CAPITAL ASSET PRICING AND ARBITRAGE PRICING THEORY

The Risk Reward Relationship

KEY ISSUES

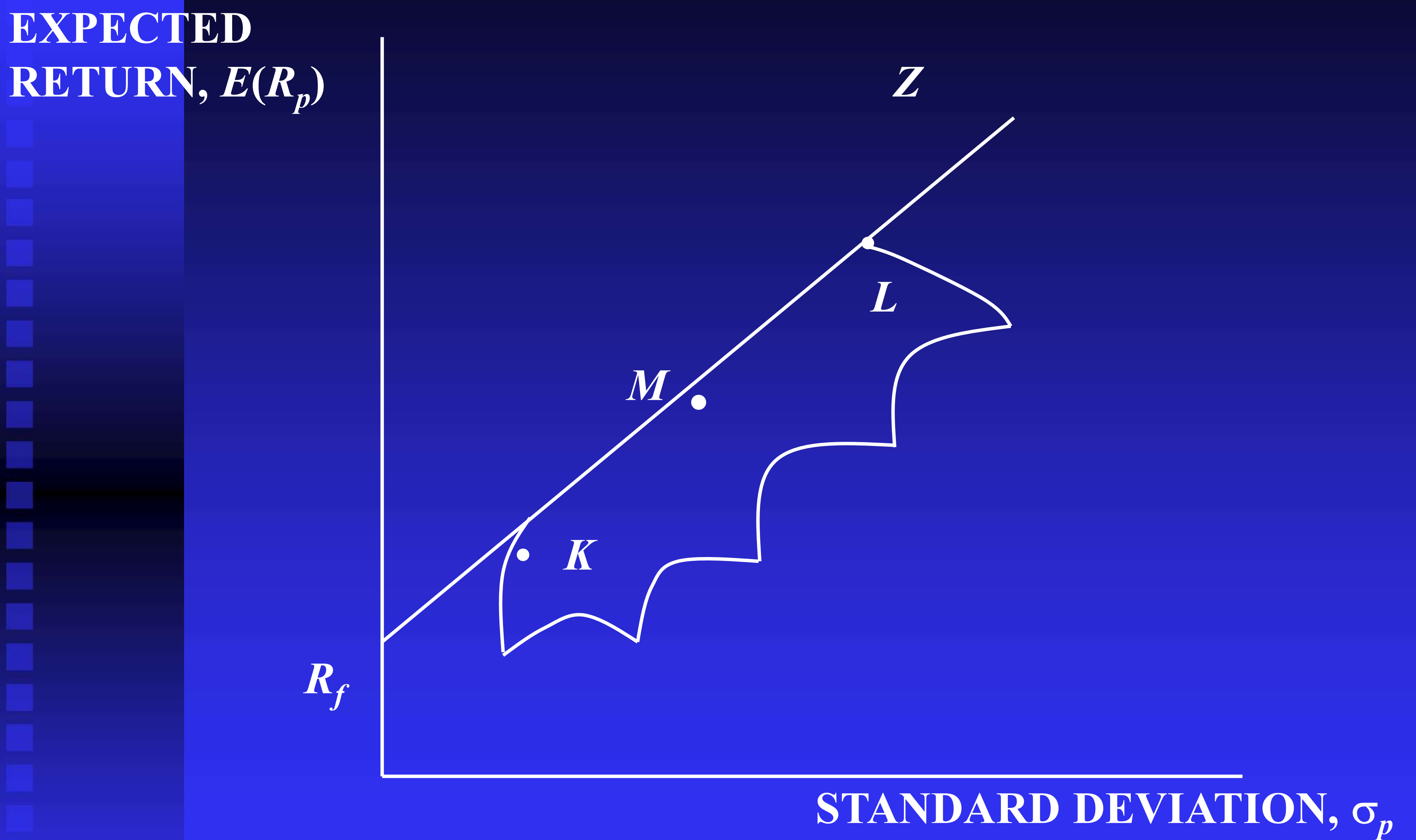
Essentially, the capital asset pricing model (CAPM) is concerned with two questions:

- **What is the relationship between risk and return for an efficient portfolio?**
- **What is the relationship between risk and return for an individual security?**

BASIC ASSUMPTIONS

- RISK - AVERSION
- MAXIMISATION . . EXPECTED UTILITY
- HOMOGENEOUS EXPECTATION
- PERFECT MARKETS

CAPITAL MARKET LINE



$$E(R_j) = R_f + \lambda \sigma_j$$

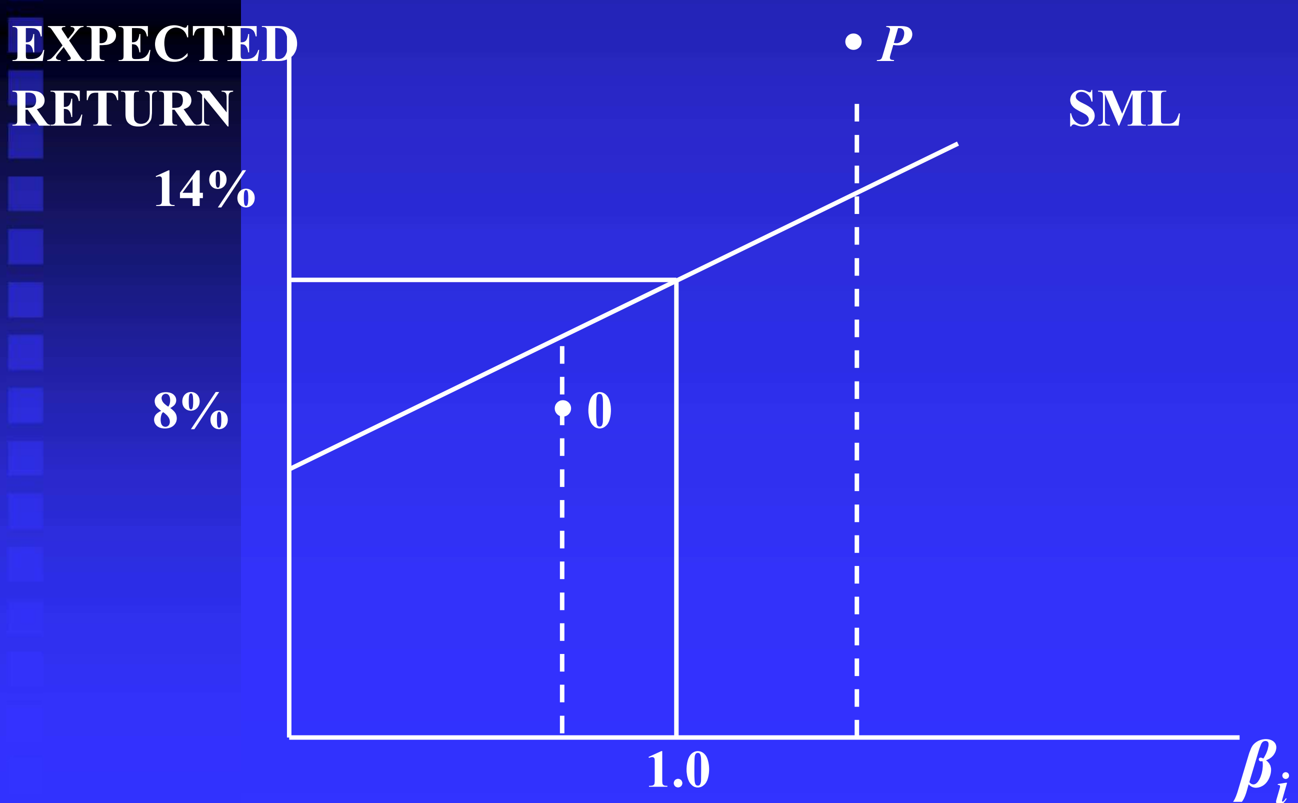
$$\lambda = \frac{E(R_M) - R_f}{\sigma_M}$$

SECURITY MARKET LINE

$$E(R_i) = R_f + \left(\frac{E(R_M) - R_f}{\sigma_M^2} \right) C_{iM}$$

$$\beta_i = \frac{\sigma_{iM}}{\sigma_M^2}$$

$$E(R_i) = R_f + [E(R_M) - R_f]\beta_i$$



ALPHA = EXPECTED RETURN - FAIR RETURN

RELATIONSHIP BETWEEN SML AND CML

SML

$$E(R_i) = R_f + \left(\frac{E(R_M) - R_f}{\sigma_M^{\,2}} \right) \sigma_{iM}$$

SINCE $\sigma_{iM} = \rho_{iM}\ \sigma_i\ \sigma_M$

$$E(R_i) = R_f + \left(\frac{E(R_M) - R_f}{\sigma_M} \right) \rho_{iM}\ \sigma_i$$

IF *i* AND *M* ARE PERFECTLY CORRELATED $\rho_{iM} = 1$. SO

$$E(R_i) = R_f + \left(\frac{E(R_M) - R_f}{\sigma_M} \right) \sigma_i$$

THUS

CML IS A SPECIAL CASE OF SML

INPUTS REQUIRED FOR APPLYING CAPM

RISK-FREE RETURN

- RATE ON A SHORT-TERM GOVT SECURITY
- RATE ON A LONG TERM GOVT BOND

MARKET RISK PREMIUM

- HISTORICAL
- DIFFERENCE BETWEEN THE AVERAGE RETURN ON STOCKS AND THE AVERAGE RISK - FREE RETURN

PERIOD : AS LONG AS POSSIBLE

AVERAGE : A.M VS. G.M.

DETERMINANTS OF RISK PREMIUM

- VARIANCE IN THE UNDERLYING ECONOMY
- POLITICAL RISK
- MARKET STRUCTURE

FINANCIAL MARKET CHARACTERISTICS	EXAMPLES	PREMIUM OVER THE GOVT BOND RATE (%)
EMERGING MARKET, WITH POLITICAL RISK	SOUTH AMERICAN MARKETS, CHINA, RUSSIA	7.5 - 9.5
EMERGING MARKETS WITH LIMITED POLITICAL RISK	SINGAPORE, MALAYSIA, THAILAND, INDIA, SOME EAST EUROPEAN MARKETS	7.5
DEVELOPED MARKETS WITH WIDE STOCK LISTINGS	UNITED STATES, JAPAN, U.K., FRANCE, ITALY	5.5
DEVELOPED MARKETS WITH LIMITED LISTINGS AND STABLE ECONOMIES	GERMANY, SWITZERLAND	3.5 - 4.5

* Source : Aswath Damodaran Corporate Finance Theory and Practice, John Wiley.

TRIUMPH OF OPTIMISTS

ELROY DIMSON, PAUL MARCH, AND MICHAEL STANTON … *TRIUMPH OF THE OPTIMISTS*, (2001)

- **EQUITY RETURNS … 16 RICH COUNTRIES … DATA … 1900**

- **GLOBAL HISTORICAL RISK PREMIUM … 20TH CENTURY .. 4.6%**

- **BEST ESTIMATE OF EQUITY PREMIUM WORLDWIDE IN FUTURE IS 4 TO 5 PERCENT**

CALCULATION OF BETA

$$R_{it} = \alpha_i + \beta_i R_{Mt} + e_{it}$$

$$\beta_i = \frac{\sigma_{iM}}{\sigma_M^{\ 2}}$$

CALCULATION OF BETA

Period	Return on stock *A*, R_A	Return on market portfolio, R_M	Deviation of return on stock *A* from its mean $(R_A - \overline{R_A})$	Deviation of return on market portfolio from its mean $(R_M - \overline{R_M})$	Product of the deviation, $(R_A - \overline{R_A})(R_M - \overline{R_M})$	Square of the deviation of return on market portfolio from its mean $(R_M - \overline{R_M})^2$
1	10	12	0	3	0	9
2	15	14	5	5	25	25
3	18	13	8	4	32	16
4	14	10	4	1	4	1
5	16	9	6	0	0	0
6	16	13	6	4	24	16
7	18	14	8	5	40	25
8	4	7	-6	-2	12	4
9	- 9	1	-19	-8	152	64
10	14	12	4	3	12	9
11	15	-11	5	-20	-100	400
12	14	16	4	7	28	49
13	6	8	-4	-1	4	1
14	7	7	-3	-2	6	4
15	- 8	10	-18	1	-18	1
	$\Sigma R_A = 150$ $\overline{R_A} = 10$	$\Sigma R_M = 135$ $\overline{R_M} = 9$			$\Sigma(R_A - \overline{R_A})(R_M - \overline{R_M}) = 221$	$\Sigma (R_M - \overline{R_M})^2 = 624$

ESTIMATION ISSUES

- ESTIMATION PERIOD
  - A LONGER ESTIMATION PERIOD PROVIDES MORE DATA BUT THE RISK PROFILE .. FIRM MAY CHANGE
  - 5 YEARS
- RETURN INTERVAL

  DAILY, WEEKLY, MONTHLY
- MARKET INDEX

  STANDARD PRACTICE

ADJUSTING HISTORICAL BETA

- HISTORICAL ALIGNMENT … CHANCE FACTOR
- A COMPANY’S BETA MAY CHANGE OVER TIME

MERILL LYNCH …	0.66 …	HISTORICAL BETA
	O.34 …	MARKET BETA

BETAS BASED ON FUNDAMENTAL INFORMATION

KEY FACTORS EMPLOYED ARE

- **INDUSTRY AFFILIATION**
- **CORPORATE GROWTH**
- **EARNINGS VARIABILITY**
- **FINANCIAL LEVERAGE**
- **SIZE**

BETAS BASED ON ACCOUNTING EARNINGS

REGRESS THE CHANGES IN COMPANY EARNINGS (ON A QUARTERLY OR ANNUAL BASIS) AGAINST CHANGES IN THE AGGREGATE EARNINGS OF ALL THE COMPANIES INCLUDED IN A MARKET INDEX.

LIMITATIONS

- **ACCOUNTING EARNINGS .. GENERALLY SMOOTHED OUT .. RELATIVE .. VALUE OF THE COMPANY**
- **ACCOUNTING EARNINGS … INFLUENCED BY NON - OPERATING FACTORS**
- **LESS FREQUENT MEASUREMENT**

BETAS FROM CROSS SECTIONAL REGRESSIONS

1. ESTIMATE A CROSS - SECTIONAL REGRESSION RELATIONSHIP FOR PUBLICLY TRADED FIRMS:

   BETA = 0.6507 + 0.27 COEFFICIENT OF VARIATION IN OPERATING INCOME + 0.09 D/E + 0.54 EARNINGS - .00009 TOTAL ASSETS (MILLION $)

2. PLUG THE CHARACTERISTICS OF THE PROJECT, DIVISION, OR UNLISTED COMPANY IN THE REGR'N REL'N TO ARRIVE AT AN ESTIMATE OF BETA

   BETA = 0.6507 + 0.27 (1.85) + 0.09 (0.90) + 0.54 (0.12) - 0.00009 (150) = 1.2095

EMPIRICAL EVIDENCE ON CAPM

1. SET UP THE SAMPLE DATA

$$R_{it}\ ,\ R_{Mt}\ ,\ R_{ft}$$

2. ESTIMATE THE SECURITY CHARACTERISTIC LINES

$$R_{it} - R_{ft} = a_i + b_i(R_{Mt} - R_{ft}) + e_{it}$$

3. ESTIMATE THE SECURITY MARKET LINE

$$\bar{R}_i = \gamma_0 + \gamma_1\ b_i + e_i \qquad ,\ i = 1, \ldots 75$$

EVIDENCE

IF CAPM HOLDS

- THE RELATION … LINEAR .. TERMS LIKE b_i^2 .. NO EXPLANATORY POWER
- $\gamma_0 \simeq \overline{R_f}$
- $\gamma_1 \simeq \overline{R_M - R_f}$
- NO OTHER FACTORS, SUCH AS COMPANY SIZE OR TOTAL VARIANCE, SHOULD AFFECT R_i
- THE MODEL SHOULD EXPLAIN A SIGNIFICANT PORTION OF VARIATION IN RETURNS AMONG SECURITIES

GENERAL FINDINGS

- THE RELATION … APPEARS .. LINEAR
- $\gamma_0 > \overline{R_f}$
- $\gamma_1 < \overline{R_M - R_f}$
- IN ADDITION TO BETA, SOME OTHER FACTORS, SUCH AS STANDARD DEVIATION OF RETURNS AND COMPANY SIZE, TOO HAVE A BEARING ON RETURN
- BETA DOES NOT EXPLAIN A VERY HIGH PERCENTAGE OF THE VARIANCE IN RETURN

CONCLUSIONS

PROBLEMS

- STUDIES USE HISTORICAL RETURNS AS PROXIES FOR EXPECTATIONS
- STUDIES USE A MARKET INDEX AS A PROXY

POPULARITY

- SOME OBJECTIVE ESTIMATE OF RISK PREMIUM .. BETTER THAN A COMPLETELY SUBJECTIVE ESTIMATE
- BASIC MESSAGE .. ACCEPTED BY ALL
- NO CONSENSUS ON ALTERNATIVE

ARBITRAGE - PRICING THEORY

RETURN GENERATING PROCESS

$$R_i \;=\; a_i + b_{i\,1}\, I_1 + b_{i2}\, I_2 \ldots + b_{ij}\, I_1 + e_i$$

EQUILIBRIUM RISK - RETURN RELATIONSHIP

$$E(R_i) = \lambda_0 + b_{i1}\lambda_1 + b_{i2}\lambda_2 + \ldots\; b_{ij}\lambda_j$$

λ_j = **RISK PREMIUM FOR THE TYPE OF RISK ASSOCIATED WITH FACTOR** j