

Introduction Fourier transform and the frequency domain

The background is a dark blue with a fine, repeating grid pattern. On the right side, there are several white, parallel diagonal lines that create a sense of motion or depth, extending from the bottom right towards the top right.

Fourier Transform

- Fourier Transform , named after Joseph Fourier, is a mathematical transformation employed to transform signals between time(or spatial domain and frequency domain.
- It is a tool that breaks a waveform(a function or signal)into an alternate representation, characterized by sine and cosines.
- Fourier transformation is used in a wide range of application such as image filtering, image compression. Image analysis and image reconstruction etc.



Fourier

The formula for Fourier transformation

$$F(u) = \int_{-\infty}^{\infty} f(x)e^{-j2\pi ux} dx,$$
$$f(x) = \int_{-\infty}^{\infty} F(u)e^{j2\pi ux} du$$

Example:

$$f(x) = \begin{cases} 1, & |x| < \frac{X}{2}, \\ 0, & |x| \geq \frac{X}{2}. \end{cases}$$

$$F(u) = \int_{-\infty}^{\infty} f(x) e^{-j2\pi ux} dx$$

$$= \int_{-X/2}^{X/2} e^{-j2\pi ux} dx$$

$$= \frac{1}{-j2\pi u} [e^{-j2\pi u X/2} - e^{j2\pi u X/2}]$$

$$= X \frac{\sin(\pi Xu)}{(\pi Xu)} = X \operatorname{sinc}(\pi Xu).$$

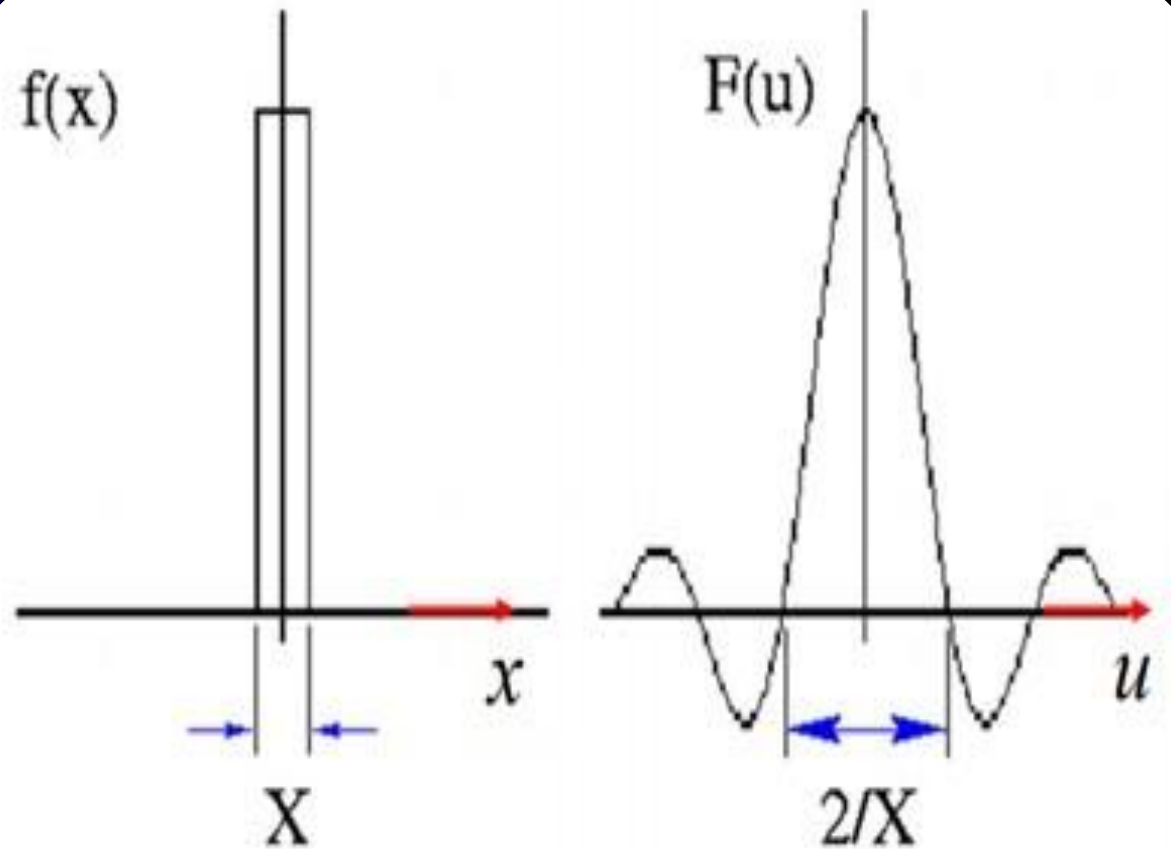


IMAGE ENHANCEMENT TECHNIQUES

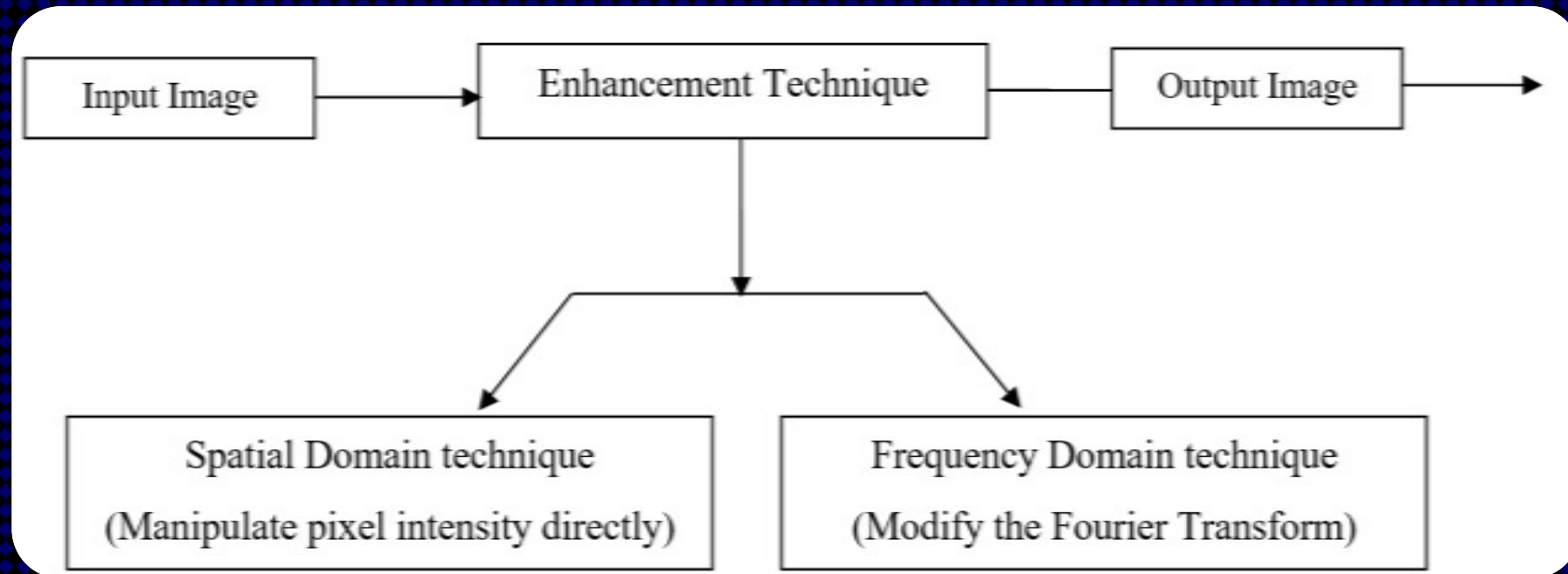


Fig. 1. Types of Enhancement Technique

The enhancement technique differs from one field to another according to its objective. Advancement in the technology brings the development in the digital image processing techniques in both domains:

DIFFERENCE BETWEEN SPATIAL DOMAIN AND FREQUENCY DOMAIN

► Spatial domain

1. Spatial domain deals with image plane itself.
2. Spatial domain works based on direct manipulation of pixels.
3. Spatial domain can be more easier to understand.
4. Spatial domain is more cheaper.
5. Spatial domain takes time to computer.
6. Used for filtering basics, smoothing filters, sharpening filters, unsharp masking and Laplacian.

► Frequency domain

1. Frequency domain deals with the rate of pixel change.
2. Frequency domain works based on modifying Fourier transform.
3. Frequency domain can be less easier to understand.
4. Frequency domain is less cheaper.
5. Frequency domain takes more time to compute.
6. Used to filtering basics, Smooth an image with a Gaussian kernel.

A. Spatial domain

The term spatial domain refers to the image plane itself, and approaches in this category are based on direct manipulation of pixel values of an image. It enhances the whole image in a uniform manner. The value of the pixels with coordinates (x, y) in an enhanced image 'F' is the result of performing some operation on the pixels with the neighbourhood of (x, y) in the input image 'f'. This method is straightforward and are chiefly utilized in real time applications. But it lags in producing adequate robustness and imperceptibility requirement.

B. Frequency domain.

The frequency domain processing techniques are based on modifying the Fourier transform of an image. The basic idea in using this technique is to enhance the image by manipulating the transform coefficient of the image, such as Discrete Fourier Transform (DFT), Discrete Wavelet Transform (DWT), and Discrete Cosine Transform (DCT). This methods advantages includes low complexity of computations, ease of viewing and manipulating the frequency composition of the image and the easy applicability of special transformed domain properties.

IMAGE ENHANCEMENT USING FREQUENCY DOMAIN TECHNIQUE

In frequency domain methods, the image is first transferred into frequency domain. All the enhancement operations are performed on the Fourier transform of the. Image enhancement function in the frequency domain is denoted by the expression: $\mathbf{g(x, y) = T[f(x, y)]}$ where $\mathbf{f(x, y)}$ is the input image, $\mathbf{g(x, y)}$ is an enhanced image formed by the result of performing some operation, T on the frequency component of the transformed image.

Filtering in the Frequency Domain The procedures required to enhance an image using frequency domain technique are:

- i. Transform the input image into the Fourier domain.
- ii. Multiply the Fourier transformed image by a filter.
- iii. Take the inverse Fourier transform of the image to get the resulting enhanced image.

Basic Steps for Filtering in the Frequency Domain:

1. Given an input image $f(x, y)$ of size $M \times N$.
2. Compute $F(u, v)$, the DFT of the image.
3. Multiply $F(u, v)$ by a filter function $H(u, v)$, i.e., **$G(u, v) = H(u, v)F(u, v)$**
4. Compute inverse DFT of the $G(u, v)$.
5. Obtain the real part of the result.

Step-1 Input Image An input image may be defined as a two-dimensional function, $f(x, y)$, where x and y are spatial (plane) coordinates, and the amplitude of f at any pair of coordinates (x, y) is called the intensity or grey level of the image at that point.

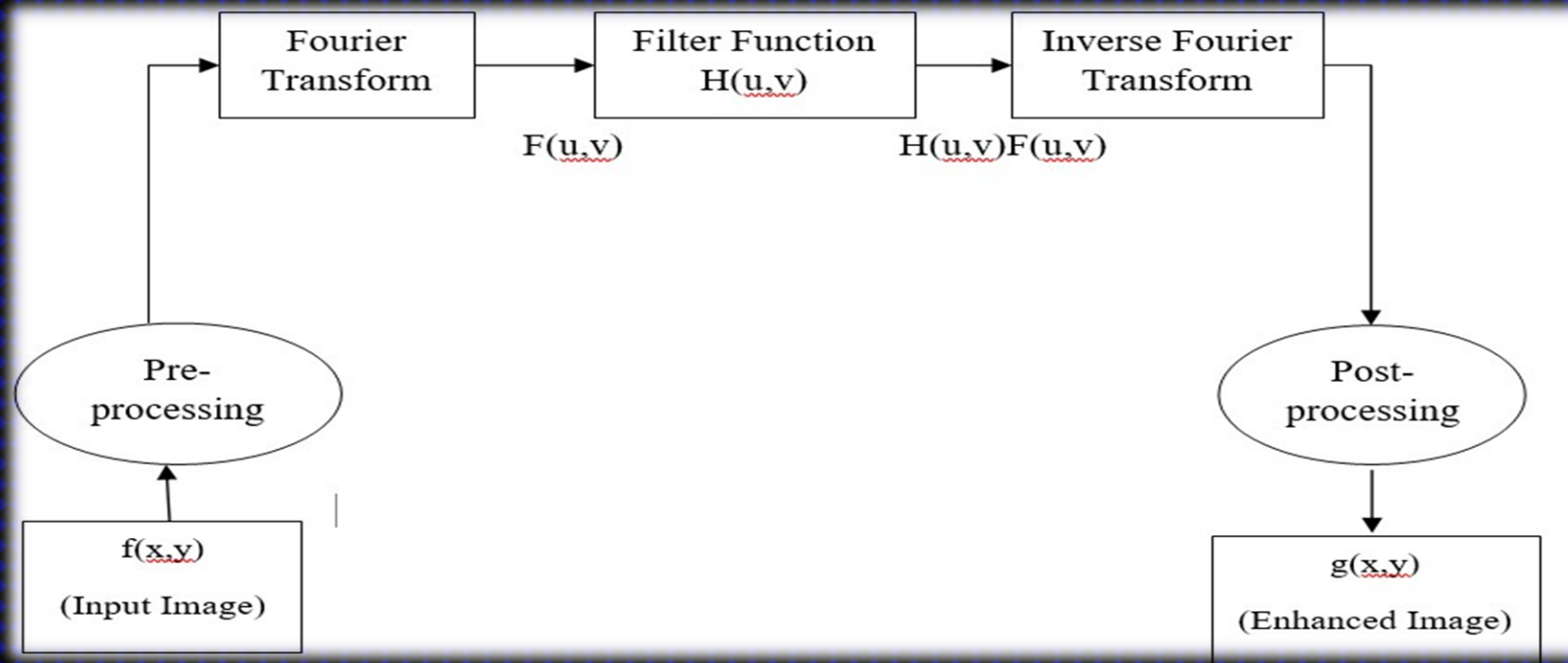


Fig. 2. Frequency Domain Filtering Operations

Step-2 Compute Fourier Transform of the input image. The image $f(x, y)$ of size $M \times N$ will be represented in the frequency domain $F(u, v)$ using Discrete Fourier Transform (DFT). The concept behind the Fourier transform is that any waveform that can be constructed using a sum of sine and cosine waves of different frequencies. The Discrete Fourier Transform (DFT) of an image takes a discrete signal and transforms it into its discrete frequency domain representation. The Fourier transform $F(u)$, of a single variable continuous function $f(x)$, is defined by:

$$F(u) = \frac{1}{M} \sum_{x=0}^{M-1} f(x) e^{-j2\pi ux / M}$$

where, u represents the frequency and x represents time/space. The exponential in the above formula can be expanded into sines and cosines with the variables u and v determining these frequencies.

Step-3

Filtering of the Fourier Transformed image. A filter is a tool designed to suppress certain frequency components of an input image and return the image in a modified format. They are used to compensate for image imperfections such as noise, and insufficient sharpness. By filter design we can create filters that pass signals with frequency components in some bands, and attenuate signals with content in other frequency bands. The general formula for filtering is given as: $G(u, v) = F(u, v) \cdot H(u, v)$ where the $H(u, v)$ is the transfer function, and $F(u, v)$ is the Fourier transform of the image function. The $G(u, v)$ is the filtered final function. In all the filters, it is important to find the right filter function $H(u, v)$ as it amplifies some frequencies and suppresses certain frequency components in an image. There are many filters that are used for blurring/smoothing, sharpening and edge detection in an image.

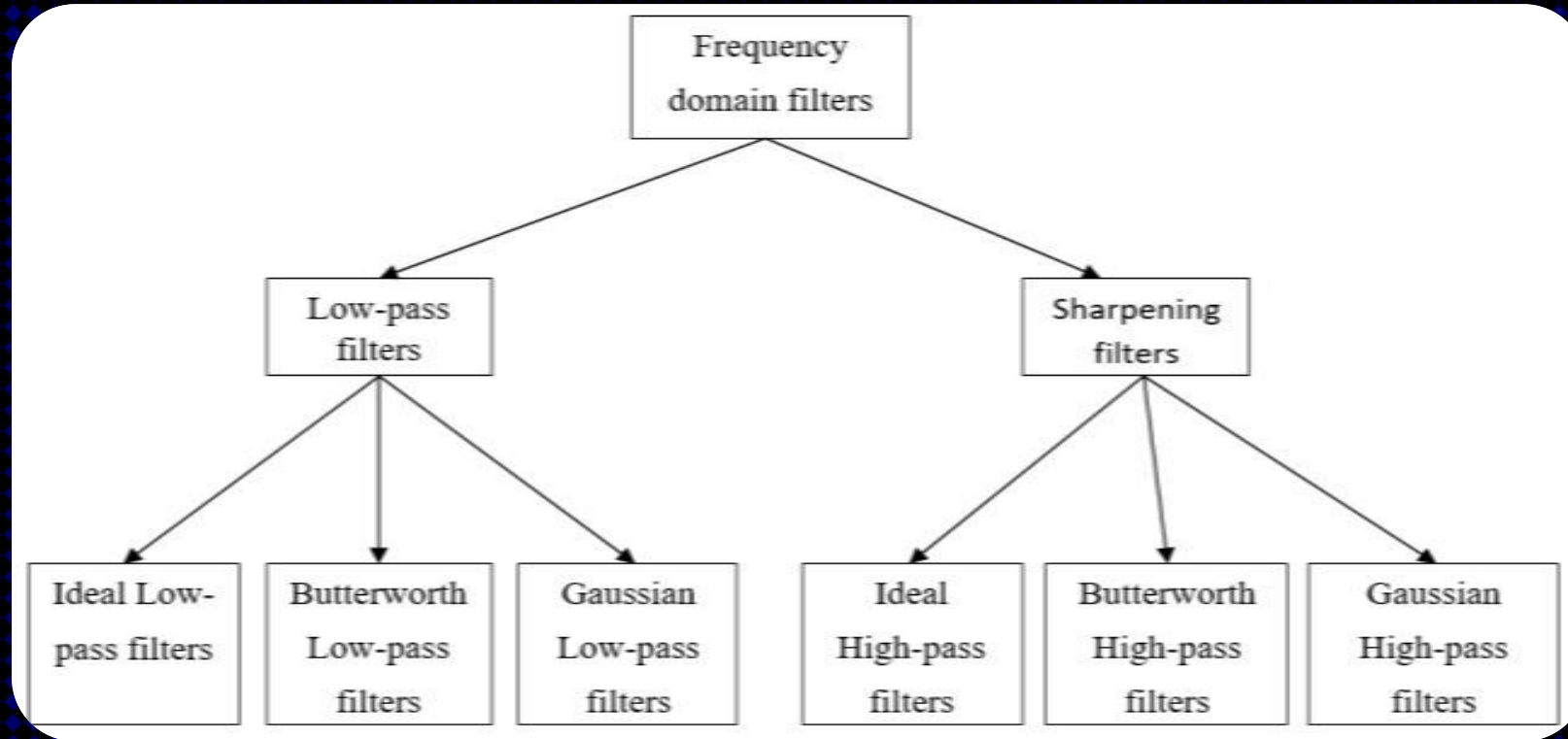


Fig. 3. Types of Frequency Domain Filters

Based on the property of using the frequency domain the image filters are broadly classified into two categories:

1. LOW-PASS FILTERS / SMOOTHING FILTERS.
2. HIGH-PASS FILTERS / SHARPENING FILTERS

Lowpass filter (smoothing)

A low pass filter is used to pass low-frequency signals.

The strength of the signal is reduced and frequencies which are passed is higher than the cut-off frequency.

The amount of strength reduced for each frequency depends on the design of the filter. Smoothing is low pass operation in the frequency domain.

Following are some lowpass filters:



1. Ideal Lowpass Filters

The ideal lowpass filter is used to cut off all the high-frequency components of Fourier transformation.

Below is the transfer function of an ideal lowpass filter.

$$H(u, v) = \begin{cases} 1 & \text{if } D(u, v) \leq D_0 \\ 0 & \text{if } D(u, v) > D_0 \end{cases}$$

$$D(u, v) = \left[\left(u - \frac{M}{2} \right)^2 + \left(v - \frac{N}{2} \right)^2 \right]^{\frac{1}{2}}$$



ideal lowpass

2. Butterworth Lowpass Filters

Butterworth Lowpass Filter is used to remove high-frequency noise with very minimal loss of signal components.

$$H(u, v) = \frac{1}{1 + \left[\frac{D(u, v)}{D_0} \right]^{2n}}$$



Butterworth Lowpass

High pass filters (sharpening)

A high pass filter is used for passing high frequencies but the strength of the frequency is lower as compared to cut off frequency. Sharpening is a high pass operation in the frequency domain. As lowpass filter, it also has standard forms such as Ideal high pass filter, Butterworth high pass filter, Gaussian high pass filter.



Original image



Gaussian highpass



Butterworth highpass

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