Ex

max $z = 3x_1 + 5x_2$

subject to

$3x_1 + 2x_2 = 18$

$x_1 \le 4$

$x_2 \le 6$

$x_1, x_2 \ge 0$

Standard LPP

max $z = 3x_1 + 5x_2 - Ma_1 + 0s_1 + 0s_2$

subject to

$3x_1 + 2x_2 + a_1 = 18$

$x_1 + s_1 = 4$

$x_2 + s_2 = 6$

$x_1, x_2 \ge 0$, $s_1, s_2 \ge 0$, $a_1 \ge 0$

		C_j	3	5	$-M$	0	0	
C_B	BV	X_B	x_1	x_2	a_1	s_1	s_2	min ratio X_B/x_1
$-M$	a_1	18	3	2	1	0	0	6
0	s_1	4	$\boxed{1}$	0	0	1	0	4 →
0	s_2	6	0 ↑	1	0	0	1	—
		$C_j - Z_j$	$3+3M$	$5+2M$	0	0	0	min ratio X_B/x_2
$-M$	a_1	6	0	$\boxed{2}$	1	-3	0	3 →
3	x_1	4	1	0	0	1	0	—
0	s_2	6	0	1 ↑	0	0	1	6
		$C_j - Z_j$	0	$5+2M$	0	$-3M$	0	X_B/s_1
5	x_2	3	0	1		$-3/2$	0	—
3	x_1	4	1	0		1	0	4
0	s_2	3	0	0		$\boxed{3/2}$	1	2 →
		$C_j - Z_j$	0	0		↑ $+9/2$	0	
5	x_2	6	0	1		0	1	
3	x_1	2	1	0		0	$-2/3$	
0	s_1	2	0	0		1	$2/3$	
		$C_j - Z_j$	0	0		0	-3	

Since all $C_j - Z_j \le 0$ ∴ optimal solution is reached

$x_1 = 2 \quad x_2 = 6 \quad z_{max} = 3(2) + 5(6) = 36$ — Ans

* Here artificial variable is not present in solution

Standard LPP

Ex. $\max z = x_1 + x_2$

subject to

$x_1 \geq 5$

$x_2 \leq 10$

$x_1, x_2 \geq 0$

$\max z = x_1 + x_2 - 0s_1 - Ma_1 + 0s_2$

subject to

$x_1 - s_1 + a_1 = 5$

$x_2 + s_2 = 10$

$x_1, x_2, s_1, s_2, a_1 \geq 0$

		C_j	1	1	0	$-M$	0	
C_B	BV	X_B	x_1	x_2	s_1	a_1	s_2	min ratio (x_B/x_1)
$-M$	a_1	5	$\boxed{1}$	0	-1	1	0	5 →
0	s_2	10	0	1	0	0	1	–
		$C_j - Z_j$	$1+M$	0	$-M$	0	0	

Entering is x_1 & leaving is a_1

		C_j	1	1	0	0	
C_B	BV	X_B	x_1	x_2	s_1	s_2	min ratio
1	x_1	5	1	0	-1	0	–
0	s_2	10	0	$\boxed{1}$	0	1	10 →
		$C_j - Z_j$	0	1	1	0	
1	x_1	5	1	0	-1	0	–
1	x_2	10	0	1	0	1	–
		$C_j - Z_j$	0	0	1	-1	

Entering is s_1 but outgoing variable cannot be found as all entries in s_1 column are ≤ 0

∴ Unbounded solution.

Standard LPP

Ex $\min z = 3x_1 + 5x_2$

s.t. $x_1 + x_2 \leq 4$

$2x_1 + 2x_2 \geq 32$

$x_1, x_2 \geq 0$

$\min z = 3x_1 + 5x_2 + 0s_1 + 0s_2 + Ma_1$

s.t. $x_1 + x_2 + s_1 = 4$

$2x_1 + 2x_2 - s_2 + a_1 = 32$

$x_1, x_2, a_1, s_1, s_2 \geq 0$

		c_j	3	5	0	0	M	
C_B	BV	X_B	x_1	x_2	s_1	s_2	a_1	min ratio
0	s_1	4	$\boxed{1}$	1	1	0	0	4
M	a_1	32	2	2	0	-1	1	16
		$c_j - z_j$	$3-2M$	$5-2M$	0	M	0	
3	x_1	4	1	1	1	0	0	
M	a_1	24	0	0	-2	-1	1	
		$c_j - z_j$	0	2	$-3+2M$	M	0	

Since all $c_j - z_j \geq 0$ but artificial variable a_1 is present in basic solution with positive value $(a_1 = 24)$. Hence infeasible solution