

Ex → A DMS 'X' has four symbols  $x_1, x_2, x_3$  and  $x_4$  with  $P(x_1) = \frac{1}{2}, P(x_2) = \frac{1}{4}, P(x_3) = P(x_4) = \frac{1}{8}$

① Construct a Shannon-Fano code for 'X'

② Code efficiency.

Sol<sup>n</sup> →

Message	Prob. of message	Codeword for message	Length (n)
$x_1$	$\frac{1}{2}$ ] 0	0	1
$x_2$	$\frac{1}{4}$ ] ] 0	10	2
$x_3$	$\frac{1}{8}$ ] ] ] 0	110	3
$x_4$	$\frac{1}{8}$ ] ] ] ] 1	111	3

$$H = \sum_{k=1}^n P_k \log_2 \left( \frac{1}{P_k} \right)$$

$$H = \sum_{k=1}^4 P_k \log_2 \left( \frac{1}{P_k} \right)$$

$$H = \frac{1}{2} \log_2 (2) + \frac{1}{4} \log_2 (4) + \frac{1}{8} \log_2 (8) + \frac{1}{8} \log_2 (8)$$

$$H = \frac{1}{2} + \frac{2}{4} + \frac{3}{8} + \frac{3}{8}$$

$$H = 1.75 \text{ bits/symbols}$$

$$L = \sum_{k=1}^m p_k \eta_k$$

$$L = \frac{1}{2} \times 1 + \frac{1}{4} \times 2 + \frac{1}{8} \times 3 + \frac{1}{8} \times 3$$

$$L = 1.75$$

$$\eta = \frac{H(x)}{L} = \frac{1.75}{1.75}$$

$$\boxed{\eta = 100\%}$$

Ans