

### Information →

In communication information is define in terms of message, like, audio, video & image.

### Information Theory →

It is the science for measuring, preserving, transmitting and estimating information in random data. It was initially proposed by Shannon as a mathematical theory of communication more than five decades ago.

Uncertainty → Let us consider a message  $X$  which is combination of different types of messages  $x_1, x_2, x_3, \dots, x_n$  and probabilities of these messages are given by  $p_1, p_2, \dots, p_n$  respectively.  $\Pi$

$$\text{i.e } X = \{x_1, x_2, \dots, x_n\}$$

$$P = \{p_1, p_2, \dots, p_n\}$$

Total Probability  $P = \sum_{i=1}^n p_i$

if  $x_1 \rightarrow p_1 = 0$  (event never occur) } NO  
 $x_1 \rightarrow p_1 = 1$  (occurring of event is 100%) } uncertainty

But if  $x_1 \rightarrow p_1 = 0.5$  } high uncertainty  
 $x_1 \rightarrow p_1 = 0.1$  }

## Information Source →

It may be viewed as an object which produces an event, the outcome of which is selected at random according to a probability distribution.

## Discrete memoryless Source (DMS)

A discrete memoryless source can be characterized by the list of the symbols, the probability assignment to these symbols and the specification of the rate of generating these symbols by the source.

## Measure of Information →

Let us consider communication system which transmits messages  $m_1, m_2, m_3, \dots$  with probabilities of occurrence  $p_1, p_2, \dots$ . The amount of information transmitted through the message is given by →

Amount of Information  $I_k = \log_2 \left( \frac{1}{p_k} \right) = \frac{\log_{10} \left( \frac{1}{p_k} \right)}{\log_{10} 2}$

unit of Information is bit.

## Properties of Information

- ① If there is more uncertainty about the message, information carried is also more. (i.e. if  $U_1 > U_2$  then  $I_1 > I_2$  for any two messages)
- ② If receiver knows the message being transmitted, the amount of information carried is zero. ( $P=1$ )
- ③ If  $I_1$  is the information carried by message  $m_1$ , and  $I_2$  is the information carried by  $m_2$ , then amount of information carried ~~compositely~~ due to  $m_1$  &  $m_2$  is  $= I_1 + I_2$
- ④ If there are  $m = 2^N$  equally likely messages, then amount of information carried by each message will be  $= N$  bits

### Proof of property ① →

Let us consider there are two messages  $m_1$  &  $m_2$  and their probabilities are given by  $P_1 = \frac{1}{4}$  &  $P_2 = \frac{1}{2}$  respectively then

$$P_1 = \frac{1}{4}$$

$$P_2 = \frac{1}{2}$$

$$U_1$$

$$U_2$$

$$U_1 > U_2$$

$$I_1 = \log_2 \left( \frac{1}{P_1} \right)$$

$$= \log_2 \left( \frac{1}{\frac{1}{4}} \right)$$

$$\boxed{I_1 = 2 \text{ bits}}$$

$$I_2 = \log_2 \left( \frac{1}{P_2} \right)$$

$$I_2 = \log_2 2$$

$$\boxed{I_2 = 1 \text{ bit}}$$

So  $I_1 > I_2$  which we have to prove.

Property ② → If receiver knows message being transmitted then its probability will be  $P=1$

$$\text{So } I = \log_2\left(\frac{1}{P}\right)$$

$$I = \log_2 1$$

$$\boxed{I = 0 \text{ bit}} \quad \underline{\text{proved}}$$

Property ③ → In this case let us consider probability of  $m_1$  is  $P_1$  & probability of  $m_2$  is  $P_2$  then.

$$I_1 = \log_2\left(\frac{1}{P_1}\right)$$

$$I_2 = \log_2\left(\frac{1}{P_2}\right)$$

Since messages  $m_1$  &  $m_2$  are independent, so the probability of combined message is  $P_1 P_2$ . Therefore information carried ~~due~~ due to  $m_1$  &  $m_2$  is →

$$\cancel{I = I_1 + I_2} \quad I = \log_2\left(\frac{1}{P}\right) = \log_2\left(\frac{1}{P_1 P_2}\right)$$

$$I = \log_2\left(\frac{1}{P_2}\right) + \log_2\left(\frac{1}{P_1}\right)$$

$$\log_2\left(\frac{1}{P_1 P_2}\right) = I_1 + I_2$$

$$\log_2\left(\frac{1}{P_1}\right) + \log_2\left(\frac{1}{P_2}\right) = I$$

$$\boxed{I = I_1 + I_2}$$

④ Since all the  $m$  messages are equally likely and independent, probability of occurrence of each message will be  $\frac{1}{m}$ .

$$\text{we know that } I_K = \log_2 \left( \frac{1}{P_K} \right) \rightarrow \textcircled{1}$$

$$\text{But } P_K = \frac{1}{m} \text{ so}$$

$$\text{so from eqn } \textcircled{1} \quad I_K = \log_2 M \rightarrow \textcircled{2}$$

we know that  $M = 2^N$  so put this value in

eqn  $\textcircled{2}$  we get

$$I_K = \log_2 2^N$$

$$I_K = N \log_2 2$$

$$\boxed{I_K = N \text{ bits}} \quad \underline{\text{proved}}$$