PSD of NRZ unipolar Line coding technique →

Power spectral density as per Winer – Khintchine relationship

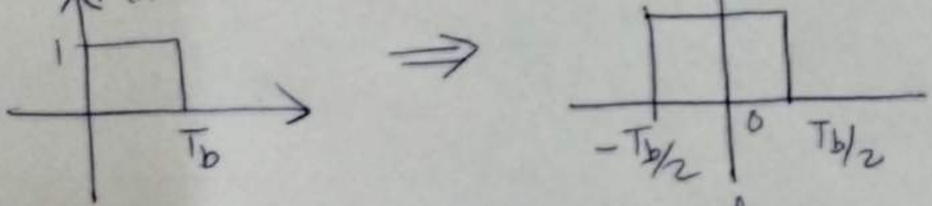
$$P(f) = \frac{1}{T_b} |X(f)|^2 \sum_{n=\infty}^{\infty} R_A(n)\, e^{-j2\pi f n T_b} \rightarrow (1)$$

wher $R_A(n) \rightarrow$ auto correlation of unipolar

$X(f) \rightarrow$ Fourier transform of NRZ pulse

So to calculate PSD we have to find out the $X(f)$ of NRZ pulse and auto correlation of unipolar.

Step I →



$$X(f) = \int_{\infty}^{\infty} x(t)\, e^{-j2\pi f t}\, dt$$

Apply fourier transform

$$X(f) = \int_{-T_b/2}^{T_b/2} 1\, e^{j2\pi f t}\, dt$$

$$X(f) = \left[\frac{e^{-j2\pi f t}}{-j2\pi f}\right]_{T_b/2}^{-T_b/2}$$

$$= \frac{e^{j2\pi f T_b} - e^{-j2\pi f T_b}}{2j\pi f}$$

$$= \frac{\sin(\pi f T_b)}{\pi f T_b} \times T_b$$

$\left\{ \text{Sin}\theta_c = \frac{\text{Si}\,\pi\theta}{\pi\theta} \right.$

$$\boxed{X(f) = T_b \, \text{Sinc}(f T_b)} \rightarrow (2)$$

Step IInd $\rightarrow$ For unipolar format ~~amplitude~~ :

$$A_L = \begin{cases} a & , \text{Symbol } 1 \\ 0 & , \text{Symbol } 0 \end{cases}$$

For a large binary seqn

$$P[A_L = a] = P[A_L = 0] = 1/2$$

Auto correlation $\rightarrow$

$$R_A[n] = E[A_L \; A_{L-n}]$$

if $n = 0$

$$R_A[0] = E[A_L^2]$$

A_L	A_L	A_L^2	Probability
0	0	0	1/2
a	a	a^2	1/2

$$E[A_L^2] = R_A[0] = \sum A_L^2 \, P[\cancel{x}]$$

$$E[A_L^2] = R_A[0] = a^2 \, P[A_L = a] + 0 \, P[A_L = 0]$$

$$R_A[0] = a^2/2$$

for discrete funcn

$$E[X^2] = \sum x^2 P(x)$$

If $n \neq 0$

$R_A[n] = E[A_L \, A_{L-n}]$

A_L	A_{L-n}	$A_L A_{L-n}$	P
0	0	0	1/4
0	a	0	1/4
a	0	0	1/4
a	a	a^2	1/4

$$E[A_L \, A_{L-n}] = R_A[n]$$

$$= \sum A_L^2 \, P(X=x)$$

$$= \frac{a^2}{4}$$

Auto correlation $\rightarrow$

$$R_A(n) = \begin{cases} a^2/2 & , n=0 \\ a^2/4 & , n \neq 0 \end{cases} \rightarrow (3)$$

from eqn (1) $\rightarrow$

$$P(f) = \frac{1}{T_b} |X(f)|^2 \sum_{n=-\infty}^{\infty} R_A(n) \, e^{-j2\pi f n T_b}$$

put the value of $X(f)$ & $R_A(n)$ from eqn (2) & (3)

to ~~eq~~ above eqn we get

$$P(f) = \frac{1}{T_b} \times T_b^2 \text{Sin}^2\text{c}(fT_b) \left\{ \frac{a^2}{2} + \sum_{\substack{n=-\infty \\ n\neq 0}}^{\infty} \frac{a^2}{4} e^{-j2\pi f n T_b} \right\}$$

$$P(f) = \frac{a^2}{2} T_b \text{Sin}^2\text{c}(fT_b) + \frac{a^2}{4} T_b \text{Sin}^2\text{c}(fT_b) \left\{ \sum_{n=\infty}^{\infty} e^{-j2\pi f n T_b} - 1 \right\}$$

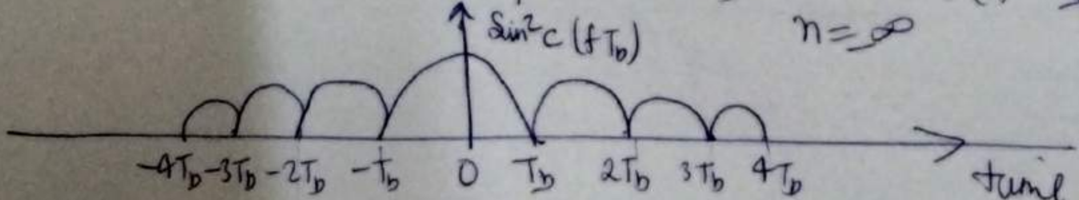
$$P(f) = \frac{a^2 T_b}{4} \text{Sin}^2\text{c}(fT_b) + \frac{a^2}{4} T_b \text{Sin}^2\text{c}(fT_b) \sum_{n=\infty}^{\infty} e^{-j2\pi f n T_b}$$

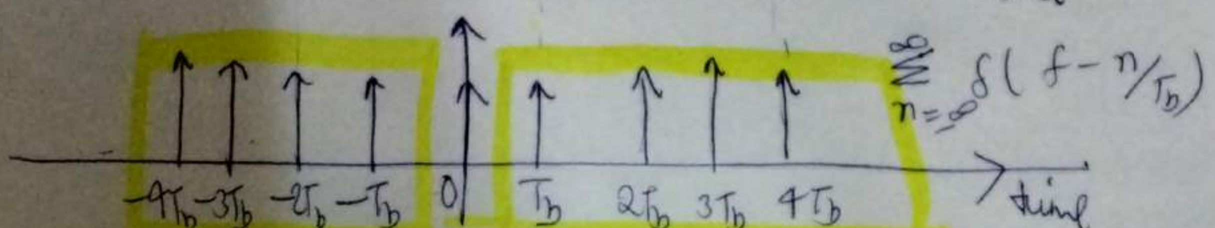
By Poission formula $\rightarrow \sum_{n=\infty}^{\infty} e^{-j2\pi f n T_b} = \frac{1}{T_b} \sum_{m=\infty}^{\infty} \delta\left(f - \frac{m}{T_b}\right)$

PSD will be $\rightarrow$

$$P(f) = \frac{a^2}{4} T_b \text{Sin}^2\text{c}(fT_b) + \frac{a^2 T_b}{4} \left[\frac{\text{Sin}^2\text{c}(fT_b)}{T_b} \sum_{n=-\infty}^{\infty} \delta(f - m/T_b) \right)$$

$$= \frac{a^2}{4} T_b \text{Sin}^2\text{c}(fT_b) + \frac{a^2}{4} \text{Sin}^2\text{c}(fT_b) \sum_{n=\infty}^{\infty} \delta\left(f - \frac{m}{T_b}\right)$$





$$\boxed{P(f) = \frac{a^2}{4} T_b \, \mathrm{sinc}^2(f T_b) + \frac{a^2}{4} \delta(f)}$$

DC component (This term will lead to distortion ~~sa~~ in signal