RISK AND RETURN

Two Sides of the Investment Coin

No Investor is obliged to take any risk on his or her investment.

It appeals to common sense that one will invest in a risky venture only if one expects to earn a return in excess of the risk free return.

RETURN

In the world of security analysis the term return is associated with a percentage (for example, ROI of 15 %) and not a mere amount (like, profit of Rs.100).

Exactly what is return?

Is it the **return on assets**?

Is it the **return on capital employed**?

Is it a **return on equity**?

Is it a **Internal rate of return** or **accounting rate of return**?

Solution: We are primarily concerned with returns from the investor's perspective i.e. productivity of a financial asset.

SINGLE PERIOD RETURN

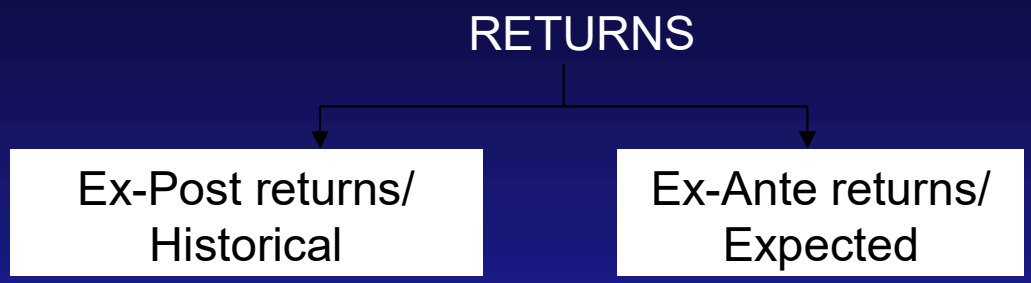
PERIOD	P_0	P_1	P_2	P_3	P_4	P_5	P_6	P_7	P_8	P_9
PRICES	100	110	108	130	154	145	160	170	193	220

Solution: $1/n \sum r_i = r_1 + r_2 + r_3 + \ldots\ldots\ldots + r_n / n$

MULTI PERIOD RETURN

Lets consider returns when more than a single period in under consideration.

PERIOD	P_0	P_1	P_2
PRICE	100	200	110



HISTORICAL (EX-POST) RETURNS

The return on a stock is calculated by including annual gains. If an investor receives Rs.2 per share in dividends and earns Rs.3 per share per year in capital gains and has an average investment of Rs. 25, the return on stock would be 25%.

	ABC Ltd.	XYZ Ltd.
Price as on 31.03.02	20	10
Price as on 31.03.03	15	15
Dividends for the year 2002-03	1	1

HOLDING PERIOD RETURN = $(P_i - P_0) + D_I / P_0$

Assume a bond with cash flows of Rs.100 each year and a principal payment of Rs.1000 in five years and a current price of Rs.960. What is:
Its current yield?
Its yield to maturity?

Year	Annual Returns(%)
1	16.98
2	-11.36
3	7.64
4	21.12
5	-22.14
6	-30.01
7	37.68
8	28.27
9	1.76
10	13.98

HISTORICAL RETURN ?

EXPECTED (EX ANTE) RETURNS

CONDITIONAL RETURNS (%)	PROBABILITY (P_I)
-24	.05
-10	.15
0	.15
12	.20
18	.20
22	.15
30	.10

EXPECTED RETURN OF A PORTFOLIO OF SECURITIES

$$E_p = W_1E_1 + W_2E_2 + \ldots\ldots\ldots + W_nE_n$$

OR

$$E_p = \Sigma W_iE_i$$

STOCK	PRICE AS ON 1.4.02	PRICE AS ON 31.3.04	ANNUAL DIVIDEND
X	20	30	2
Y	30	40	3
Z	50	60	5

HOLDING PERIOD RETURN

Assume a bond with cash flows of Rs.100 each year and a principal payment of Rs.1000 in five years and a current price of Rs.960. What is:

Its current yield?

Its yield to maturity?

CAPITAL ASSSET PRICING MODEL

$$r_{it} = \alpha_i + \beta_i r_{mt} + e_{it}$$

Given $\sigma_{i2} = 100$ and $cov_{im} = 110$, calculate β_{im}.

Classify the stock as defensive or aggressive.

If $\rho_{im} = .8$, what is the standard deviation of the stock i?

RETURN

- Return is the primary motivating force that drives investment.
- The return of an investment consists of two components:
  - Current return
  - Capital return

RISK

- Risk refers to the possibility that the actual outcome of an investment will deviate from its expected outcome.
- The three major sources of risk are : business risk, interest rate risk, and market risk.
- Modern portfolio theory looks at risk from a different perspective. It divides total risk as follows.

$$\text{Total risk} = \text{Unique risk} + \text{Market risk}$$

MEASURING HISTORICAL RETURN

- **TOTAL RETURN**

$$R = \frac{C + (P_E - P_B)}{P_B}$$

- **RETURN RELATIVE**

$$\text{RETURN RELATIVE} = \frac{C + P_E}{P_B}$$

- **CUMULATIVE WEALTH INDEX**

$$CWI_n = WI_0 \quad (1+R_1)\,(1+R_2)\,\ldots\,(1+R_n)$$

- **ARITHMETIC RETURN**

$$\bar{R} = \frac{\sum_{t=1}^{n} R_i}{n}$$

- **GEOMETRIC RETURN**

$$GM = [\,(1+R_1)\,(1+R_2)\,\ldots\,(1+R_n)\,]^{1/n} - 1$$

$$\left(1+\text{GEOMETRIC MEAN}\right)^2 \simeq \left(1+\text{ARITHMETIC MEAN}\right)^2 - \left(\text{STANDARD DEVIATION}\right)^2$$

THE CHOICE BETWEEN A.M. AND G.M.

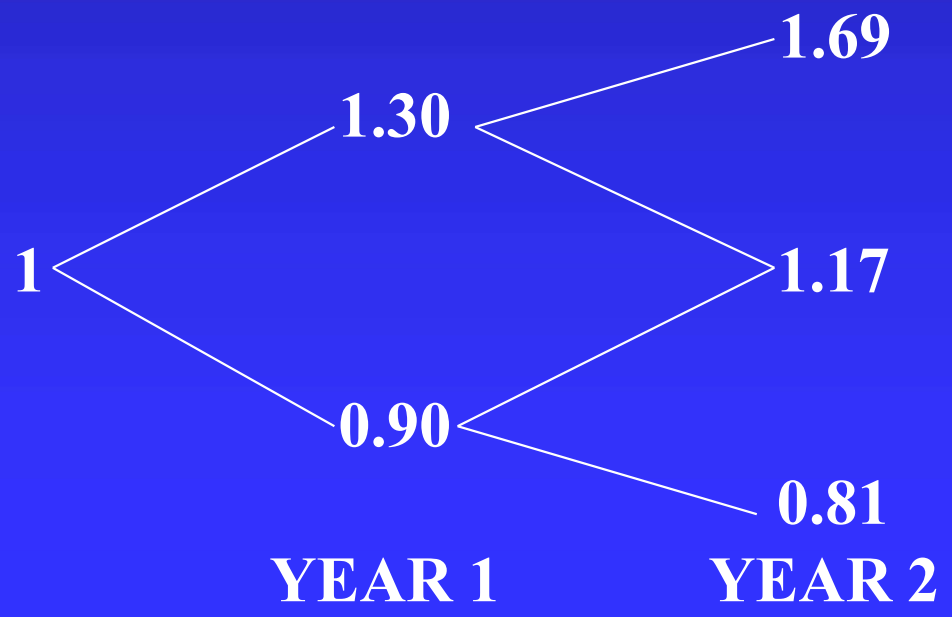
- A.M … MORE APPROPRIATE MEASURE OF AVERAGE PERFORMANCE OVER SINGLE PERIOD
- G.M IS A BETTER MEASURE OF GROWTH IN WEALTH OVER TIME

$$\text{REAL RETURN} = \frac{1 + \text{NOMINAL RETURN}}{1 + \text{INFLATION RATE}} - 1$$

ARITHMETIC VS. GEOMETRIC MEAN

- **CAPM … ADDITIVE MODEL**

  ∴ CAPM EXPECTED EQUITY RISK PREMIUM MUST BE DERIVED BY ARITHMETIC, NOT GEOMETRIC, SUBTRACTION
- **THE A.M … RATE OF RETURN, WHICH, WHEN COMPOUNDED OVER MULTIPLE PERIODS GIVES THE MEAN OF THE PROB. DISTR'NS OF ENDING WEALTH**



$$A.M = 0.5 \times 30\% - 0.5 \times 10\% = 10\%$$
$$G.M = [(1.30)(0.90)]^{1/2} = 8.2\%$$

THE *EXPECTED VALUE*, OR PROBABILITY - WEIGHTED AVERAGE OF ALL POSSIBLE OUTCOMES IS EQUAL TO:

$$(0.25) \times 1.69 + (0.5) \times 1.17 + (0.25) \times 0.81 = 1.21$$

NOW THE RATE THAT MUST BE COMPOUNDED UP TO ACHIEVE A TERMINAL WEALTH OF 1.21 AFTER 2 YEARS IS 10% (THE A.M … NOT .. GM)

- IN THE INVESTMENT MARKETS, WHERE RETURNS ARE DESCRIBED BY A PROB. DISTR'N, THE A.M. IS THE MEASURE THAT ACCOUNTS FOR UNCERTAINTY, AND IS THE APPROPRIATE ONE FOR ESTIMATING DISCOUNT RATES AND COST OF CAPITAL

MEASURING HISTORICAL RISK

$$\sigma = \left(\frac{\sum_{t=1}^{n} (R_i - \bar{R})^2}{n-1} \right)^{1/2}$$

PERIOD	RETURN R_i	DEVIATAION $(R_i - \bar{R})$	SQUARE OF DEVIATION $(R_i - \bar{R})^2$
1	15	5	25
2	12	2	4
3	20	10	100
4	-10	-20	400
5	14	4	16
6	9	-1	1
	$\Sigma R_i = 60$		$\Sigma (R_i - \bar{R})^2 = 546$

$\bar{R} = 10$

$$\sigma^2 = \left(\frac{\Sigma (R_i - \bar{R})^2}{n-1} \right) = 107.2$$

$\sigma = [107.2]^{1/2} = 10.4$

CRITIQUE & DEFENCE OF VARIANCE (AND S.D.)

CRITIQUE

1. VARIANCE CONSIDERS ALL DEVIATIONS, NEGATIVE AS WELL AS POSITIVE
2. WHEN THE PROBABILITY DISTRIBUTION IS NOT SYMMETRICAL AROUND ITS EXPECTED VALUE, VARIANCE ALONE DOES NOT SUFFICE. IN ADDITION, THE SKEWNESS OF THE DISTRIBUTION SHOULD BE CONSIDERED.

DEFENCE

1. IF A VARIABLE IS NORMALLY DISTRIBUTED … μ AND σ CAPTURE ALL INFORMATION
2. IF UTILITY OF MONEY … QUADRATIC FUNCTION … EXPECTED UTILITY .. $f(\mu, \sigma)$
3. STANDARD DEVIATION ANALYTICALLY MORE EASILY TRACTABLE.

RISK AND RETURN OF FINANCIAL ASSETS IN THE U.S. OVER 75 YEARS (1926-2000)

In general, investors are risk-averse. Hence risky investments must offer higher expected returns than less risky investments

Portfolio	*Average Annual Rate of Return (%)*	*Standard Deviation (%)*
Treasury bills	3.9	3.2
Government bonds	5.7	9.4
Corporate bonds	6.0	8.7
Common stocks (S&P 500)	13.0	20.2
Small-firm common stock	17.3	33.4

RISK PREMIUMS

- EQUITY RISK PREMIUM
- BOND HORIZON PREMIUM
- BOND DEFAULT PREMIUM

MEASURING EXPECTED (EX ANTE) RETURN AND RISK

EXPECTED RATE OF RETURN

$$E(R) = \sum_{i=1}^{n} p_i R_i$$

STANDARD DEVIATION OF RETURN

$$\sigma = [\Sigma p_i (R_i - E(R))^2]$$

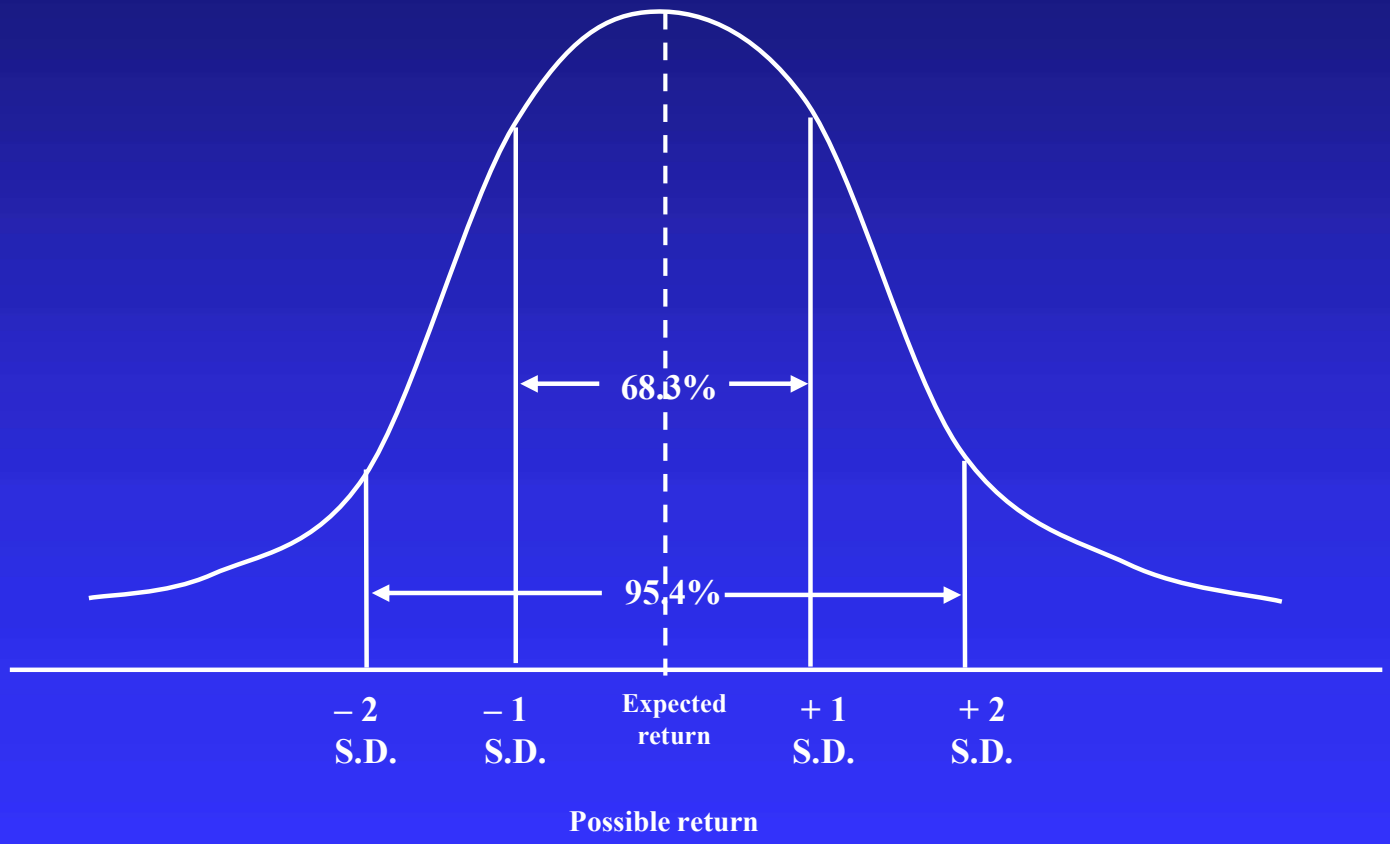
Bharat Foods Stock

i. State of the Economy	p_i	R_i	$p_i R_i$	$R_i - E(R)$	$(R_i - E(R))^2$	$p_i (R_i - E(R))^2$
1. Boom	0.30	16	4.8	4.5	20.25	6.075
2. Normal	0.50	11	5.5	-0.5	0.25	0.125
3. Recession	0.20	6	1.2	-5.5	30.25	6.050

$E(R) = \Sigma p_i R_i = 11.5$ $\quad\quad$ $\Sigma p_i (R_i - E(R))^2 = 12.25$

$$\sigma = [\Sigma p_i (R_i - E(R))^2]^{1/2} = (12.25)^{1/2} = 3.5\%$$

NORMAL DISTRIBUTION



SUMMING UP

- For earning returns investors have to almost invariably bear some risk. While investors like returns they abhor risk. Investment decisions therefore involve a tradeoff between risk and return.
- The total return on an investment for a given period is :

$$R = \frac{C + (P_E - P_B)}{P_B}$$

- The return relative is defined as:

$$\text{Return relative} = \frac{C + P_E}{P_B}$$

- The cumulative wealth index captures the cumulative effect of total returns. It is calculated as follows:

$$CWI_n = WI_0 (1 + R_1) (1+ R_2) \ldots (1+ R_n)$$

- The arithmetic mean of a series of returns is defined as:

$$\bar{R} = \frac{\sum_{i=1}^{n} R_i}{n}$$

- The geometric mean of a series of returns is defined as:

$$GM = [1+ R_1) (1+ R_2)\ldots.(1+ R_n)]^{1/n} - 1$$

- The arithmetic mean is a more appropriate measure of average performance over a single period. The geometric mean is a better measure of growth in wealth over time
- The real return is defined as:

$$\frac{\text{1+ Nominal return}}{\text{1+ Inflation rate}} - 1$$

- The most commonly used measures of risk in finance are variance or its square root the standard deviation. The standard deviation of a historical series of returns is calculated as follows:

$$\sigma = \left(\frac{\sum_{t=1}^{n} (R_i - \overline{R})^2}{n - 1} \right)^{1/2}$$

- Risk premium may be defined as the additional return investors expect to get for assuming additional risk. There are three well known risk premiums: equity risk premium, bond horizon premium, and bond default premium.
- The expected rate of return on a stock is:

$$E(R) = \sum_{i=1}^{n} p_i R_i$$

- The standard deviation of return is:

$$\sigma^2 = (\Sigma p_i (R_i - E(R)^2) ^{1/2}$$