Connection of graphical method & Simplex method

See Ex1 (graphical method).

LPP

$\max z = 40x + 80y$

subject to

$2x + 3y \le 48$

$x \le 15$

$y \le 10$

$x, y \ge 0$

standard LPP

$\max z = 40x + 80y + 0s_1 + 0s_2 + 0s_3$

subject to

$2x + 3y + s_1 = 48$

$x + s_2 = 15$

$y + s_3 = 10$

$x, y, s_1, s_2, s_3 \ge 0$

(0,0)

		C_j	40	80	0	0	0	
C_B	BV	X_B	x	y	s_1	s_2	s_3	min ratio (X_B/y)
0	s_1	48	2	3	1	0	0	$48/3 = 16$
0	s_2	15	1	0	0	1	0	—
0	s_3	10	0	$\boxed{1}$	0	0	1	$10/1 = 10$
		$C_j - Z_j$	40	80	0	0	0	

x y (0,9)

		C_j	40	80	0	0	0	
C_B	BV	X_B	x	y	s_1	s_2	s_3	min ratio (X_B/x)
0	s_1	18	$\boxed{2}$	0	1	0	-3	$18/2 = 9$
0	s_2	15	1	0	0	1	0	$15/1 = 15$
80	y	10	0	1	0	0	1	—
		$C_j - Z_j$	40	0	0	0	-80	

$R_1 - 3R_3$

x y (9,10)

		C_j	40	80	0	0	0	
C_B	BV	X_B	x	y	s_1	s_2	s_3	min Ratio
40	x	9	1	0	$1/2$	0	$-3/2$	
0	s_2	6	0	0	$-1/2$	1	$3/2$	
80	y	10	0	1	0	0	1	
		$C_j - Z_j$	0	0	-20	0	-20	

$R_1/2$,

$R_2 - R_1$

All $C_j - Z_j \le 0$ ∴ optimum solution is reached

$\boxed{x = 9 \quad y = 10}$ $z_{max} = 9(40) + 80(10) = 1160$. ↳ Ans

So, we need not test at other extreme points in feasible region. Also, see the sequence:

$(0,0) \to (0,9) \to (9,10)$.

Ex Solve the given LPP using simplex method:

$$\max z = 10x_1 + x_2 + 2x_3$$

subject to

$$12x_1 + 3x_2 - 6x_3 + 3x_4 = 6$$
$$16x_1 + x_2 - 6x_3 \le 6$$
$$3x_1 - x_2 - x_3 \le 1$$
$$x_1, x_2, x_3 \ge 0$$

Solution Note! x_4 variable is in 1st constraint only and it does not make any contribution in objective function.
So, if we divide 1st eqn by 3, coeff. of x_4 will be one and we can treat it as slack variable.

Standard LPP

$$\max z = 10x_1 + x_2 + 2x_3 + 0x_4 + 0x_5 + 0x_6$$

subject to

$$12x_1 + 3x_2 - 6x_3 + 3x_4 = 6 \Rightarrow 4x_1 + x_2 - 2x_3 + x_4 = 2$$
$$16x_1 + x_2 - 6x_3 + x_5 = 6$$
$$3x_1 - x_2 - x_3 + x_6 = 1$$
$$x_i \ge 0 \quad 1 \le i \le 6$$

		C_j	10	1	2	0	0	0		
C_B	BV	X_B	x_1	x_2	x_3	x_4	x_5	x_6	min Ratio (X_B/x_1)	
0	x_4	2	4	1	-2	1	0	0	$2/4 = 0.5$	
0	x_5	6	16	1	-6	0	1	0	$16/6 = 2.6$	
0	x_6	1	$\boxed{3}$	-1	-1	0	0	1	$1/3 = 0.3$ →	
		$C_j - Z_j$	10↑	1	2	0	0	0	min Ratio (X_B/x_3)	
0	x_4	$2/3$	0	$7/3$	$-2/3$	1	1	$-4/3$	-	$R_3/3$
0	x_5	$2/3$	0	$19/3$	$-2/3$	0	1	$-16/3$	-	$R_2 - 16R_3$
10	x_1	$1/3$	1	$-1/3$	$-1/3$	0	0	$1/3$	-	$R_1 - 4R_3$
		$C_j - Z_j$	0	$13/3$	$16/3$↑	0	0	$-10/3$		

Here x_3 ↑. But all entries in x_3 column are -ive. So outgoing variable cannot be determined. Hence, unbounded solution.

Note: $c_j - z_j > 0$. $\Rightarrow$ If we increase x_j variable by one unit profit or value of z increases by by $c_j - z_j$ units. Here $c_2 - z_2$ & $c_3 - z_3$ are both positive. Hence as we go on increasing value of x_2 & x_3 the value of z will go on increasing by $((c_2 - z_2)x_2 + (c_3 - z_3)x_3$ units. Hence unbounded.

Simplex Method (Minimization Case):

If LPP is of the form $\min z = cx$

subject to $AX \le b$, $x \ge 0$.

then, method is exactly same as maximization case. The ~~only~~ difference is (1) for entering variable we take largest -ive value of $c_j - z_j$

(2) optimality is reached when all $c_j - z_j \ge 0$.

Example Solve using simplex method:

$\min z = x_1 - 3x_2 + 2x_3$

subject to

$3x_1 - x_2 + 3x_3 \le 7$

$-2x_1 + 4x_2 \le 12$

$-4x_1 + 3x_2 + 8x_3 \le 10$

$x_1, x_2, x_3 \ge 0$

Standard form

$\min z = x_1 - 3x_2 + 2x_3 + 0s_1 + 0s_2 + 0s_3$

subject to

$3x_1 - x_2 + 3x_3 + s_1 = 7$

$-4x_1 + 3x_2 + 8x_3 + s_3 = 10$

$-2x_1 + 4x_2 + s_2 = 12$

$x_1, x_2, x_3, s_1, s_2, s_3 \ge 0$

		c_j	1	-3	2	0	0	0	
C_B	BV	X_B	x_1	x_2	x_3	s_1	s_2	s_3	min ratio X_B/x_2
0	s_1	7	3	-1	3	1	0	0	–
0	s_2	12	-2	$\boxed{4}$	0	0	1	0	$12/4 = 3$ →
0	s_3	10	-4	3	8	0	0	1	$10/3 = 3.33$
		$c_j - z_j$	1	-3 ↑	2	0	0	0	X_B/x_1
0	s_1	10	$\boxed{5/2}$	0	3	1	1/4	0	4 →
-3	x_2	3	-1/2	1	0	0	1/4	0	–
0	s_3	1	-5/2	0	8	0	-3/4	1	–
		$c_j - z_j$	-1/2 ↑	0	2	0	3/4	0	
1	x_1	4	1	0	6/5	2/5	1/10	0	
-3	x_2	5	0	1	3/5	1/5	3/10	0	
0	s_3	11	0	0	11	1	-1/2	1	
		$c_j - z_j$	0	0	13/5	1/5	8/10	0	

Since all $c_j - z_j \geq 0$ (minimization problem) optimality is reached. The optimal solution is

$x_1 = 4$ $x_2 = 5$. $x_3 = 0$ $z_{min} = -11$. — Ans

In case you have a minimization problem in which constraints are inq in the form $AX \geq b$ or $AX = b$ or a combination of these then above method cannot be applied. For that we need artificial variables. These do not represent any quantity and have no physical meaning. They do not appear in final solution. This is done by assigning very large cost M to them in objective function (in minimization case) and $-M$ in maximization problem. There are two methods:

(1) Big-M method or Method of penalities

(2) Two phase Method

Big-M method:

Step 1: Express problem in standard form by introducing surplus and artificial variable and accordingly writing the objective function

Step 2: Solve the problem using simplex method. One of the three cases may arise:

(1) No artificial variable appears in basic solution and optimality is satisfied. Hence optimal solution is obtained

(2) If there is atleast one artificial variable in basic ~~solution~~ variable with zero value & optimality condition is satisfied, then the current solution is <u>degenerate solution</u>

(3) If atleast one ~~b~~ artificial variable is a basic variable and has positive value and optimality condition is satisfied, then problem has <u>infeasible solution</u> 'or' feasible solution D.N.E (why??)

<u>Note:</u> Artificial variables are used to obtain only initial solution. In simplex table once an artificial variable leaves the basis, drop the artificial variable and all the entries in that column from the next simplex table.

<u>Ex 1</u> Solve LPP ~~min z = 2x_1+3x_2~~

min $z = 2x_1 + 3x_2$

subject to

$x_1 + x_2 \geq 5$

$x_1 + 2x_2 \geq 6$

$x_1, x_2 \geq 0$

<u>standard form</u>

min $z = 2x_1 + 3x_2 + 0s_1 + 0s_2 + Ma_1 + Ma_2$

subject

$x_1 + x_2 - s_1 + a_1 = 5$

$x_1 + 2x_2 - s_2 + a_2 = 6$

$x_1, x_2, s_1, s_2, a_1, a_2 \geq 0$

$M > 0$ (large number)

		c_j	2	3	0	0	+M	+M	min ratio (X_B/x_2)
C_B	BV	X_B	x_1	x_2	s_1	s_2	a_1	a_2	
+M	a_1	5	1	1	-1	0	1	0	5/1 = 5
+M	a_2	6	1	$\boxed{2}$	0	-1	0	1	6/2 = 3 →
		$c_j - z_j$	2-2M	3-3M ↑	+M	+M	0	0	min ratio X_B/x_1
+M	a_1	2	$\boxed{1/2}$	0	-1	1/2	1	~~-1/2~~	4 →
3	x_2	3	1/2	1	0	-1/2	0	~~1/2~~	6
		$c_j - z_j$	$\left(\frac{-M+1}{2}\right)$ ↑	0	+M	$(3-M)/2$	0		

↳ a_2 leaves

∴ we remove column corresponding to a_2

		c_j	2	3	0	0	
C_B	BV	X_B	x_1	x_2	s_1	s_2	min ratio
2	x_1	4	1	0	-2	1	
3	x_2	1	0	1	1	-1	
		$c_j - z_j$	0	0	1	1	

a_1 column is excluded as a_1 left in last iteration

• Since all $c_j - z_j \geq 0$, hence optimality is reached. Optimal solution is

$x_1 = 4 \quad x_2 = 1 \quad z_{min} = 2(4) + 3(1) = 11.$ — Ans