Unrestricted Variables: These can take positive, negative or zero value.

* If it occurs in LPP, then it cannot be solved by simplex method, because in simplex method all variables are non-negative.
* So if say x is unrestricted we write $x = x' - x''$ where $x' \geq 0$ & $x'' \geq 0$.
* If $x' > x''$ then $x > 0$

  $x' < x''$ then $x < 0$

  $x' = x''$ then $x = 0$.

Eg solve the LPP $\max z = 2x_1 + x_2$

subject to $x_1 + 2x_2 \leq 1$

$2x_1 - x_2 \leq 7$

$x_1 \geq 0$, x_2 unrestricted.

Let $x_2 = x_2' - x_2''$, $x_2', x_2'' \geq 0$

Standard form of LPP

$$\max z = 2x_1 + 2(x_2' - x_2'') + 0s_1 + 0s_2$$

subject to

$$x_1 + 2(x_2' - x_2'') + s_1 = 1 \Rightarrow x_1 + 2x_2' - 2x_2'' + s_1 = 1$$

$$2x_1 - (x_2' - x_2'') + s_2 = 7 \Rightarrow 2x_1 - x_2' + x_2'' + s_2 = 7$$

$$x_1, x_2', x_2'', s_1, s_2 \geq 0$$

Now all variables are non-negative so we can apply simplex method.

C_B	BV	X_B	x_1	x_2'	x_2''	s_1	s_2	min ratio
		c_j	2	1	-1	0	0	
0	s_1	1	$\boxed{1}$	2	-2	1	0	1 →
0	s_2	7	2	-1	1	0	1	7/2
		$c_j - z_j$	2 ↑	1	-1	0	0	

C_B	BV	X_B	x_1	x_2'	x_2''	s_1	s_2	min ratio
		c_j	2	1	-1	0	0	
2	x_1	1	1	2	-2	1	0	—
0	s_1	5	0	-5	$\boxed{5}$	-2	1	1 →
		$c_j - z_j$	0	-3	3 ↑	-2	0	

C_B	BV	X_B	x_1	x_2'	x_2''	s_1	s_2	min ratio
		c_j	2	1	-1	0	0	
2	x_1	3	1	0	0	-1/5	1	
-1	x_2''	1	0	-1	1	-2/5	1/5	
		$c_j - z_j$	0	0	0	0	-9/5	

$\because$ All $c_j - z_j \le 0$, therefore optimality is reached

Optimal soln is

$$x_1 = 3 \quad x_2'' = 1 \quad x_2' = 0$$

$$\therefore x_2 = 0 - 1 = -1$$

or, $\boxed{x_1 = 3 \quad x_2 = -1 \quad z_{max} = 5}$

Ex max $z = \cancel{8x_1}\ \cancel{x_1}$

subject to $\cancel{x_1 + 4x_2 \le 1}$

$\cancel{2x_1 + 8x_2 \ge -6}$

max $z = x + 4y$

subject to

$y - x \leq 4$

$y + 2x \leq -5$

x unristicted, $y \geq 0$

max $z = x + \cancel{4y'} - 4y''$

s.t.

$\cancel{y' - y'' - x + s_1 \geq 4}$

$\cancel{y' - y'' + 2x - s_2 + a_1 = -5}$

r

max $z = x' - x'' + 4y - Ma_1 + 0s_1 + 0s_2$

s.t. $y - (x' - x'') + s_1 = 4$

$-y + 2(x' - x'') - s_1 + a_1 = 5$

$x', x'', y \geq 0$

C_B	BV	c_j / x_B	x' (1)	x'' (-1)	y (4)	s_1 (0)	s_2 (0)	a_1 (-M)	min ratio
0	s_1	4	-1	1	$\boxed{1}$	1	0	0	4 →
-M	a_1	5	-2	2	-1	0	-1	1	-
		$c_j - z_j$	1	-1	4 ↑	0	0	0	
4	y	4	-1	1	1	1	0	0	4
-M	a_1	9	-3	$\boxed{3}$	0	1	-1	1	3 →
		$c_j - z_j$	5-3M	-5+3M ↑	0	M-4	-M	0	
4	y	1	0	0	1	2/3	1/3		
-1	x''	3	-1	1	0	1/3	-1/3		
		$c_j - z_j$	0	0	0	-3	-1		

∵ All $c_j - z_j \leq 0$ ∴ optimal soln.

$x' = 0 \quad x'' = 3 \quad y = 1 \quad \therefore x = x' - x'' = -3$

∴ $\boxed{x = -3 \quad y = 1 \quad z_{max} = 1}$ - Ans