Laminar (viscous) flow.

Newtonian fluid, the flows can be classified as.

i) Laminar   (ii) turbulent (Turbulent)

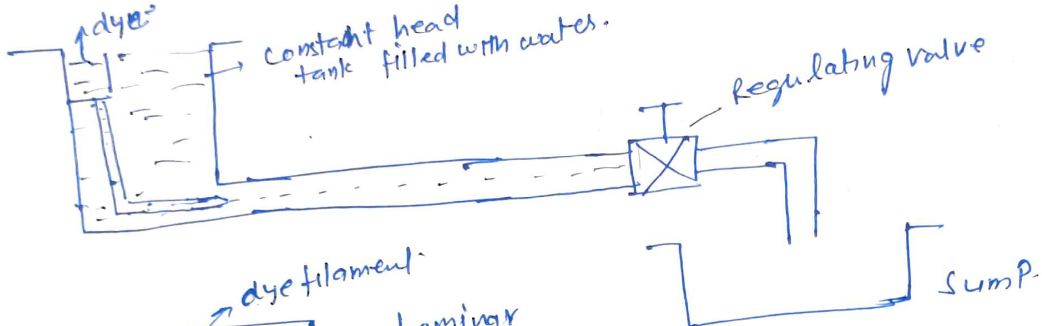
example of laminar flow-

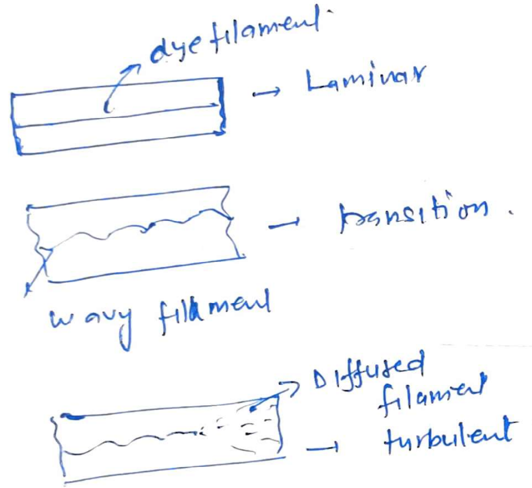
i) Movement of blood in the arteries of human body.

ii) Rise of water in plants through their roots

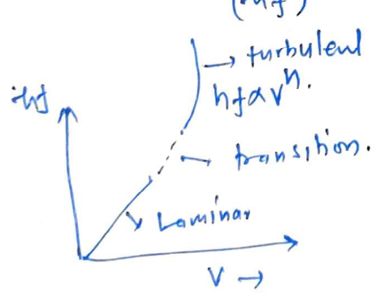
iii) underground flow.

Reynolds experiment:- (osborne Reynold's in 1883)





in this the loss of head $\propto v^n$ (h_f) [n varies 1.75 to 2]



Reynold's no ⇒ Its the ratio of inertia force to viscous force.

$$= \frac{\text{inertia force}}{\text{viscous force}}$$

$$= \frac{m \cdot a}{\mu \cdot \frac{du}{dy} \cdot A}$$

according Newton's law of viscosity

$$\tau = \mu \cdot \frac{du}{dy}$$

$\tau \times A = F$ (shear force)

$$= \frac{\rho \times \forall \times \frac{dv}{dt}}{\mu \cdot \frac{du}{dy} \times A}$$

$$= \frac{\rho \times \forall \times \frac{V}{dt}}{\mu \frac{V}{L} \times A}$$

$dy = L \Rightarrow$ characteristic length.

$$Re = \frac{\rho \underbrace{\left(\frac{\forall}{dt}\right)}_{=Q} \times V}{\mu \times \frac{V}{L} \times A} \Rightarrow \frac{\rho \times Q \times L}{\mu \times A} = \frac{\rho \times \cancel{A} \times V \times L}{\mu \times \cancel{A}} = \boxed{\frac{\rho V L}{\mu}}$$ (*)

$V \rightarrow$ mean velocity, L - characteristic length

ρ - mass density, $\mu \rightarrow$ dynamic viscosity

ν = (kinematic viscosity) = $\frac{\mu}{\rho}$

$$\boxed{Re = \frac{V \cdot L}{\nu}}$$ (*)

For pipe flow - $Re < 2000$ — Laminar (viscous flow).

$Re > 4000$ — turbulent

$2000 < Re < 4000$ — transition.

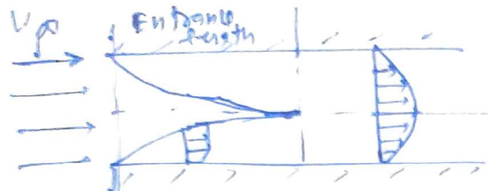
Laminar $\rightarrow$ velocity is considerably small.

$$Re = \frac{\text{inertia}}{\text{viscous}} \Rightarrow Re \propto \frac{1}{\text{viscous force}}$$

in laminar flow

Viscous flow in pipe:-

• Assumption: - Flow is steady, Flow is axisymmetric $\left(\frac{\partial u_\theta}{\partial t} = 0\right)$ fully developed.

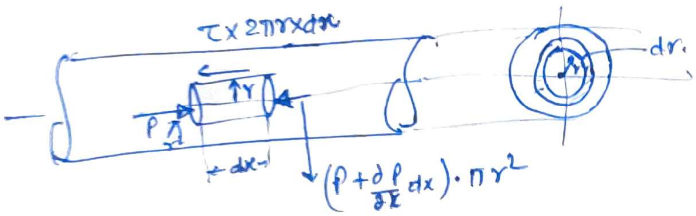


fully developed flow - velocity profile does not changes along the dir^n of flow.

* Possible force.

$$F_x = \underbrace{(F_x)_g + (F_x)_p + (F_x)_v}_{\text{Navier stokes}} + (F_x)_t + (F_x)_{com.}$$

$$F_x = (F_x)_p + (F_x)_v \rightarrow \text{Hagen Poiseuille eq}^n.$$



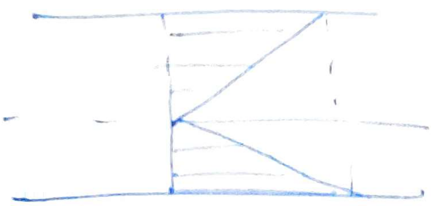
Steady = $\frac{dv}{dt} = 0$ ← Local acc^n

$$P \times \pi r^2 - \left(P + \frac{\partial P}{\partial r} \times dx\right) \cdot \pi r^2 - \tau \times 2\pi r \cdot dx = 0$$

$$P \times r - \left(P + \frac{\partial P}{\partial x} \cdot dx\right) r - 2\tau\, dx = 0$$

$$-\frac{\partial P}{\partial x} \cdot dx \times r = 2\tau\, dx$$

$$\boxed{\tau = \left(-\frac{\partial P}{\partial x}\right) \times \frac{r}{2}}$$



$\tau \propto r.$

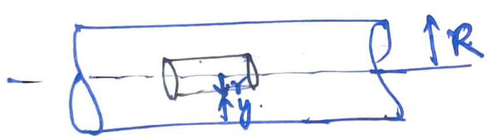
Shear stress increases linearly from zero to to the center line of the pipe (max^m at the pipe wall).

As the shear stress is zero at the center line of the pipe bernoulli's eq^n is applicable at the center line of pipe

Velocity Distribution!

Assumption – (1) The fluid follows the newton's law of viscosity

(2) There is no slip condition at the boundary.



$R = r + y.$

$y \to$ distance measured from boundary.

$$dr + dy = 0 \Rightarrow dy = -dr. \quad \text{—— (I)}$$

$$\tau = \mu \frac{du}{dy} \Rightarrow \tau = -\mu \cdot \frac{du}{dr} \quad \text{—— (II)}$$

$$\tau = -\left(\frac{\partial p}{\partial x}\right) \cdot \frac{r}{2} \quad \text{—— (III)}$$

from eq^n (II) & eq^n (III)

$$-\mu \left(\frac{du}{dr}\right) = \left(\frac{\partial p}{\partial x}\right) \cdot \frac{r}{2}.$$

$$du = \left(\frac{\partial p}{\partial x}\right) \cdot \frac{r}{2} \cdot \frac{dr}{\mu}.$$

integrating above eq^n w.r.to

$$u = \frac{1}{2}\left(\frac{\partial p}{\partial x}\right) \cdot \frac{r^2}{2} + c.$$

$r = R,\ u = 0,$ Then $C = \frac{-1}{4\mu} \left(\frac{\partial p}{\partial x}\right) \cdot R^2$

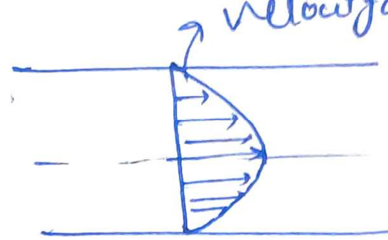
$$u = \frac{1}{4\mu}\left(\frac{\partial p}{\partial x}\right) r^2 - \frac{1}{4\mu}\left(\frac{\partial p}{\partial x}\right) \cdot R^2$$

$$u = -\frac{1}{4\mu}\left(\frac{\partial p}{\partial x}\right)\left[R^2 - r^2\right]$$ → Parabolic velocity distribution.

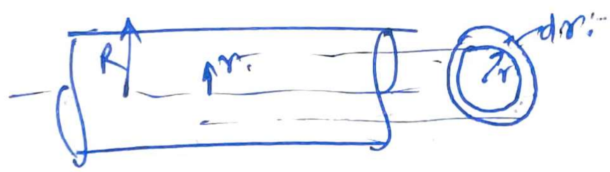
max^m velocity, at $r = 0$

$$\boxed{u_{max} = -\frac{1}{4\mu}\left(\frac{\partial p}{\partial x}\right) \cdot R^2}$$

$$\boxed{u = u_{max}\left[1 - \frac{r^2}{R^2}\right]}$$



Discharge -



discharge through elementary ring of Thickness dr.

$$dQ = 2\pi r \cdot dr \times u$$

$$= u_{max}\left[1 - \frac{r^2}{R^2}\right] \times 2\pi r \cdot dr$$

Total discharge. $Q = \int_0^R dQ$.

$$= \int_0^R (u_{max})\, 2\pi \left(r - \frac{r^3}{R^2}\right) \cdot dr$$

$$= 2\pi\, u_{max}\left[\frac{r^2}{2} - \frac{r^4}{4R^2}\right]_0^R$$

$$= 2\pi\, u_{max}\left[\frac{R^2}{2} - \frac{R^4}{4R^2}\right]$$

$$= 2\pi\, u_{max} \frac{R^2}{4}$$

$$= \pi\, u_{max} \cdot \frac{R^2}{2}$$

$$\boxed{Q = \pi\, U_{max} \cdot \frac{R^2}{2}}$$