger, compared to K_A in part (a). Substituting the value of K_A in the expression for K_v, we have,

$$50 = \frac{415.872}{0.1+20\times 415.872\times b}$$

b can be calculated as,

$$b = 0.001$$

The natural frequency $\omega_n = \sqrt{\dfrac{K_A}{0.2}} = \sqrt{\dfrac{415.872}{0.2}} = 45.6$ rad/sec

Comparing ω_n in part (a), we see that the natural frequency has increased. Thus the setlling time is redued to a value give by,

$$t_s = \frac{4}{\delta\omega_n} = \frac{4}{0.456\times 45.6} = 0.1924 \text{ sec.}$$

This problem clearly illustrates the effects of P, P D and derivative output controls.

Example 3.6

Consider the control system shown in Fig. 3.25.

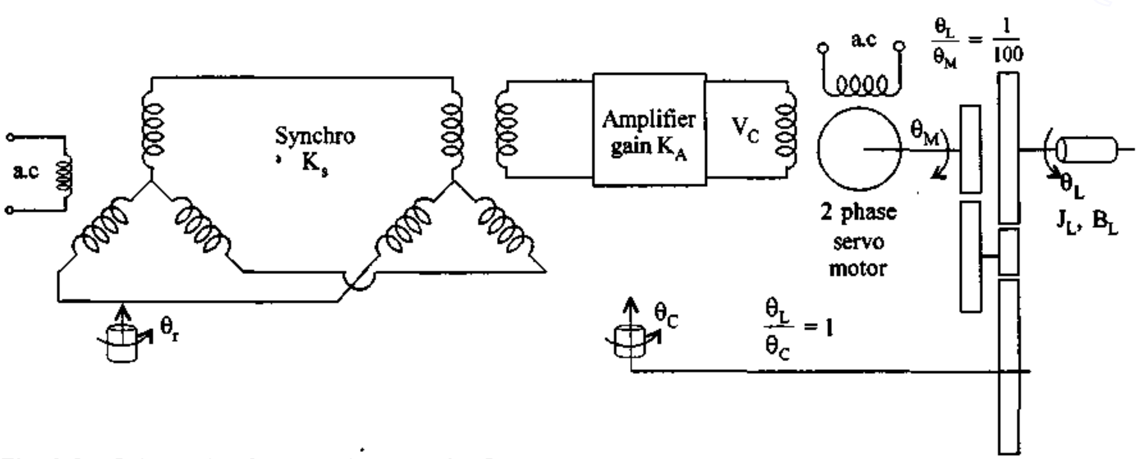

Fig. 3.25 Schematic of a control system for Ex. 3.6

The sensitivity of synchro error detector $K_S = 1$ V | deg. The transfer function of the two phase servo motor is given by,

$$\frac{\theta_M(s)}{V_C(s)} = \frac{10}{s(1+0.1s)}$$

(a) It is required that the load be driven at a constant speed of 25 rpm at steady state. What should be the gain of the amplifier, K_A, so that the error between output and input position does not exceed 2 deg under steadystate. For this gain what are the values of damping factor, natural frequency and settling time.

(b) To improve the transient behavior of the system, the amplifier is modified to include a derivative term, so that the output of the amplifier is given by,

$$v_C(t) = K_A\, e(t) + K_A T_d\, \dot{e}(t)$$

Determine the value of T_D so that the damping ratio is improved to 0.5. What is the settling time in this case.

Solution : The block diagram of the system is given in Fig. 3.26

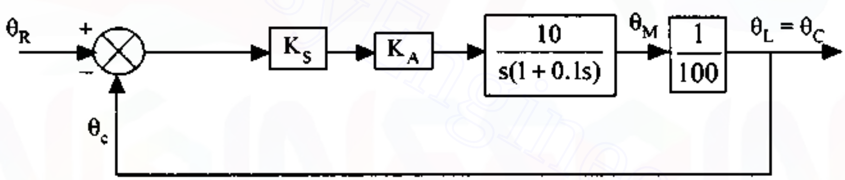

Fig. 3.26 The block diagram of system shown in Fig. 3.25

The forward path transfer function of the system is,

$$G(s) = \frac{K_S K_A}{10s(1+0.1s)}$$

$$K_S = 1 \text{ V/deg} = \frac{180}{\pi} \text{ V/rad}$$

$$\therefore \qquad G(s) = \frac{180 \times K_A}{10\pi s(1+0.1s)}$$

Steady state speed $= 25 \text{ rpm} = \dfrac{25 \times 2\pi}{60}$ rad | sec. $= \dfrac{5\pi}{6}$ rad | sec

Steady state error, $e_{SS} = 2 \text{ deg} = \dfrac{2}{180} \times \pi = \dfrac{\pi}{90}$ rad

For the given system,

$$K_v = \underset{s \to 0}{\text{Lt}} \; s\,G(s)$$

$$= \underset{s \to 0}{\text{Lt}} \frac{s \times 180 \times K_A}{\pi s(1+0.1s)10}$$

$$= \frac{18K_A}{\pi}$$

$$e_{ss} = \frac{5\pi}{6K_v} = \frac{5\pi \times \pi}{6 \times 18K_A}$$

But $$e_{ss} = \frac{\pi}{90}$$

$$\therefore \frac{5\pi^2}{108K_A} = \frac{\pi}{90}$$

From which, we get, $K_A = 13.1$

$$G_c(s) = \frac{75}{s(1+0.1s)}$$

$$T(s) = \frac{\theta_C(s)}{\theta_R(s)} = \frac{75}{0.1s^2 + s + 75}$$

$$= \frac{750}{s^2 + 10s + 750}$$

$$\omega_n = \sqrt{750} = 27.39$$

$$\delta = \frac{1}{2\sqrt{K_v \tau}} = \frac{1}{2\sqrt{75 \times 0.1}} = 0.1826$$

$$t_s = \frac{4}{\delta\omega_n} = \frac{4}{0.1826 \times 27.39} = 0.8 \text{ sec}$$

(b) When the amplifier is modified as,

$$v_C(t) = K_A e(t) + K_A T_D \dot{e}(t)$$

The open loop transfer function becomes,

$$G(s) = \frac{K_S K_A (1 + T_D s)}{10s(0.1s+1)}$$

$$T(s) = \frac{\theta_C(s)}{\theta_R(s)} = \frac{K_S K_A(1+T_D s)}{s^2+(10+K_S K_A T_D)s+K_S K_A}$$

$$= \frac{180\times 13.1(1+T_D s)}{s^2+\left(10+\frac{180}{\pi}\times 13.1T_D\right)s+\frac{180}{\pi}\times 13.1} \times \frac{1}{\pi}$$

$$= \frac{750(1+T_D s)}{s^2+(10+750T_D)s+750}$$

$$\omega_n = \sqrt{750} = 27.39$$

$$2\,\delta\,\omega_n = 10 + 750\,T_D$$

$$T_D = \frac{2\delta\omega_n - 10}{750} = \frac{2\times 0.5\times 27.39-10}{750} = 0.0232$$

$$t_s = \frac{4}{\delta\omega_n} = \frac{4}{0.5\times 27.39} = 0.292 \text{ see}$$

Thus, we see that, by including a derivative term in the amplifier, the transient perofrmance is improved. It may also be noted that K_v is not changed and hence the steady state error remains the same.

Problems

3.1 Draw the schematic of the position control system described below.

Two potentiometers are used as error detector with θ_R driving the reference shaft and the load shaft driving the secind potentiometer shaft. The error signal is amplified and drives a d c. Servomotor armature. Field current of the motor is kept constant. The motor drives the load through a gear.

Draw the block diagram of the system and obtain the closed loop transfer function. Find the natural frequency, damping factor, peak time, peak overshoot and settling time for a unit step input, when the amplifier gain K_A = 1500. The parameters of the system are as follows:

Potentiometer sensitivity	K_P = 1V/rad
Resistance of the armature	$R_a = 2\Omega$
Equivalent Moment of Inertia at motor shaft	$J = 5 \times 10^{-3}$ kg–m^2
Equivalent friction at the motor shaft	$B = 1 \times 10^{-3}$ NW/rad/sec
Motor torque constant	K_T = 1.5 Nω m/A
Gear ratio	$n = \frac{1}{10}$
Motor back e.m.f constant	K_b = 1.5 V/rad/sec

3.2 A position control system is shown in Fig. P 3.2

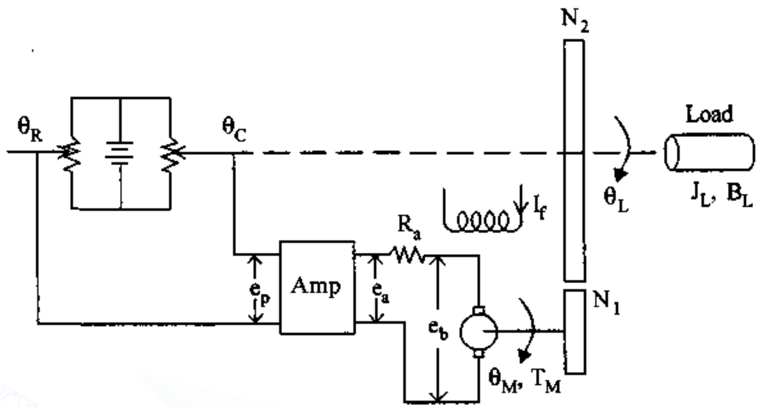

Fig. P 3.2

Sensitivity of the potentiometer error detection $K_P = \frac{24}{\pi}$ V/rad

Amplifier gain	K_A V/V
Armature resistance	$R_a = 0.2\ \Omega$
Motor back e.m.f constant	$K_b = 5.5 \times 10^{-2}$ V/rad/sec
Motor torque constant	$K_T = 6 \times 10^{-5}$ N-m/A
Moment of Inertia of motor referred to motor shaft	$J_m = 10^{-5}$ Kg m^2
Moment of Inertia of load referred to the output shaft	$J_L = 4.4 \times 10^{-3}$ Kgm^2
Friction coefficient of the load referred to the output shaft	$B_v = 4 \times 10^{-2}$ Nm/rad/sec

Gear ratio $\frac{N_1}{N_2} = \frac{1}{10}$

(i) If the amplifier gain is 10V/V obtain the transfer function of the system $\frac{C(s)}{R(s)} = \frac{\theta_L(s)}{\theta_R(s)}$.

Find the peak overshoot, peak time, and settling time of the system for a unit step input.

(ii) What values of K_A will improve the damping factor to 0.707

(iii) What value of K_A will give the frequency of oscillations of 9.23 rad/sec to a step input.

3.3 The open loop transfer function of a unity feed back control system is given by,

$$G(s) = \frac{K}{s(Ts+1)}$$

If the maximum response is obtained at t = 4 sec and the maximum value is 1.26, find the values if K and T.

3.4 A unity feedback system has the plant transfer function

$$G_1(s) = \frac{C(s)}{M(s)} = \frac{10}{s(s+2)}$$

A Proportional derivative control is employed to control the dynamics of the system. The controller characteristics are given by,

$$m(t) = e(t) + K_D \frac{de(t)}{dt}$$

where e(t) is the error.

Determine

(i) The damping factor and undamped natural frequency when $K_D = 0$

(ii) The value of K_D so that the damping factor is increased to 0.6.

3.5 Consider the system shown in Fig. P 3.5.

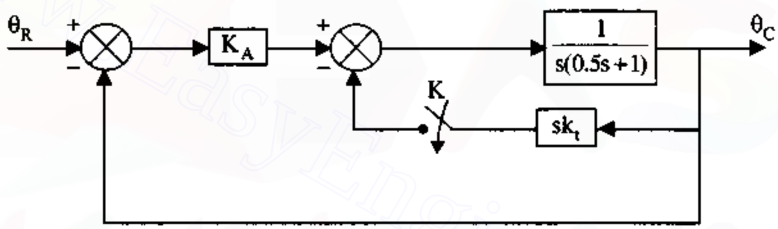

Fig. P 3.5

(i) With switch K open, determine the damping factor and the natural frequency of the system. If a unit ramp input is applied to the system, find the steady state output. Take $K_A = 5$

(ii) The damping factor is to be increases to 0.7 by including a derivative output compensation. Find the value of k_t to achieve this. Find the value of undamped natural frequency and the steady state error due to a unit ramp input.

(iii) It is possible to maintain the same steady state error for a unit ramp input as in part (i) by choosing proper values of K_A and k_t. Find these values.

3.6 In the system shown in Fig. P 3.6 find the values of *K* and *a* so that the peak overshoot for a step input is 25% and peak time is 2 sec.

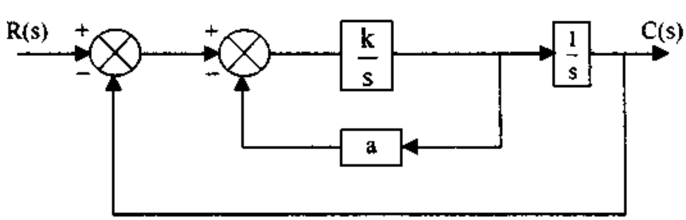

Fig. P 3.6

3.7 Determine the values of K and a such that the damping factor is 0.6 and a settling time of 1.67sec.

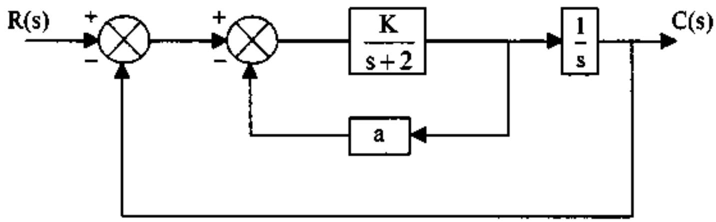

Fig. P 3.7

Also find the step response of the system.

3.8 Find the steadystate errors for unit step, unit velocity and unit acceleration inputs for the following systems.

(i) $\dfrac{15}{s(s+1)(s+5)}$ (ii) $\dfrac{10(0.1s+1)}{(0.02s+1)(0.2s+1)}$

(iii) $\dfrac{100}{s^2(0.1s+1)(.01s+1)}$ (iv) $\dfrac{(s+2)(s+5)}{s(0.2s+1)(0.6s+1)}$

3.9 In the system shown in Fig P 3.7 find the value of K and a such that the damping factor of the system is 0.6 and the steady state error due to a unit ramp input is 0.25.

3.10 For the unity feedback system with,

$$G(s) = \frac{10}{s+10}$$

Find the error series for the input,

$$v(t) = 1 + 2t + \frac{3t^2}{2}$$

3.11 Find the steadystate error as a function of time for the unity feedback system,

$$G(s) = \frac{100}{s(1+0.1s)}$$

for the following inputs.

(a) $r(t) = \dfrac{t^2}{2}\, u(t)$ (b) $r(t) = 1 + 2t + \dfrac{t^2}{2}$.