Ex 2 Solve the following LPP:

$\min z = 2x_1 + 5x_2 + 3x_3$

subject to

$3x_1 + x_3 \geq 10$

$5x_1 + x_2 + 2x_3 \geq 15$

$x_1 + 2x_2 \geq 8$

$x_1, x_2, x_3 \geq 0$

Standard LPP

$\min z = 2x_1 + 5x_2 + 3x_3 + 0s_1 + 0s_2 + 0s_3 + Ma_1 + Ma_2 + Ma_3$

subject to

$3x_1 + x_3 - s_1 + a_1 = 10$

$5x_1 + x_2 + 2x_3 - s_2 + a_2 = 15$

$x_1 + 2x_2 - s_3 + a_3 = 8$

$x_1, x_2, x_3, s_1, s_2, s_3 \geq 0$

$a_1, a_2, a_3 \geq 0$

Soln 2:

Initial BFS is $a_1 = 10$, $a_2 = 15$, $a_3 = 8$

		c_j	2	5	3	0	0	0	M	M	M	X_B/x_1
C_B	BV	X_B	x_1	x_2	x_3	s_1	s_2	s_3	a_1	a_2	a_3	Min Ratio
M	a_1	10	3	0	1	−1	0	0	1	0	0	10/3
M	a_2	15	[5]	1	2	0	−1	0	0	1	0	3 →
M	a_3	8	1 ↑	2	0	0	0	−1	0	0	1	8
						M	M	M	0	0	0	
		$c_j - z_j$	$2-9M$	$5-3M$	$3-3M$							

C_B	BV	X_B	x_1	x_2	x_3	s_1	s_2	s_3	a_1	a_3	Min Ratio
M	a_1	1	0	$-3/5$	$-1/5$	−1	$3/5$	0	1	0	—
2	x_1	3	1	$1/5$	$2/5$	0	$-1/5$	0	0	0	15
M	a_3	5	0	[$9/5$] ↑	$-2/5$	0	$1/5$	−1	0	1	$\frac{25}{9}$ →
		$c_j - z_j$	0	$(23-6M)/5$	$\left(\frac{1+3M}{5}\right)$	M	$\left(\frac{2-4M}{5}\right)$	M	0	0	

Note: a_2 left in 1st table, so column a_2 is removed in 2nd table.

		c_j	2	5	3	0	0	0	M	
C_B	BV	X_B	x_1	x_2	x_3	s_1	s_2	s_3	a_1	Min Ratio
M	a_1	$8/3$	0	0	$-1/3$	−1	[$2/3$]	$-1/3$	1	4 →
2	x_1	$22/9$	1	0	$4/9$	0	$-2/9$	$1/9$	0	—
5	x_2	$25/9$	0	1	$-2/9$	0	$1/9$ ↑	$-\frac{5}{9}$	0	25
		$c_j - z_j$	0	0	$\frac{29}{9} + \frac{M}{3}$	M	$-\frac{1}{9} - \frac{2M}{3}$	$\frac{M}{3} + \frac{23}{9}$	0	

Note: a_3 left in 2nd table, so, column a_3 is removed in 3rd table.

		c_j	2	5	3	0	0	0	
C_B	BV	X_B	x_1	x_2	x_3	s_1	s_2	s_3	min ratio
0	s_2	4	0	0	$-1/2$	$-3/2$	1	$-1/2$	–
2	x_1	10/3	1	0	1/3	$-1/3$	0	0	–
5	x_2	7/3	0	1	$-1/6$	**1/6**	0	$-1/2$	14 →
		$c_j - z_j$	0	0	3/2	$-1/6$ ↑	0	5/2	

		c_j	2	5	3	0	0	0	
C_B	BV	X_B	x_1	x_2	x_3	s_1	s_2	s_3	min ratio
0	s_2	11	0	9	-2	0	1	-5	
2	x_1	8	1	2	0	0	0	-1	
0	s_1	14	0	6	-1	1	0	-3	
		$c_j - z_j$	0	1	3	0	0	2	

All $c_j - z_j \geq 0$, so optimality is reached.

Optimal solution is:

$x_1 = 8$, $x_2 = 0$, $x_3 = 0$. $z_{min} = 16$ — **Ans**

Note: This example shows that a slack/surplus variable can replace a decision variable in basis.

Ex3. Solve the LPP:

$\max z = x_1 + 2x_2 + 3x_3$

subject to

$x_1 - x_2 + x_3 \geq 4$

$x_1 + x_2 + 2x_3 \leq 8$

$x_1 - x_3 \geq 2$

$x_1, x_2, x_3 \geq 0$

Standard LPP

$\max z = x_1 + 2x_2 + 3x_3 + 0s_1 + 0s_2 + 0s_3 - Ma_1 - Ma_2$

subject to

$x_1 - x_2 + x_3 - s_1 + a_1 = 4$

$x_1 + x_2 + 2x_3 + s_2 = 8$

$x_1 - x_3 - s_3 + a_2 = 2$

$x_1, x_2, x_3, s_1, s_2, s_3 \geq 0$

$a_1, a_2 \geq 0$

soln 3

		C_j	1	2	3	0	0	0	-M	-M	
C_B	BV	X_B	x_1	x_2	x_3	s_1	s_2	s_3	a_1	a_2	X_B/x_1 min ratio
-M	a_1	4	1	-1	1	-1	0	0	1	0	4
0	s_2	8	1	1	2	0	1	0	0	0	8
-M	a_2	2	[1] ↑	0	-1	0	0	-1	0	1	2 →
		$C_j - Z_j$	1+2M	2-M	3	-M	0	-M	0	0	

		C_j	1	2	3	0	0	0	-M	
C_B	BV	X_B	x_1	x_2	x_3	s_1	s_2	s_3	a_1	min Ratio X_B/x_3
-M	a_1	2	0	-1	[2]	-1	0	1	1	1 →
0	s_2	6	0	1	3	0	1	1	0	2
1	x_1	2	1	0	-1 ↑	0	0	-1	0	—
		$C_j - Z_j$	0	2-M	4+2M	-M	0	M+1	0	

		C_j	1	2	3	0	0	0	
C_B	BV	X_B	x_1	x_2	x_3	s_1	s_2	s_3	min ratio
3	x_3	1	0	-1/2	1	-1/2	0	1/2	—
0	s_2	3	0	[5/2]	0	3/2	1	-1/2	6/5 →
1	x_1	3	1	-1/2 ↑	0	-1/2	0	-1/2	—
		$C_j - Z_j$	0	4	0	2	0	-1	

		C_j	1	2	3	0	0	0	
C_B	BV	X_B	x_1	x_2	x_3	s_1	s_2	s_3	min Ratio
3	x_3	8/5	0	0	1	-1/5	1/5	2/5	
2	x_2	6/5	0	1	0	3/5	2/5	-1/5	
1	x_1	18/5	1	0	0	-1/5	1/5	-3/5	
		$C_j - Z_j$	0	0	0	-2/5	-8/5	-1/5	

Since all $C_j - Z_j \le 0$ (maximization) optimality is reached. Optimal soln is

$x_1 = \frac{18}{5}$ $x_2 = 6/5$ $x_3 = 8/5$ $z_{max} = \frac{54}{5}$ Ans

Ex4! Solve LPP!

max $z = 60x_1 + 40x_2$

subject to

$4x_1 + 2x_2 \le 100$

$4x_1 + 6x_2 \le 180$

$x_1 + x_2 = 40$

$x_1 \le 20$

$x_2 \ge 10$

$x_1, x_2 \ge 0$

Standard LPP

max $z = 60x_1 + 40x_2 + 0s_1 + 0s_2 - Ma_1 + 0s_3 - Ma_2$

subject to

$4x_1 + 2x_2 + s_1 = 100$

$4x_1 + 6x_2 + s_2 = 180$

$x_1 + x_2 + a_1 = 40$

$x_1 + s_3 = 20$

$x_2 - s_4 + a_2 = 10$

$x_1, x_2, s_1, s_2, s_3, a_1, a_2 \ge 0$

		c_j	60	40	0	0	0	−M	−M	
CB	BV	X_B	x_1	x_2	s_1	s_2	s_3	a_1	a_2	min Ratio
0	s_1	100	4	2	1	0	0	0	0	50
0	s_2	180	4	6	0	1	0	0	0	30
−M	a_1	40	1	1	0	0	0	1	0	40
0	s_3	20	1	0	0	0	1	0	0	—
−M	a_2	10	0	[1] ↑	0	0	0	0	1	10 →
	$c_j - z_j$		60+M	40+2M	0	0	0	0	0	

CB	BV	X_B	x_1	x_2	s_1	s_2	s_3	a_1	min ratio
0	s_1	80	[4]	0	1	0	0	0	20 →
0	s_2	120	4	0	0	1	0	−1	30
−M	a_1	30	1	0	0	0	1	0	30
0	s_3	20	1	0	0	0	0	0	20
40	x_2	10	0 ↑	1	0	0	0	a	—
	$c_j - z_j$		60+M	0	0	0	0	0	

CB	BV	X_B	x_1	x_2	s_1	s_2	s_3	a_1	min Ratio
60	x_1	20	1	0	1/4	0	0	0	
0	s_2	40	0	0	−1	1	0	0	
−M	a_1	10	0	0	−1/4	0	0	1	
0	s_3	0	0	0	−1/4	0	1	0	
40	x_2	10	0	1	0	0	0	0	
	$c_j - z_j$		0	0	$\frac{-60-M}{4}$	0	0	0	

Since all $c_j - z_j \le 0$ ∴ optimality is reached. But there is a artificial variable (a_1) in basis with positive value. Hence solution is infeasible.

Ex5 In above problem change the R.H.S of constraint 3 i.e, instead of $b_3 = 40$ take $b_3 = 30$. Recompute the problem and check the result.

Soln5 In the last table we will find the artificial variable although present in basis will have value zero. Hence, solution will be feasible but degenerate.

$x_1 = 20, x_2 = 0 \quad z_{max} = 1200$.

Big-M method:

Ex Solve using Big-M method

$\max z = 3x_1 + 2x_2$

subject to

$2x_1 + x_2 \leq 2$

$3x_1 + 4x_2 \geq 12$

$x_1, x_2 \geq 0$

soln

C_B	BV	X_B	x_1	x_2	s_1	s_2	a_1	min ratio
0	s_1	2	2	[1]	1	0	0	2 →
-M	a_1	12	3	4 ↑	0	-1	1	3
		$c_j - z_j$	3+3M	2+4M	0	-M	0	
2	x_2	2	2	1	1	0	0	
-M	a_1	4	-1	0	-4	-1	1	
		$c_j - z_j$	-1-M	0	-2-4M	-M	0	

All $c_j - z_j \leq 0$ But artificial variable is present in basis with +ive value ($a_1 = 4$)

Hence solution is infeasible.