$$Q = \pi \times \frac{1}{4\mu}\left(-\frac{\partial P}{\partial x}\right) \cdot R^2 \cdot \frac{R^2}{2}$$

$$Q = \frac{\pi}{8\mu}\left(-\frac{\partial P}{\partial x}\right) R^4$$

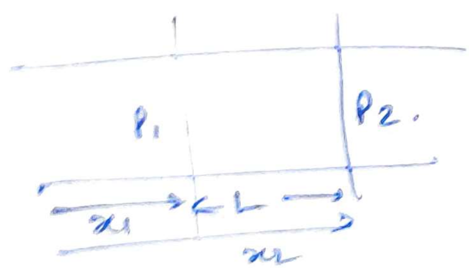
$$\boxed{Q = \frac{\pi}{128\mu}\left(-\frac{\partial P}{\partial x}\right) \cdot D^4}$$ Hagen Poiseuille eqn.

Av. velocity = It is the ratio of total discharge to total area of flow

$$\bar{u} = \frac{Q}{A} = \frac{\frac{\pi}{2} \times u_{max} R^2}{\pi R^2} = \frac{u_{max}}{2}$$

$$\boxed{\bar{u}_{av} = \frac{u_{max}}{2}}$$

Pressure drop in given length:

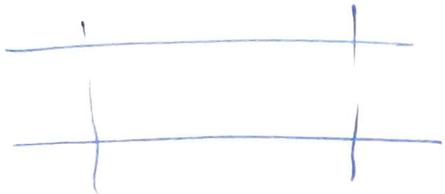


$$\bar{u}_{av} = \frac{u_{max}}{2} = \frac{-\frac{1}{4\mu} \times \left(\frac{\partial P}{\partial x}\right) \cdot R^2}{2}$$

$$-\int_{P_1}^{P_2} \partial P = \frac{8\mu \bar{u}}{R^2} \int_{x_1}^{x_2} \partial x$$

$$(P_1 - P_2) = \frac{8\mu\bar{u}}{R^2}\left[x_2 - x_1\right] \quad - (x_2 - x_1 = L)$$

$$\boxed{(P_1 - P_2) = \frac{32\mu\bar{u}}{D^2}}$$



$z_1 = z_1$

$A_1 V_1 = A_2 V_2$

$A_1 = A_2$

$V_1 = V_2 = V$

$$\frac{P_1}{\rho g} + \frac{v^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{v^2}{2g} + z_2 + h_L.$$

$$\boxed{\frac{P_1 - P_2}{\rho g} = h_L.}$$

$$P_1 - P_2 = w \times h_L$$

$$P_1 - P_2 = \rho g \times \frac{f \cdot L v^2}{2 g \cdot D}.$$

$$P_1 - P_2 = 32 \frac{\mu \bar{u} L}{D^2}$$

$$\frac{32 \mu \bar{u} L}{D^2} = \frac{\rho g \times f L u^2}{2 g D}.$$

$$\frac{64 \mu}{D} = \rho f \cdot V$$

$$f = \frac{64 \cdot \mu}{\rho D u} \quad Re$$

$$f = \frac{64}{Re} \rightarrow [\text{frictin factor}].$$

Shear velocity :- (V_*)

$$V_* = \sqrt{\frac{\tau_0}{\rho}}$$

$$\tau = -\left(\frac{\partial p}{\partial x}\right)\cdot\frac{r}{2} \Rightarrow \tau_0 = \left(\frac{-\partial p}{\partial x}\right)\cdot\frac{R}{2}.$$

$$\tau_0 = -\frac{(P_2 - P_1)}{(x_2 - x_1)} \times \frac{D}{4}$$

$$\frac{P_1 - P_2}{L} \times \frac{D}{4} = \tau_0. \qquad \left(P_1 - P_2 = \frac{\rho g f L V^2}{2gD}\right)$$

$$\frac{D}{4L} \times \frac{\rho g f L V^2}{2gD} = \tau_0.$$

$$\tau_0 = \frac{\rho f V^2}{8}$$

$$\frac{\tau_0}{\rho} = \frac{f V^2}{8}.$$

$$V_* = \sqrt{\frac{\tau_0}{\rho}} \Rightarrow \sqrt{\frac{f \cdot V^2}{8}} = \sqrt{\frac{f}{8}} \cdot V$$

$$\boxed{V_* \text{ (Shear velocity)} = \sqrt{\frac{f}{8}} \times V}$$

* The minimum value of friction factor that can occour in laminar flow through circular pipe.

$$f = \frac{64}{Re} \Rightarrow \frac{64}{2000} = 32 \times 10^{-3}$$

$$\boxed{f = .032}$$

Q. Oil flows from A to B through 100 meter long horizontal pipe (steel) of 150 mm diameter. The pressure at A is 1.08 MPa and pressure at B is 0.95 MPa. $\nu = 412.6 \times 10^{-6}\ m^2/s$, $\rho = 918\ kg/m^3$. Find the Reynold's number and discharge if the flow is laminar

(9)

Sol^n

$P_a = 1.08\ MPa$, $P_B = .95\ MPa$ $\nu = 412.6 \times 10^{-6}\ m^2/s$.

$\rho = 918\ kg/m^3$.

$$P_1 - P_2 = \frac{32\ \mu V L}{D^2}$$

$$.13 \times 10^{6} = \frac{32 \times 918 \times 412.6 \times 10^{-6} \times V \times 100}{(150 \times 10^{-3})^2}$$

$$V = 2.41\ m/s$$

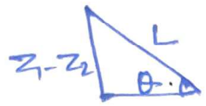
$$Q = A \cdot V = \frac{\pi}{4} \times (.15)^2 \times 2.41$$

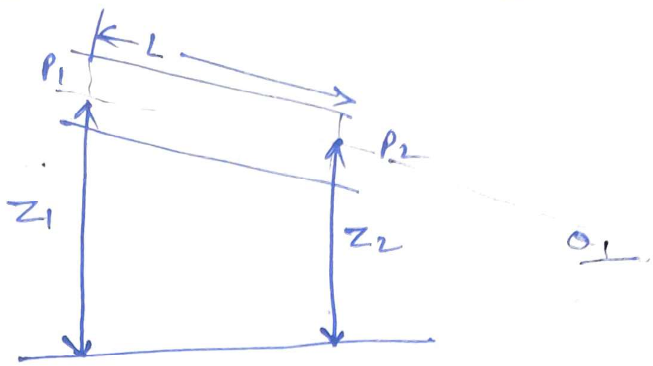
$$= .043\ m^3/s. \ (43\ Lit/s)$$

$$Re = \frac{\rho V D}{\mu} = \frac{V D}{\nu}$$

$$= \cancel{\frac{918 \times .15}{412.6 \times 10^{-6}}} = \frac{2.41 \times .15}{412.6 \times 10^{-6}} = \underline{876.}$$

Head loss in inclined pipe:-





$$\sin\theta = \frac{z_1 - z_2}{L}$$

Apply Bernoulli's eqn. between (1) & (2)

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2 + h_L$$

$$\boxed{\begin{array}{l} A_1 V_1 = A_2 V_2 \\ A_1 = A_2 \\ V_1 = V_2 \end{array}}$$

$$\frac{P_1 - P_2}{\rho g} + (z_1 - z_2) = h_L$$

(*)

$$\boxed{\frac{\Delta P}{\rho g} + L\sin\theta = h_L}$$

Q- A liquid with a specific gravity 2.8 and a viscosity 0.8 poise flows through a smooth pipe of unknown diameter, resulting a pressure drop of 800 N/m² in 2 km length of the pipe. What is the pipe diameter if the mass flow rate is 2500 kg/hr. flow is laminar.

Sol^n

$\mu = .8$ poise $= .08$ N/m², $\Delta P = 800$ N/m²

$L = 2000$ m, $m = 2500$ kg/hr $= \frac{2500}{3600} = .6944$ kg/s

$$m = \rho A \bar{u} \Rightarrow \bar{u} = \frac{m}{\rho A} = \frac{.6944}{(2.8\times10^3)\times(\frac{\pi}{4}\times D^2)} = \frac{3.158\times10^{-4}}{D^2}$$

~~Assuming~~ The flow is laminar

$$\Delta P = \frac{32\mu\bar{u}L}{D^2}$$

$$800 = 32\times.08\times\frac{3.158\times10^{-4}}{D^2}\times\frac{2000}{D^2}$$

$$\boxed{D = .212\,m}$$