Uniquely decodable & Instantaneous Code:

Uniquely Decodable Code (UDC):

A distinct code is uniquely decodable if the original source sequence can be reconstructed perfectly from the encoded binary sequence.

Prefix-free condition is sufficient condition to ensure that a code is uniquely decodable but it is not a necessary condition.

eg: ① A→0
B→1
C→10
D→11

Let us consider

100 → It can be created as,

$\underbrace{100}_{CA}$ & $\underbrace{100}_{BAA}$

Since it is created in two ways, so it is not a uniquely decodable code.

② A→0 , B→10 , C→110 , D→111

Let us consider,

100 → It can be created as only

$\underbrace{100}_{BA}$

It can be created in a unique way, so it is UDC.

2) Instantaneous Code:

A necessary and sufficient condition for a code to be instantaneous is that none of the codeword in the code should be a prefix of another code.

Examples of instantaneous Code →

Let us consider a source 'S' consisting of five Symbols $S = \{S_1, S_2, S_3, S_4, S_5\}$, so the instaneous code,

$S_1 \to 0$	$S_1 \to 00$
$S_2 \to 10$	$S_2 \to 01$
$S_3 \to 110$	$S_3 \to 10$
$S_4 \to 1110$	$S_4 \to 110$
$S_5 \to 1111$	$S_5 \to 111$

$S_5 \rightarrow 1111$

* **Examples of Kraft Inequality:**

Q) Consider a DMS 'X' with symbols x_i with $i = 1, 2, 3, 4$. Below table links four possible binary codes.

x_i	Code A	Code B	Code C	Code D
x_1	00	0	0	0
x_2	01	10	11	100
x_3	10	11	100	110
x_4	11	110	110	111

Show that all the codes except code B satisfies the Kraft Inequality.

Sol$^n \rightarrow$

For Code A $\rightarrow$ $n_1 = n_2 = n_3 = n_4 = 2$

$$K = \sum_{i=1}^{4} 2^{-n_i}$$

$$= \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4}$$

$\boxed{K = 1} \leq 1$ (So Kraft Inequality satisfies)

For Code B $\Rightarrow$

$n_1 = 1$, $n_2 = n_3 = 2$, $n_4 = 3$

$$K = \sum_{i=1}^{4} 2^{-n_i} = \frac{1}{2} + \frac{1}{4} + \frac{1}{4} + \frac{1}{8} = \frac{9}{8} \approx 1.1$$

Since, $K > 1$ (So Kraft Inequality doesn't satisfies here).

For Code C

$n_1 = 1$, $n_2 = 2$, $n_3 = n_4 = 3$

$$K = \sum_{i=1}^{4} 2^{-n_i} = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{8}$$

$\boxed{K = 1 \leq 1}$ (So Kraft Inequality satisfies here).

For Code D

$$n_1 = 1, \; n_2 = n_3 = n_4 = 3$$

$$K = \sum_{i=1}^{4} 2^{-n_i} = \frac{1}{2} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8}$$

$$\boxed{K = \frac{7}{8} \leq 1}$$ (so Kraft Inequality satisfies here)

Entropy Coding

The design of a variable length ~~of~~ Code Such that its average codeword length approaches the Entropy of the DMS is often referred to as Entropy Coding.

It is divided in two parts →

1) Shannon fano Coding.
2) Huffman Coding.