B.Tech - 1st Year, Branch - MEE

Course Code : PHY-S101 (Theory + Lab)

Course Name : Physics - I

Credit : 03 (For Theory)

Credit : 02 (For Lab)

Grading pattern of PHY-S101 Course

- Quiz : 15 Marks
- Class performance + Attendance : 05 Marks
- Mid Semester Exam : 20 Marks
- End Semester Exam : 40 Marks
- Lab : 20 Marks

Total : 100 Marks

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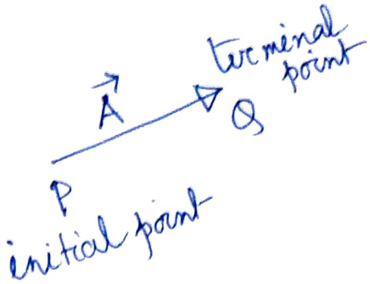
Department : Physics, UIET.

Vectors and Scalars



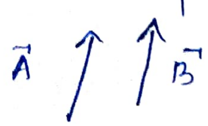
Vector : A vector is a quantity having both magnitude and direction such as force, velocity, acceleration etc.

Geometrically it is represented by an arrow PQ where P is called the initial point and Q is called the terminal point. Analytically it is represented by a letter $\vec{A}$ with an arrow over it. The magnitude of this vector is defined as $|\vec{A}|$.

Scalar : A scalar is a quantity having magnitude but no direction eg, mass, temp, time etc.

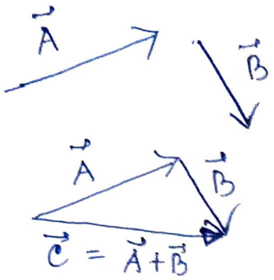
Vector Algebra : It deals with the operations of addition, subtraction and multiplication of vectors

1. Two vectors $\vec{A}$ and $\vec{B}$ are said to be equal if they have the same magnitude and direction regardless of the position of their initial points. Thus $\vec{A} = \vec{B}$



2. Two vectors which have the same magnitude but opposite in direction are denoted by $\vec{A}$ and $-\vec{A}$.

3. Addition of two vectors



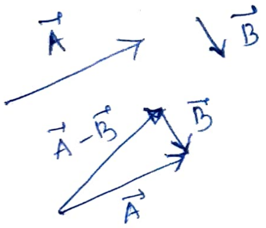
The sum or resultant of two vectors $\vec{A}$ and $\vec{B}$ is a vector $\vec{C}$ which is obtained by placing the initial point of $\vec{B}$ on the terminal point of $\vec{A}$ and then joining the initial point of $\vec{A}$ to the terminal point of $\vec{B}$.

This sum is written as $\vec{A}+\vec{B}=\vec{C}$.

We can also find out the resultant vector of two vectors by using the parallelogram law for vector addition.

Extensions to sums of more than two vectors are immediate.

4. Subtraction of two vectors



The difference of two vectors $\vec{A}$ and $\vec{B}$ is represented by $\vec{A}-\vec{B}=\vec{C}$. Equivalently, $\vec{A}-\vec{B}$ can be defined as the sum $\vec{A}+(-\vec{B})$.

To construct $\vec{A}-\vec{B}$, place the terminal pt. of $\vec{B}$ at the terminal point of $\vec{A}$ then join the initial point of $\vec{A}$ to the initial point of $\vec{B}$. Then $\vec{A}-\vec{B}$ extends from the initial point of $\vec{A}$ to the initial point of $\vec{B}$.

If $\vec{A}=\vec{B}$ then $\vec{A}-\vec{B}$ is defined as the null vector.

5. Multiplication of a vector by a scalar

The product of a vector $\vec{A}$ by a scalar m is a vector $m\vec{A}$ with magnitude $|m|$ times $|\vec{A}|$. The direction of this vector $m\vec{A}$ is same as or opposite to that of $\vec{A}$ depending upon m is +ve or −ve.

If $m = 0$ then the vector $m\vec{A}$ is a null vector.

Laws of vector algebra

If $\vec{A}$, $\vec{B}$ and $\vec{C}$ are vectors and m and n are scalars, then

1. $\vec{A}+\vec{B} = \vec{B}+\vec{A}$ ⟶ Commutative law for addition.
2. $\vec{A}+(\vec{B}+\vec{C}) = (\vec{A}+\vec{B})+\vec{C}$ ⟶ Associative law for addition
3. $m\vec{A} = \vec{A}m$ ⟶ Commutative law for multiplication
4. $m(n\vec{A}) = (mn)\vec{A}$ ⟶ Associative law for multiplication
5. $(m+n)\vec{A} = m\vec{A}+n\vec{A}$ ⟶ Distributive law
6. $m(\vec{A}+\vec{B}) = m\vec{A}+m\vec{B}$ ⟶ Distributive law

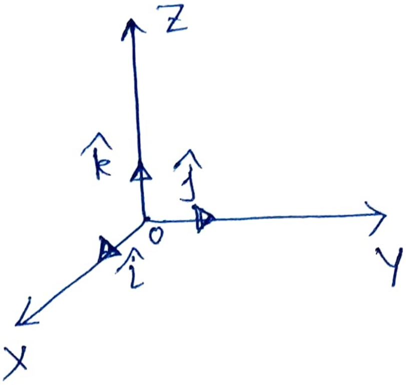
A unit vector

A unit vector is a vector having unit magnitude.
If $\vec{A}$ is a vector with magnitude $|\vec{A}| \neq 0$, then

$\dfrac{\vec{A}}{|\vec{A}|}$ is a unit vector having the same direction as $\vec{A}$.

A unit vector in the direction of vector $\vec{A}$ is denoted by $\hat{A} = \dfrac{\vec{A}}{|\vec{A}|}$ $\therefore\ \vec{A} = |\vec{A}|\,\hat{A}$.

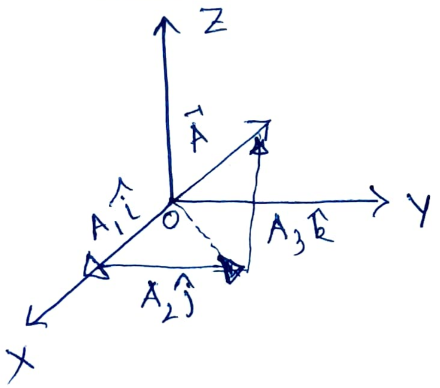
The Rectangular unit vectors



The rectangular unit vectors $\hat{i}, \hat{j}, \hat{k}$ are mutually perpendicular unit vectors having directions of the positive X, Y and Z axes respectively of a rectangular coordinate system.

Such a system derives its name from the fact that a right threaded screw rotated through 90° from OX to OY will advance in positive Z-direction as shown in the figure.

Components of a vector



Any vector $\vec{A}$ in 3-dimensions can be represented with initial point at the origin O of a rectangular coordinate system.

Let A_1, A_2, A_3 be the components of the vector $\vec{A}$ along the X, Y and Z directions respectively, then we can write

$$\vec{A} = A_1\hat{i} + A_2\hat{j} + A_3\hat{k}.$$

The magnitude of $\vec{A}$ is $|\vec{A}| = \sqrt{A_1^2 + A_2^2 + A_3^2}$

In particular the position vector $\vec{r}$ from the origin O to the point P(x, y, z) is written as $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ with $|\vec{r}| = \sqrt{x^2+y^2+z^2}$.

Multiplying a vector by a vector

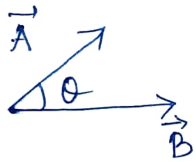
There are two ways to multiply a vector by a vector. One way produces a scalar called the scalar product or dot product and the other produces a new vector called the vector product or cross product.

- The scalar product or dot product of two vectors

The scalar product or dot product of two vectors is defined as

$$\vec{A}\cdot\vec{B} = |\vec{A}||\vec{B}|\cos\theta \Rightarrow \text{a scalar quantity}$$

where θ is the angle they form when placed tail to tail.



Note that the dot product is commutative i.e,

$$\vec{A}\cdot\vec{B} = \vec{B}\cdot\vec{A}$$

and distributive,

$$\vec{A}\cdot(\vec{B}+\vec{C}) = \vec{A}\cdot\vec{B} + \vec{A}\cdot\vec{C}$$

Geometrically, $\vec{A}\cdot\vec{B}$ is the product of magnitude of $|\vec{A}|$ times the projection of $\vec{B}$ along $\vec{A}$ (or the product of $|\vec{B}|$ times the projection of $\vec{A}$ along $\vec{B}$).

If two vectors are parallel, then $\vec{A}\cdot\vec{B} = |\vec{A}||\vec{B}|$

If $\vec{A}$ and $\vec{B}$ are perpendicular then $\vec{A}\cdot\vec{B} = 0$.

The following laws are valid for dot product

1. $\vec{A}\cdot\vec{B} = \vec{B}\cdot\vec{A} \longrightarrow$ Commutative law
2. $(\vec{A}+\vec{B})\cdot\vec{C} = \vec{A}\cdot\vec{C} + \vec{B}\cdot\vec{C} \longrightarrow$ Distributive law
3. $m(\vec{A}\cdot\vec{B}) = (m\vec{A})\cdot\vec{B} = \vec{A}\cdot(m\vec{B}) = (\vec{A}\cdot\vec{B})m$, m is a scalar
4. $\hat{i}\cdot\hat{i} = \hat{j}\cdot\hat{j} = \hat{k}\cdot\hat{k} = 1$, $\hat{i}\cdot\hat{j} = \hat{j}\cdot\hat{k} = \hat{k}\cdot\hat{i} = 0$
5. If $\vec{A} = A_1\hat{i} + A_2\hat{j} + A_3\hat{k}$ and $\vec{B} = B_1\hat{i} + B_2\hat{j} + B_3\hat{k}$

$$\therefore \vec{A}\cdot\vec{B} = A_1B_1 + A_2B_2 + A_3B_3$$

The vector product or cross product of two vectors

The cross product of two vectors is defined by

$$\vec{A}\times\vec{B} = |\vec{A}|\,|\vec{B}|\sin\theta\;\hat{n}\quad,\qquad 0\leq\theta\leq\pi$$

where $\hat{n}$ is a unit vector in a direction perpendicular to the plane containing the vector $\vec{A}$ and $\vec{B}$. Of course there are two directions perpendicular to any plane: 'in' and 'out'. The ambiguity is resolved by the right-hand rule: place your fingers along the direction of $\vec{A}$ in such a way that your fingers would ~~out~~ sweep from $\vec{A}$ to $\vec{B}$ through the smaller angle between them. Then your outstretched thumb points the direction of $\vec{C}=\vec{A}\times\vec{B}$. Note that $\vec{A}\times\vec{B}$ is itself a vector.

The cross product is distributive,

$$\vec{A}\times(\vec{B}\times\vec{C}) = (\vec{A}\times\vec{B})+(\vec{A}\times\vec{C})$$

but not commutative. In fact

$$(\vec{A}\times\vec{B}) = -(\vec{B}\times\vec{A})$$

The following laws are valid for cross product

1. $\vec{A}\times\vec{B} = -(\vec{B}\times\vec{A})$
2. $\vec{A}\times(\vec{B}+\vec{C}) = (\vec{A}\times\vec{B})+(\vec{A}\times\vec{C})$
3. $m(\vec{A}\times\vec{B}) = (m\vec{A})\times\vec{B} = \vec{A}\times(m\vec{B}) = (\vec{A}\times\vec{B})m$, m is a scalar
4. $\hat{i}\times\hat{i} = \hat{j}\times\hat{j} = \hat{k}\times\hat{k} = 0$, $\hat{i}\times\hat{j} = \hat{k}$, $\hat{j}\times\hat{k} = \hat{i}$, $\hat{k}\times\hat{i} = \hat{j}$
5. If $\vec{A} = A_1\hat{i}+A_2\hat{j}+A_3\hat{k}$ and $\vec{B} = B_1\hat{i}+B_2\hat{j}+B_3\hat{k}$

   then $$\vec{A}\times\vec{B} = \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ A_1 & A_2 & A_3\\ B_1 & B_2 & B_3\end{vmatrix}$$

6. The $|\vec{A}\times\vec{B}| = |\vec{A}|\,|\vec{B}|\sin\theta$ is same as the area of a parallelogram with sides $\vec{A}$ and $\vec{B}$.

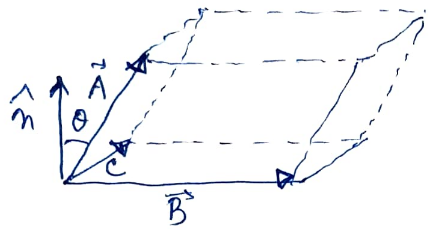
Area of the parallelogram $= h|\vec{B}| = |\vec{A}|\sin\theta\,|\vec{B}| = |\vec{A}\times\vec{B}|$, h = height

Triple products

Dot and cross multiplication of three vectors $\vec{A}, \vec{B}$ and $\vec{C}$ may produce meaningful products of the form $(\vec{A}\cdot\vec{B})\vec{C}$, $\vec{A}\cdot(\vec{B}\times\vec{C})$ and $\vec{A}\times(\vec{B}\times\vec{C})$.

The following laws are valid

1. $(\vec{A}\cdot\vec{B})\vec{C} \neq \vec{A}(\vec{B}\cdot\vec{C})$
2. $\vec{A}\cdot(\vec{B}\times\vec{C}) = \vec{B}\cdot(\vec{C}\times\vec{A}) = \vec{C}\cdot(\vec{A}\times\vec{B})$



Geometrically $|\vec{A}\cdot(\vec{B}\times\vec{C})|$ is the volume of the parallelepiped (shown in the figure) by $\vec{A}$, $\vec{B}$ and $\vec{C}$ since $|\vec{B}\times\vec{C}|$ is the area of the base and $|\vec{A}\cos\theta|$ is the altitude. $|\vec{A}\cos\theta| = |\vec{A}\cdot\hat{n}| \Rightarrow$ height from the base.

3. $\vec{A}\times(\vec{B}\times\vec{C}) \neq (\vec{A}\times\vec{B})\times\vec{C}$
4. i) $\vec{A}\times(\vec{B}\times\vec{C}) = (\vec{A}\cdot\vec{C})\vec{B} - (\vec{A}\cdot\vec{B})\vec{C} = \vec{B}(\vec{A}\cdot\vec{C}) - \vec{C}(\vec{A}\cdot\vec{B})$
   $\Downarrow$
   $\overline{B}\overline{A}\overline{C} - \overline{C}\overline{A}\overline{B}$ rule

   ii) $(\vec{A}\times\vec{B})\times\vec{C} = (\vec{A}\cdot\vec{C})\vec{B} - (\vec{B}\cdot\vec{C})\vec{A}$

The product $\vec{A}\cdot(\vec{B}\times\vec{C})$ is sometimes called the <u>scalar triple product</u> and the product $\vec{A}\times(\vec{B}\times\vec{C})$ is called <u>vector triple product</u>.

How vectors transform?

The definition of a vector as "a quantity with magnitude and direction" is not altogether satisfactory. What precisely does 'direction' mean?

We know that in 3-dimensions a vector has three components. Suppose we consider the cartesian coordinate system X-Y-Z with base vectors $\hat{i}, \hat{j}, \hat{k}$ and origin O.

In this case the vector can be written as

$\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$, where A_x, A_y, A_z are the components of the vector along three coordinate axes X, Y and Z.

Now a second coordinate system (X′-Y′-Z′) is introduced with same origin and base vectors $\hat{i}', \hat{j}'$ and $\hat{k}'$. In this second coordinate system the vector can be written as

$\vec{A} = A_{x'}\hat{i}' + A_{y'}\hat{j}' + A_{z'}\hat{k}'$.

The relationship between the components in one coordinate system and the components in a second coordinate system are called the <u>transformation equations</u>.

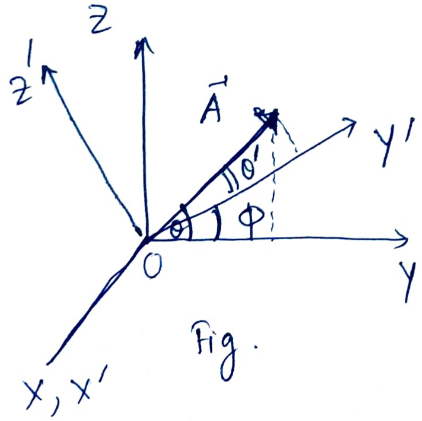


Fig.

Suppose that the second coordinate system X′, Y′, Z′ is obtained from the first one X, Y, Z by a rotation through an angle φ relative to X, Y, Z about the common X = X′ axes (as shown in the figure)

In the figure the vector $\vec{A}$ makes an angle θ with the Y-axis of the first coordinate system. Then we get

$A_y = A\cos\theta$ and $A_z = A\sin\theta$

while

$A_{y'} = A\cos\theta' = A\cos(\theta-\varphi) = A(\cos\theta\cos\varphi + \sin\theta\sin\varphi)$ $\left[\text{since } \theta' = \theta - \varphi\right]$

$\therefore A_{y'} = A_y\cos\varphi + A_z\sin\varphi$

and
$$A_{z'} = A \sin\theta' = A \sin(\theta - \phi) = A(\sin\theta \cos\phi - \cos\theta \sin\phi)$$
$$\therefore A_{z'} = -A_y \sin\phi + A_z \cos\phi.$$

and $A_x = A_{x'}$ since we have given the rotation about $x = x'$ axes.

In matrix notation we can write

$$\begin{pmatrix} A_{x'} \\ A_{y'} \\ A_{z'} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & \sin\phi \\ 0 & -\sin\phi & \cos\phi \end{pmatrix} \begin{pmatrix} A_x \\ A_y \\ A_z \end{pmatrix}$$

More generally, for rotation about an arbitrary axis in three dimensions, the transformation law takes the form

$$\begin{pmatrix} A_{x'} \\ A_{y'} \\ A_{z'} \end{pmatrix} = \begin{pmatrix} R_{xx} & R_{xy} & R_{xz} \\ R_{yx} & R_{yy} & R_{yz} \\ R_{zx} & R_{zy} & R_{zz} \end{pmatrix} \begin{pmatrix} A_x \\ A_y \\ A_z \end{pmatrix}$$

or more compactly,

$$A_i' = \sum_{j=1}^{3} R_{ij} A_j$$

where the index 1 stands for x, 2 for y and 3 for z.