University Institute of Engineering & Technology CSJMU KANPUR

Department of Electronics & Communication Engineering

Course Name- Network Analysis and Synthesis (ECE 202)

Branch B.Tech Electronics & Communication

UNIT 1: Graph Theory

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1. Introduction

Networks and graphs are well-established elements of many "new physics" approaches, and they turn out in many fast-developing fields like the theory of computer networks, Markov chains, biochemical reaction networks, the statistical mechanics of disordered media, neuronal networks, food webs, traffic networks, and social networks [1, 2, 3, 4, 5, 6, 7, 8]. Unfortunately, the important concepts of graph theory are still missing from many undergraduate curricula. However, electrical networks are an important part of the standard physics curricula, and they provide an easy access route to the fascinating world of graphs and networks [8]. Indeed, graph-theoretical concepts are at the very basis of many programs for the resolution of electrical and electronic circuits – like, e.g., SPICE – and here we describe a student project to construct from scratch one such program. In this paper we give a bare-bones description of the project, and suggest a few possible developments, with the hope that we may be able to kick off a number of interesting projects at the interface of physics and graph theory.

1. Graphs

The approach that we suggest in this paper requires only a basic knowledge of graph theory. There are many excellent introductions to the theory of graphs , and the instructor should follow her/his preferences in the exposition of graph theory. In this section we review the basic definitions and concepts that are necessary and useful for a proper development of the student project.

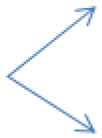
A graph is a collection of points (called vertices or nodes) and lines joining some of these points (called arcs or links). The links can be either directed or not, and the corresponding graphs are directed or non-directed.

A. Terminology used in network graph:-

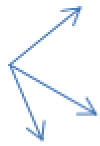
(i) Path:-A sequence of branches traversed in going from one node to another is called a path.

(ii) Node:-A node point is defined as an end point of a line segment and exits at the junction between two branches or at the end of an isolated branch.

(iii) Degree of a node:- It is the no. of branches incident to it.



2-degree



3-degree

(iv) Tree:- It is an interconnected open set of branches which include all the nodes of the given graph. In a tree of the graph there can't be any closed loop.

(v) Tree branch (Twig):- It is the branch of a tree. It is also named as twig.

(vi) Tree link (or chord):-It is the branch of a graph that does not belong to the particular tree.

(vii) Loop:- This is the closed contour selected in a graph.

(viii) Cut-Set: - It is that set of elements or branches of a graph that separated two parts of a network. If any branch of the cut-set is not removed, the network remains connected. The term cut-set is derived from the property designated by the way by which the network can be divided in to two parts.

(ix) Tie-Set:- It is a unique set with respect to a given tree at a connected graph containing on chord and all of the free branches contained in the free path formed between two vertices of the chord.

(x) Network variables:- A network consists of passive elements as well as sources of energy . In order to find out the response of the network the through current and voltages across each branch of the network are to be obtained.

(xi) Directed (or Oriented) graph:- A graph is said to be directed (or oriented) when all the nodes and branches are numbered or direction assigned to the branches by arrow.

(xii) Sub graph:- A graph said to be sub-graph of a graph G if every node of is a node of G and every branch of is also a branch of G.

(xiii) Connected Graph:- When at least one path along branches between every pair of a graph exits , it is called a connected graph.

(xiv) Incidence matrix:- Any oriented graph can be described completely in a compact matrix form. Here we specify the orientation of each branch in the graph and the nodes at which this branch is incident. This branch is called incident matrix. When one row is completely deleted from the matrix the remaining matrix is called a reduced incidence matrix.

(xv) Isomorphism:- It is the property between two graphs so that both have got same incidence matrix.

B. Relation between twigs and links- Let N=no. of nodes L= total no. of links B= total no. of branches No. of twigs= N-1 Then, L= B-(N-1)

C. Properties of a Tree-

(i) It consists of all the nodes of the graph.

(ii) If the graph has N nodes, then the tree has (N-1) branch.

(iii) There will be no closed path in a tree.

(iv) There can be many possible different trees for a given graph depending on the no. of nodes and branches.

1. FORMATION OF INCIDENCE MATRIX:- • This matrix shows which branch is incident to which node. • Each row of the matrix being representing the corresponding node of the graph. • Each column corresponds to a branch. • If a graph contain N- nodes and B branches then the size of the incidence matrix [A] will be NXB.

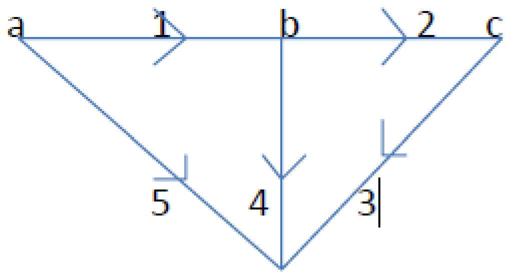
A. Procedure:-

(i) If the branch j is incident at the node I and oriented away from the node, =1. In other words, when =1, branch j leaves away node i.

(ii) If branch j is incident at node j and is oriented towards node i, =-1. In other words j enters node i.

(iii) If branch j is not incident at node i. =0. The complete set of incidence matrix is called augmented incidence matrix.

Ex-1:- Obtain the incidence matrix of the following graph.



Node-a:- Branches connected are 1& 5 and both are away from the node.

Node-b:- Branches incident at this node are 1,2 &4. Here branch is oriented towards the node whereas branches 2 &4 are directed away from the node.

Node-c:- Branches 2 &3 are incident on this node. Here, branch 2 is oriented towards the node whereas the branch 3 is directed away from the node.

Node-d:- Branch 3,4 &5 are incident on the node. Here all the branches are directed towards the node.

Node	1	2	3	4	5
1	1	0	0	0	1
$[A_i]$=2	-1	1	0	1	0
3	0	-1	1	0	0
4	0	0	-1	-1	-1

B. Properties:-

(i) Algebraic sum of the column entries of an incidence matrix is zero.

(ii) Determinant of the incidence matrix of a closed loop is zero.

C. Reduced Incidence Matrix:-

If any row of a matrix is completely deleted, then the remaining matrix is known as Reduced Incidence matrix. For the above example, after deleting row, we get,

$$[A_i'] = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 \\ -1 & 1 & 0 & 1 & 0 \\ 0 & -1 & 1 & 0 & 0 \end{bmatrix}$$

Ai' is the reduced matrix of *Ai* .

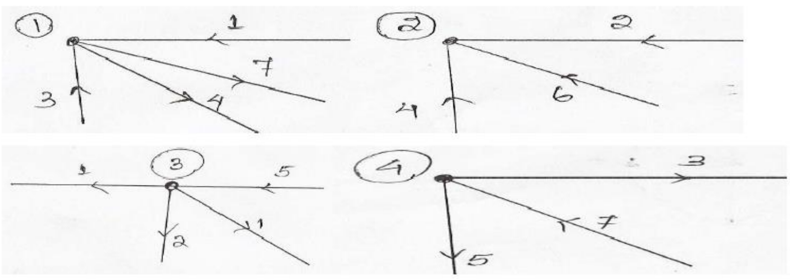
Ex-2: Draw the directed graph for the following incidence matrix.

Branch

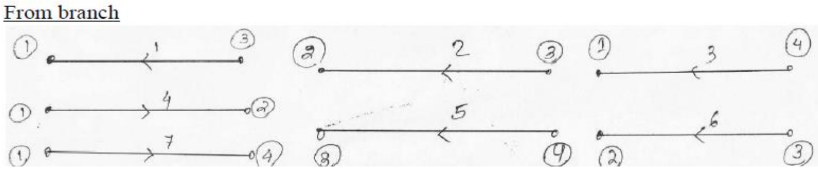
Node	1	2	3	4	5	6	7
1	-1	0	-1	1	0	0	1
2	0	-1	0	-1	0	-1	0
3	1	1	0	0	-1	1	0
4	0	0	1	0	1	0	-1

Solution:-

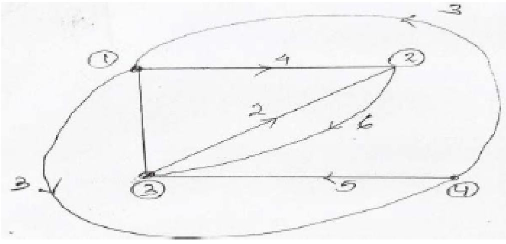
From node

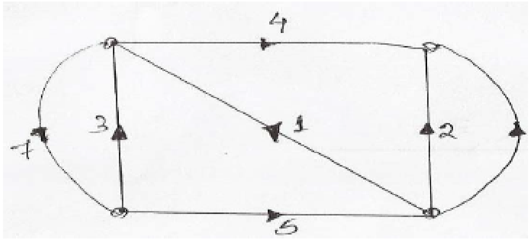


From branch



Sol.-





Tie-set Matrix:

		Branch				
		1	2	3	4	5
Loop currents	I_1	1	0	0	1	1
	I_2	-1	-1	1	0	-1

$$B_i = \begin{vmatrix} 1 & 0 & 0 & 1 & 1 \\ -1 & -1 & 1 & 0 & -1 \end{vmatrix} = \begin{vmatrix} 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & -1 & 0 & 1 \end{vmatrix}$$

Let V_1, V_2, V_3, V_4& V_5 be the voltage of branch 1,2,3,4,5 respectively and j_1, j_2, j_3, j_4, $j_5$are current through the branch 1,2,3,4,5 respectively.
So, algebraic sum of branch voltages in a loop is zero.

Now, we can write,

$V_1+ V_4+ V_5= 0$

$V_1+ V_2-V_3 + V_5=0$

Similarly, $j_1 = I_1 - I_2$ $j_2 = -I_2$ $j_3 = I_2$ $j_4 = I_1$

$j_5 = I_1 - I_2$

Fundamental of cut-set matrix:-

A fundamental cut-set of a graph w.r.t a tree is a cut-set formed by one
twig and a set of links. Thus in a graph for each twig of a chosen tree there would be a Fundamental cut set.

No. of cut-sets=No. of twigs=N-1.

Procedure of obtaining cut-set matrix:-

(i) Arbitrarily at tree is selected in a graph.
(ii) From fundamental cut-sets with each twig in the graph for the entire tree.
(iii) Assume directions of the cut-sets oriented in the same direction of the concerned twig.
(iv) Fundamental cut-set matrix [ ]
=+1; when branch has the same orientation of the cut-set
=-1; when branch has the opposite orientation of the cut-set
=0; when branch is not in the cut-set

Fundamental of Tie-set matrix:-

A fundamental tie-set of a graph w.r.t a tree is a loop formed by only one link associated with Other twigs.

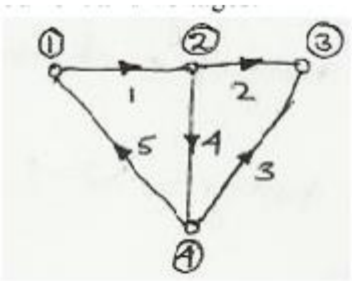
No. of fundamental loops=No. of links=B-(N-1)

Procedure of obtaining Tie-set matrix:-

(i) Arbitrarily a tree is selected in the graph.
(ii) From fundamental loops with each link in the graph for the entire tree.

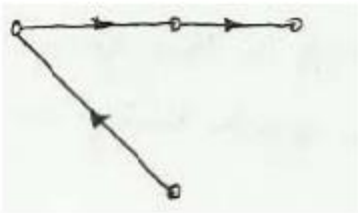
(iii)Assume directions of loop currents oriented in the same direction as that of the link.
(iv)From fundamental tie-set matrix [ ] where
=1; when branch b_j is in the fundamental loop i and their reference directions are Oriented same.
=-1; when branch b_j is in the fundamental loop i but, their reference directions are oriented oppositely.
=0; when branch b_j is not in the fundamental loop i .

Ex-3: Determine the tie set matrix of the following graph. Also find the equation of branch current and voltages.



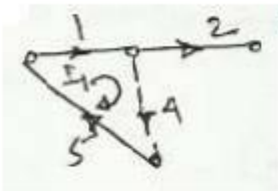
Solution

Tree



No. of loops= No. of links= 2

Loop 1



Loop 2