(5) $\sqrt{-7+24i}$ $\quad z=-7+24i$

$\therefore |z| = \sqrt{7^2+24^2} = 25$

$x=-7,\ y=24>0 \ \therefore \text{sign}\, y = 1$

$$\therefore \sqrt{-7+24i} = \pm\left[\sqrt{\tfrac{1}{2}(25+(-7))} + i\sqrt{\tfrac{1}{2}(25-(-7))}\right].$$

(See formula for $\sqrt{z}$)

$$= \pm[3+i4] \rightarrow \text{Ans}.$$

Ex Solve the equations:

① $z^2-(5+i)z+(8+i)=0$.

$$z = \frac{(5+i) \pm \sqrt{(5+i)^2-4(8+i)}}{2} \Rightarrow z = \frac{(5+i) \pm \sqrt{(-8+6i)}}{2}.$$

Now, $\sqrt{z} = \pm\left[\sqrt{\tfrac{1}{2}(|z|+x)} + i(\text{sign}\, y)\sqrt{\tfrac{1}{2}(|z|-x)}\right]$

$$\therefore \sqrt{-8+6i} = \pm\left[\sqrt{\tfrac{1}{2}(10-8)} + i\sqrt{\tfrac{1}{2}(10+8)}\right] = \pm[1+i3]$$

$$\therefore z = \frac{(5+i) \pm (1+i3)}{2} = 3+2i,\ 2-i \rightarrow \text{Ans}$$

② $z^2+z+1-i=0$

$$z = \frac{-1 \pm \sqrt{1-4(1-i)}}{2} = \frac{-1 \pm \sqrt{-3+4i}}{2}$$

$$\sqrt{-3+4i} = \pm\left[\sqrt{\tfrac{1}{2}(5-3)} + i\sqrt{\tfrac{1}{2}(5+3)}\right] = \pm(1+i2)$$

$$\therefore z = \frac{-1 \pm (1+2i)}{2} = i,\ -1-i \text{ — Ans}$$

③ $z^4-(3+6i)z^2-8+6i=0$

Let $z^2=t$. Then $t^2-(3+6i)t-8+6i=0$

$t = 3+4i,\ 2i$

$\therefore z = \pm\sqrt{3+4i},\ \sqrt{2i}$

or, $z = \pm(2+i),\ \pm(1+i)$ — Ans

## Definitions:

**Circle:** $\boxed{|z-a| = \rho}$ → circle with center at 'a' and radius $\rho$

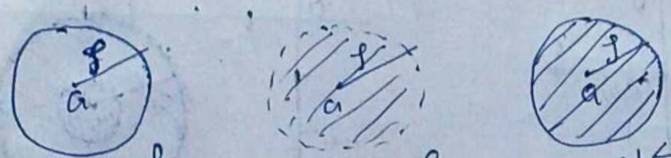
[If $a = a_1 + ia_2$ then $|z-a| = |(x-a_1) + i(y-a_2)|$

$= \sqrt{(x-a_1)^2 + (y-a_2)^2}$

$\therefore |z-a|^2 = \rho^2 \Rightarrow (x-a_1)^2 + (y-a_2)^2 = \rho^2$ → Eqn of circle in xy plane]

**Open circular disk:** $|z-a| < \rho$ (boundary points are not included)

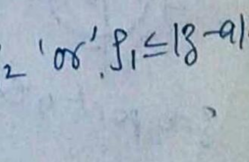
**Closed circular disk:** $|z-a| \leq \rho$ (boundary points are included)



$|z-a| = \rho$ $\quad$ $|z-a| < \rho$ $\quad$ $|z-a| \leq \rho$

Open ~~circular~~ circular disk $|z-a| < \rho$ is also called neighbourhood of 'a'

[Note: In nbd of point 'a' there are infinitely many points (provided $\rho > 0$).]



**Open annulus:** $\rho_1 < |z-a| < \rho_2$

**Closed annulus:** $\rho_1 \leq |z-a| \leq \rho_2$

**Semi open-closed annulus:** $\rho_1 < |z-a| \leq \rho_2$ 'or' $\rho_1 \leq |z-a| < \rho_2$

**Half planes:** Upper half plane : $y > 0$

Lower " " : $y < 0$

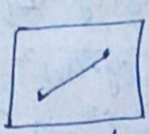
left " " : $x < 0$

right " " : $x > 0$.

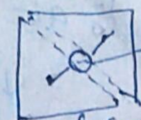
**Connected set:** ~~A ... open~~ Set S is connected if any 2 points in it can be joined by broken line of finitely many straight-line segments all of whose points belong to S.

**Domain**: An open connected set is called domain e.g. open disk, open annulus.

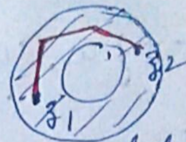
**Example:**



connected set




not connected

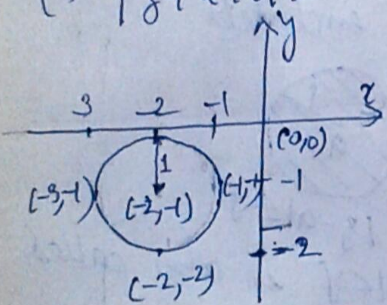


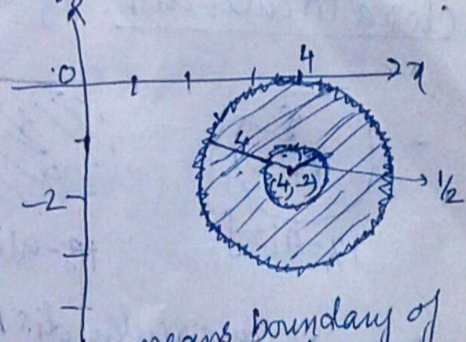

connected

**Ex** Determine and sketch the sets in complex plane given by

(1) $|z+2+i| \le 1$ → circle

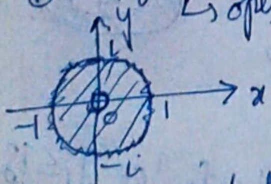
(2) $\frac{1}{2} < |z-4+2i| < 2$ → open annulus




→ means boundary of circles is not included

(3) $0 < |z| \le 1$ → open annulus

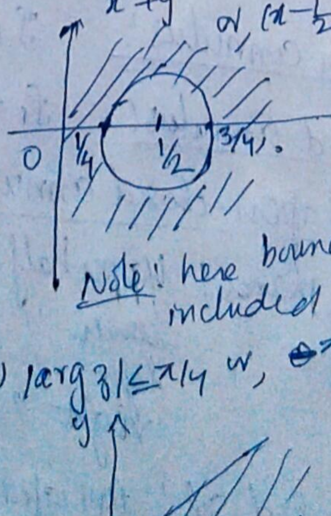



$z=0$ point is excluded.

(4) $\text{Re}\left(\frac{1}{z}\right) \le 1$ → exterior of circle

$\frac{x}{x^2+y^2} \le 1 \Rightarrow x^2+y^2-x \ge 0$

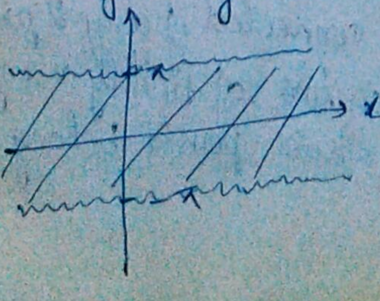
or, $(x-\frac{1}{2})^2+y^2 \ge \frac{1}{4}$




**Note**: here boundary is included

(5) $-\pi < \text{Im}\, z < \pi$

$\quad\quad\quad\quad y$




(6) $|\arg z| \le \pi/4$ or, $-\pi/4 \le \theta \le \frac{\pi}{4}$