Discrete memoryless channels →

A discrete memoryless channel is a statistical model with an input 'X' and output 'Y' shown in below fig.

During each unit of the time, the channel accepts an input symbol from 'X' and in response it generates an output symbol from 'Y'.

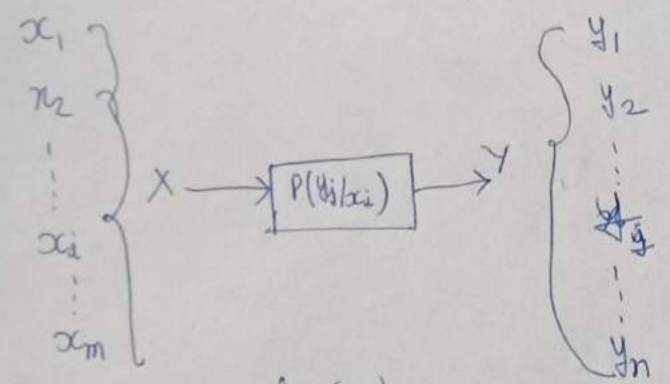


fig (a)

Above fig shows a DMC with 'm' inputs & 'n' outputs.

where $X = \{x_1, x_2 --- x_m\}$

$Y = \{y_1, y_2 --- y_n\}$

$P(y_j/x_i)$ → is the conditional probability of obtaining output y_j given that the input is x_i.

Channel Matrix →

A channel is completely specified by the complete set of transition probabilities. Accordingly the channel of fig(a) is often specified by the matrix of transition probabilities $[P(Y/X)]$ is given by

$$[P(Y/X)] = \begin{bmatrix} P(y_1/x_1) & P(y_2/x_1) & \cdots & P(y_n/x_1) \\ P(y_1/x_2) & P(y_2/x_2) & \cdots & P(y_n/x_2) \\ \cdots & \cdots & \cdots & \cdots \\ P(y_1/x_m) & P(y_2/x_m) & \cdots & P(y_n/x_m) \end{bmatrix}$$

The matrix $[P(Y/X)]$ is called the channel matrix.

Binary communication channel →

Below fig shows the diagram of a binary communication channel.

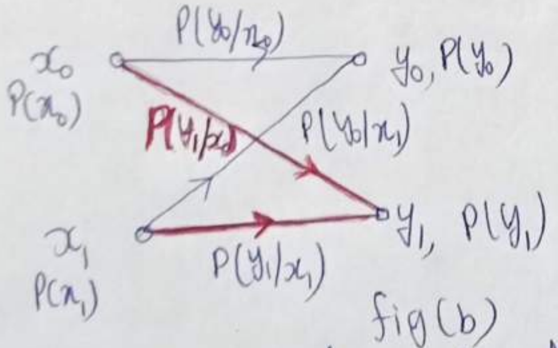


fig (b)

we can write the equations for probabilites of y_0 & y_1 as

$$P(y_0) = P(y_0/x_0)\,P(x_0) + P(x_1)\,P(y_0/x_1) \rightarrow (1)$$

$$P(y_1) = P(x_1)\,P(y_1/x_1) + P(y_1/x_0)\,P(x_0) \rightarrow (2)$$

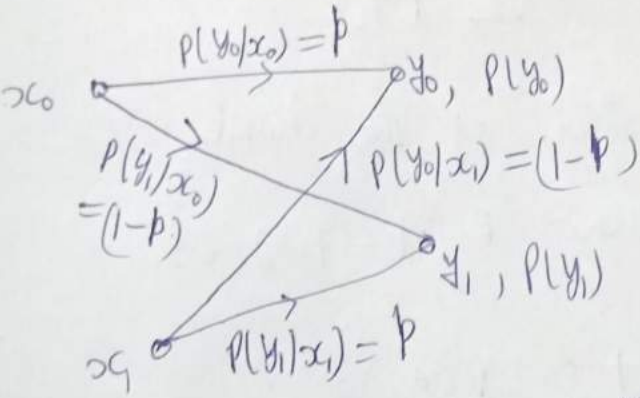
These eqn can be written in the matrix form as →

$$\begin{bmatrix} P(y_0) \\ P(y_1) \end{bmatrix} = [P(x_0) \ P(x_1)] \begin{bmatrix} P(y_0/x_0) & P(y_1/x_0) \\ P(y_0/x_1) & P(y_1/x_1) \end{bmatrix}$$

$$[P(Y/X)] = \begin{bmatrix} P(y_0/x_0) & P(y_1/x_0) \\ P(y_0/x_1) & P(y_1/x_1) \end{bmatrix}$$

Binary symmetric matrix →

The binary communication channel of fig (b) is said to be symmetric if $P(y_0/x_0) = P(y_1/x_1) = P$, such channel is shown in below fig(c).



$$P(y_0) = P(y_0/x_0)P(x_0) + P(y_0/x_1)P(x_1)$$

$$P(y_1) = P(x_1)\,P(y_1/x_1) + P(x_0)\,P(y_1/x_0)$$

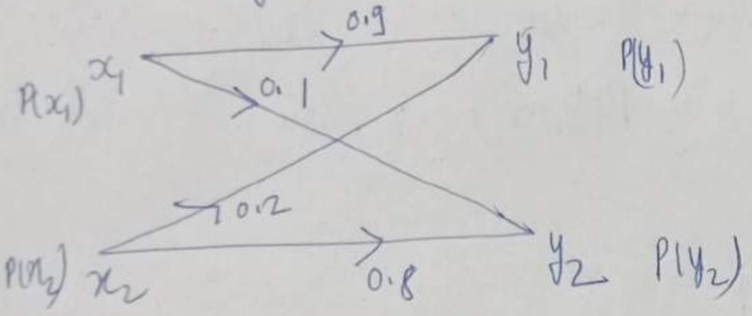
$$P(y_0) = p \cdot P(x_0) + P(x_1)\,(1-p)$$

$$P(y_1) = P(x_1)\,p + P(x_0)\,(1-p)$$

$$\begin{bmatrix} P(y_0) \\ P(y_1) \end{bmatrix} = \begin{bmatrix} P(x_0) & P(x_1) \end{bmatrix} \begin{bmatrix} p & (1-p) \\ (1-p) & p \end{bmatrix}$$

$$[P(Y/X)] = \begin{bmatrix} p & (1-p) \\ (1-p) & p \end{bmatrix}$$

ex → Given a binary channel shown in below fig.



(i) Find the channel matrix of the channel.

(ii) Find $P(y_1)$ and $P(y_2)$ when $P(x_1) = P(x_2) = 0.5$

Soln →

$$P(y_1) = 0.9\,P(x_1) + 0.2\,P(x_2)$$

$$P(y_2) = 0.1\,P(x_1) + 0.8\,P(x_2)$$

$$\begin{bmatrix} P(y_1) \\ P(y_2) \end{bmatrix} = \begin{bmatrix} 0.9 & 0.2 \\ 0.1 & 0.8 \end{bmatrix} \begin{bmatrix} P(x_1) & P(x_2) \end{bmatrix}$$

So $\left[P(Y/X)\right] = \begin{bmatrix} 0.9 & 0.1 \\ 0.2 & 0.8 \end{bmatrix}$

Ans

(iii) When $P(x_1) = P(x_2) = 0.5$ then

$$\begin{bmatrix} P(y_1) \\ P(y_2) \end{bmatrix} = \begin{bmatrix} 0.5 & 0.5 \end{bmatrix} \begin{bmatrix} 0.9 & 0.1 \\ 0.2 & 0.8 \end{bmatrix}$$

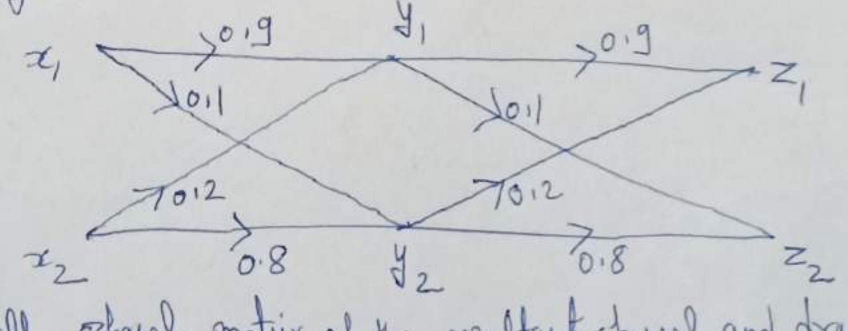
$$P(Y) = \begin{bmatrix} 0.55 \\ 0.45 \end{bmatrix}$$

$P(y_1) = 0.55$

$P(y_2) = 0.45$

Ans

ex → Two binary channels are connected in cascade as shown in below fig →



(i) Find the overall chanel matrix of the resultant channel and draw the resultant equivalent channel diagram (ii) Find $P(z_1)$ and $P(z_2)$ when $P(x_1) = P(x_2) = 0.5$