Stagnation pressure (P_0) :-

For. Perfect gas. $T \cdot P^{\frac{1-\gamma}{\gamma}} = \text{const}$

$$T_0 P_0^{\frac{1-\gamma}{\gamma}} = T_1 P_1^{\frac{1-\gamma}{\gamma}}$$

$$T_0 P_0^{\frac{1-\gamma}{\gamma}} = T \cdot P^{\frac{1-\gamma}{\gamma}}$$

$$\left(\frac{P_0}{P}\right)^{\frac{1-\gamma}{\gamma}} = \left(\frac{T}{T_0}\right) \Rightarrow \frac{P_0}{P} = \left(\frac{T}{T_0}\right)^{\frac{\gamma}{\gamma-1}}$$

$$\frac{P_0}{P} = \left[1 + \frac{\gamma-1}{2} M^2\right]^{\frac{\gamma}{\gamma-1}}$$

$$P_0 = P\left[1 + \frac{\gamma-1}{2} M^2\right]^{\frac{\gamma}{\gamma-1}}$$

$\downarrow$

stagnation Pressure

Stagnation density :-

For. perfect gas. $\frac{P}{\rho^\gamma} = \text{const}$

$$\frac{P_0}{\rho_0^\gamma} = \frac{P}{\rho^\gamma} \Rightarrow \frac{\rho_0^\gamma}{\rho^\gamma} = \frac{P_0}{P}.$$

$$\frac{\rho_0}{\rho} = \left(\frac{P_0}{P}\right)^{\frac{1}{\gamma}}.$$

$$\frac{\rho_0}{\rho} = \left(1 + \frac{\gamma-1}{2} M^2\right)^{\frac{\gamma}{\gamma-1} \times \frac{1}{\gamma}}$$

$$\boxed{\rho_0 = \rho\left(1 + \frac{\gamma-1}{2} M^2\right)^{\frac{1}{\gamma-1}}}$$

$\downarrow$

Q. An aeroplane is flying at 1000 Km/hr. through still air having a pressure of 78.5 KN/m² (Abs). and temp. −8°C. Calculate on the stagnation point on the nose of the plane

a) stagnation pressure.

b) Stagnation temperature

c) stagnation density.

$R = 287\ J/kg.K, \quad r = 1.4.$

Solⁿ

$V = 1000\ km/hr = \dfrac{1000 \times 1000}{3600} = 277.77\ m/s.$

pressure of air $P = 78.5\ KN/m^2$

$T = -8 + 273 = 265\ K$

$R = 287\ J/kg.K, \ r = 1.4$

For air $\quad C = \sqrt{r.RT} = \sqrt{1.4 \times 287 \times 265} = 326.31\ m/s.$

$M = \dfrac{V}{C} = \dfrac{277.77}{326.31} = .851$

$$P_0 = P.\left[1 + \left(\frac{r-1}{2}\right)M^2\right]^{\frac{r}{r-1}}$$

$$P_0 = 78.5\left[1 + \frac{(1.4-1)}{2}(.851)^2\right]^{\frac{1.4}{1.4-1}} = 126.1\ KN/m^2$$

$$T_0 = T\left[1 + \left(\frac{\gamma - 1}{2}\right) M^2\right].$$

$$= 265\left[1 + \frac{1.4 - 1}{2}(.851)^2\right].$$

$$= 303.4\,K.$$

$\rho_0 \Rightarrow$ stagnation density

$$\frac{P_0}{\rho_0} = R \cdot T_0.$$

$$\rho_0 = \frac{P_0}{RT_0} = \frac{126.1 \times 10^3}{287 \times 303.4} = 1.448\ kg/m^3.$$

Area-velocity relationship for compressible flow.

$$\frac{dA}{A} + \frac{dv}{V} + \frac{d\rho}{\rho} = 0 \quad \text{———} \quad (i)$$

$$\frac{dP}{\rho} + v\,dv + g\,dz = 0 \quad - \quad \text{Euler's eqn}$$

$$\frac{dP}{\rho} + v\,dv = 0.$$

$$\frac{dP \times d\rho}{\rho \cdot d\rho} + v\,dv = 0 \qquad c = \sqrt{\frac{dP}{d\rho}}$$

$$c^2 \cdot \frac{d\rho}{\rho} + v\,dv = 0$$

$$\frac{d\rho}{\rho} = -v\frac{dV}{c^2} \quad \text{———} \quad (ii)$$

Put the value in eqn (ii) (i)

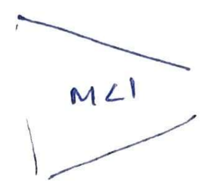
$$\frac{dA}{A} + \frac{dv}{V} - \frac{v\,dv}{c^2} = 0$$

$$\frac{dA}{A} = \frac{dv}{v}\left[\frac{v^2}{c^2}-1\right] \Rightarrow \boxed{\frac{dA}{A} = \frac{dv}{v}\left[M^2-1\right]}$$

a) For Nozzle. $\rightarrow$ $dv \longrightarrow +ve$ always.

I) for $M<1$ (subsonic):

$$\frac{dA}{A} = (+ve)(-ve) \Rightarrow \frac{dA}{A} = \underline{-ve}$$



ii) For $M>1$, Supersonic.

$$\frac{dA}{A} = (M^2-1)\frac{dV}{V} \Rightarrow (+ve)(+ve) \Rightarrow +ve$$

$$\frac{dA}{A} = (+ve)$$