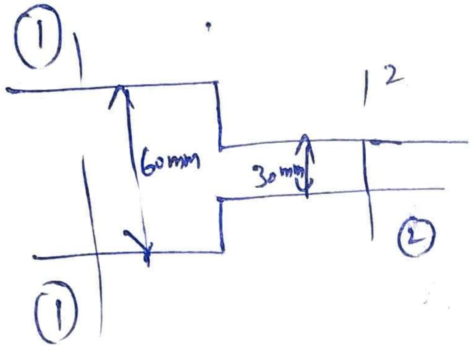
Q: A gas is flowing in a horizontal pipe at a temp. of 6°C (as shown in fig.) The pressure at the section 1 & 2 are 4 bar (gauge) and 3 bar (gauge) respectively. If R = 287 J/kg·K and atmospheric pressure is 1 bar. find the velocities of the gas at these section.

Sol^n



$D_1 = 0.06m$

$A_1 = \frac{\pi}{4} \times (.06)^2 = 2.827 \times 10^{-3} m^2$

$P_1 = 4 bar = 4 + 1 = 5$ bar

$= 5 \times 10^5 N/m^2$ (Abs.).

$P_2 = 3 + 1 = 4 \times 10^5 N/m^2$ (Abs.)

$D_2 = .03$ m

$A_2 = \frac{\pi}{4} (.03)^2 = 7.0686 \times 10^{-4} m^2$

$R = 287$ J/kg·K.

$\rho_1 A_1 V_1 = \rho_2 A_2 V_2$

$$\frac{V_2}{V_1} = \frac{\rho_1 A_1}{\rho_2 A_2} = \frac{\rho_1 \times 2.827 \times 10^{-3}}{\rho_2 \times 7.0686 \times 10^{-3}} = 4 \times \frac{\rho_1}{\rho_2}$$

For isothermal process

$$\frac{P_1}{\rho_1} = \frac{P_2}{\rho_2} \quad = \frac{\rho_1}{\rho_2} = \frac{5 \times 10^5}{4 \times 10^5} = 1.25$$

V_2 ... 4×1.25

Bernoulli's eqn b/w ① & ②

$$\frac{P_1}{\rho_1 g}\log_e P_1 + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho_2 g}\log_e P_2 + \frac{V_2^2}{2g} + Z_2$$

$$Z_1 = Z_2$$

$$\frac{P_1}{\rho_1 g}\log_e P_1 - \frac{P_2}{\rho_2 g}\log_e P_2 = \frac{V_2^2}{2g} - \frac{V_1^2}{2g}$$

$$\frac{P_1}{\rho_1} = \frac{P_2}{\rho_2} \quad \text{for } T = C$$

$$\frac{P_1}{\rho_1 g}\ln\left[\frac{5\times10^5}{4\times10^5}\right] = \frac{5V_1^2}{2g} - \frac{V_1^2}{2g} = \frac{24V_1^2}{2g} = \frac{12V_1^2}{g}$$

$$.223\,\frac{P_1}{\rho_1 g} = \frac{12V_1^2}{g}$$

$$\frac{P_1}{\rho_1} = \frac{12V_1^2}{.223} = 53.8V_1^2$$

$$P_1 = \rho_1 R T_1 \Rightarrow \frac{P_1}{\rho_1} = RT_1 = 287\times279 = 80073$$

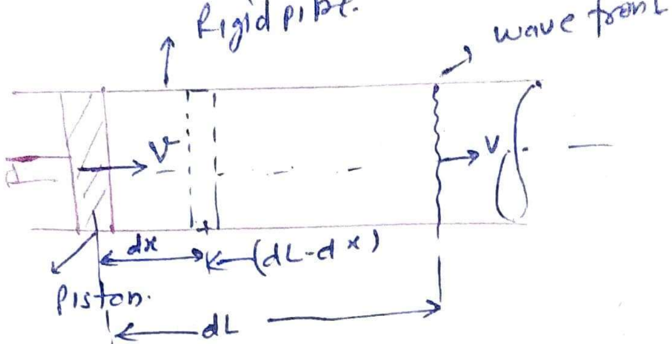
$$80073 = 53.8V_1^2$$

$$V_1^2 = 1488.34$$

$$V_1 = 38.58\,m/s$$

$$V_2 = 5V_1 = 5\times38.58 = 192.9\,m/s$$

Velocity of sound :-



A - C/s area of pipe, V - piston velocity.

P - fluid pressure in the pipe before piston movement

ρ - fluid density before the piston movement.

C → velocity of pressure wave (sound wave)

dt - Time during which piston moves.

Before movement of piston, the total mass in length dL

$= \rho \times dL \times A$.

Piston moves distance dx, in length (dL-dx) the density increases to $(\rho + d\rho)$

total mass in (dL-dx) length

$= (\rho + d\rho) \cdot A \times (dL - dx)$, $dL = C dt$, $dx = V \cdot dt$

$= (\rho + d\rho) \cdot A (C dt - V dt)$

mass before movement of piston = mass after movement of piston.

$\rho \cdot A \, dL = (\rho + d\rho) A \, dt \, (C - V)$

$\rho \cdot A \cdot C \, dt = (\rho + d\rho) A \cdot dt \, (C - V)$

$\rho C = (\rho + d\rho)(C - V)$