Vector Integration ⇒

* **Line Integral.** Let us consider a displacement from P to Q in a vector field $\vec{F}$. This displacement be divided into small elements dl_1, dl_2 etc. $\vec{F}$ is inclined at an angle θ to $d\vec{l}$ (in Fig). Such that it continuously varies in magnitude as well as direction along the curve. The Product of the length of the element and Component of $\vec{F}$ in the length direction

$$F \cos\theta \, dl = \vec{F}.dl$$

Hence the integral of $\vec{F}$ from P to Q is (Total force on P to Q)

$$\int_P^Q F \cos\theta \, dl = \int_P^Q \vec{F}.d\vec{l} \qquad \text{—(1)}$$

is Called the integral of $\vec{F}$ from P to Q.

Since the vector field $\vec{F}$ can be regarded as a gradient of some scalar function ϕ i.e. $\vec{F} = \text{grad}\,\phi$, and the gradient in fact represent the rate of change of a field quantity. Therefore the line integral is representing the sum of this rate of change and equal to the total change between P and Q so

$$\int_P^Q \vec{F}.d\vec{l} = \int_P^Q \vec{\nabla}\phi.dl = \int_P^Q \frac{\delta\phi}{\delta l}.d\vec{l} = \int_P^Q d\phi = \phi(Q) - \phi(P)$$

Here $\phi(P)$ and $\phi(Q)$ are the scalar fields at P and Q respectively. Since the value of R.H.S. i.e. $[\phi(Q) - \phi(P)]$ depends only on the values of ϕ at two points P and Q so that

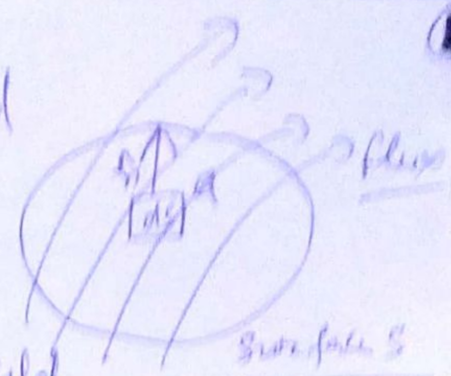
$$\int_P^Q \vec{F}.d\vec{l} = \int_P^Q \vec{\nabla}\phi.dl = \phi(Q) - \phi(P)$$

Any curve from P to Q.   Any curve from P to Q

So that the value of line integral depends only on the position of the two points in the vector field [which can be expressed as the gradient of a scalar field] and not upon the actual path taken between them, such vector field is called Conservative field. ex - electrostatics, magnetic and gravitational fields.

$$\int_P^Q \vec{F}.d\vec{l} = \int_P^Q (F_x dx + F_y dy + F_z dz) \qquad \text{—(2)}$$

2- **Surface Integral** : Let S is any surface field divided into infinitesimal elements each of which may be considered as a vector $d\vec{s}$ (in fig.)



Then if ϕ and $\vec{A}$ are scalar and vector field respectively. Then surface integral may be written as

$\iint_S \phi \, d\vec{s}$ Vector Quantity (i)

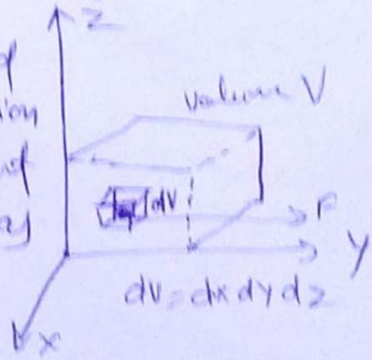
$\iint_S \vec{A}\cdot d\vec{s}$ Scalar " (ii) — (1) is called the flow or flux of the vector function $\vec{A}$ through given surface S

$\iint_S \vec{A}\times d\vec{s}$ Vector " (iii)

The area element $d\vec{s}$ written as $d\vec{s} = \hat{n}\, ds$, where $\hat{n}$ is unit normal vector to indicate the positive direction of the surface,

- If the surface is closed then outward normal taken as positive.
- If the surface is open surface then the positive normal depends on the direction in which the perimeter of the open surface is traversed

3- **Volume Integral** : Let us consider a surface enclosing volume V in a vector field $\vec{F}$ and suppose that $\vec{F}$ is a vector point function at a point in a small volume element dV (in fig). Then the $\iiint_V \vec{F}\cdot dV$ is known as integral of vector field $\vec{F}$ for entire volume V over the surface.



In Cartesian component

$$\iiint_V \vec{F}\cdot dV = \iiint_{xyz} (F_x\hat{i} + F_y\hat{j} + F_z\hat{k})\, dx\, dy\, dz \quad -(1)$$

where $dV = dx\,dy\,dz$

Gauss's divergence theorem :-

Gauss divergence theorem enable us to transform volume integral of the divergence of the vector field into surface integral of the vector field and vice versa.

So According to statement, "the flux of a vector field $\vec{F}$ over any closed surface "S" is equal to the volume integral of the divergence of the vector field over the volume V enclosed by that surface, so

$$\text{flux} = \iint_S \vec{F}\cdot\hat{n}\,ds = \iiint_V \text{div}\,\vec{F}\,dV$$

$$\boxed{\iint_S \vec{F}\cdot\hat{n}\,ds = \iiint_V (\vec{\nabla}\cdot\vec{F})\,dV}$$

Stoke's theorem →

This theorem states that the line integral of a vector $\vec{F}$ around any closed curve C is equal to surface integral of curl $\vec{F}$ taken over any surface S of which the curve C is boundary edge.

$$\boxed{\begin{aligned}\oint_C \vec{F}\cdot d\vec{l} &= \int_S (\vec{\nabla}\times\vec{F})\cdot d\vec{s} \\ \oint \vec{F}\cdot d\vec{l} &= \int_S (\text{Curl}\,\vec{F})\cdot d\vec{s}\end{aligned}}$$

Electrostatics →

Source charge   Test charge

In electrostatics all the source charge are stationary and test charge may be mobing.

Quantisation of charge [fundamental charge]

Charge is created by transfer of electron so net charge on a body is always an integral multiple of charge on an electron. Charge in a body is produced due to excess or deficiency of electron. Electron can not be divided into further smaller parts. therefore charge on a body is integral multiple of the charge on electron.

The magnitude of charge on an electron is called fundamental charge or elementary charge. Its value is 1.6×10^{-19} Coulomb and denoted by e. Therefore we may say that "Any physically existing charge is always an integral multiple of fundamental charge 'e'". This is called the principle of atomicity of charge or principle of Quantization of charge.

If q is charge on a body and n is positive or negative integral number then, $\boxed{q = \pm ne}$

e is also called quantum of charge.

Coulomb's law →

The force of attraction or repulsion between two two point charges is directly propotional to the product of the charge and inversely propotional to the square of distance between Them. The direction of this force is along the line Joining The two charges. This law is called Coulomb's inverse square law.

If two point charge q_1 and q_2 separated by a distance r, then the force acting between them is

$$F \propto q_1 q_2$$

$$\propto \frac{1}{r^2}$$

$$\boxed{F = k \frac{q_1 q_2}{r^2}} \quad -(1)$$

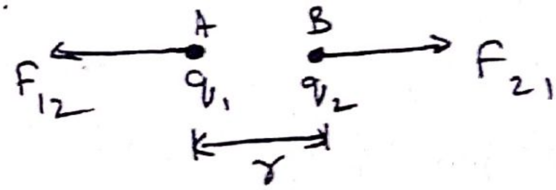
Here k is propotionality Constant $= \frac{1}{4\pi\epsilon_0 K} \approx 9\times10^9 \, N\text{-}m^2/C^2$

Here ϵ_0 is universal constant and called Permittivity of Vaccum of free space and K is dielectric Constant.

Vector form of Coulomb's law

F_{12} → force exerted on q_1 due to q_2

F_{21} → force exerted on q_2 due to q_1



$$\vec{F}_{21} = \frac{1}{4\pi\epsilon_0 K} \frac{q_1 q_2}{r^2} \hat{r}_{21} \quad -(1)$$

But $\hat{r}_{21} = \frac{\vec{r}_{21}}{|r|}$ so $\vec{F}_{21} = \frac{1}{4\pi\epsilon_0 K} \frac{q_1 q_2}{r^3} \vec{r}_{21} \quad -(2)$

Similarly

$$\vec{F}_{12} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^3} \vec{r}_{12} \quad -(3)$$

obviously $\vec{r}_{12} = -\vec{r}_{21}$ so from eq (3)

$$\vec{F}_{12} = -\frac{1}{4\pi\epsilon_0 k}\frac{q_1 q_2}{r^3}\vec{r}_{21} \quad -(4)$$

Comparing (2) and (4)

$$\boxed{\vec{F}_{21} = -\vec{F}_{12}} \quad -(5)$$

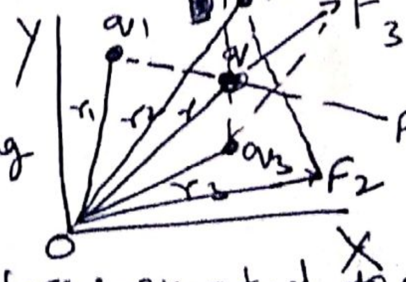
Coulomb →

(1) 1 Coulomb is a charge which when placed at a distance of 1 metre from an equal and similar charge in vacuum (air) [k=1] will repel it with force 9×10^9 Newton. [$F=\frac{1}{4\pi\epsilon_0 k}\frac{q_1q_2}{r^2} = 9\times10^9$]

(2) 1 Coulomb is the charge that is produced by removal of 6.25×10^{18} electrons from a neutral body. ($n=\frac{q}{e}=6.25\times10^{18}$)

(3) 1 Coulomb is the charge which when flowing in a conductor for 1 seconds causes a current of 1 ampere. [$q=i\cdot t$]

Principle of Superposition →



If the system contains a number of interacting charges, then the force on a given charge is equal to the vector sum of the forces exerted on it by all remaining charges.

If the force exerted due to charge q_1 on q is $\vec{F}_1$ then

Coulombs law in vector form $\vec{F}_i = \frac{1}{4\pi\epsilon_0 k}\frac{q q_1}{(r-r_i)^3}(\vec{r}-\vec{r}_i)$

The total force on q due to all n charges may be expressed as -

$$F = \sum_{i=1}^{n} F_i = \sum_{i=1}^{n}\frac{1}{4\pi\epsilon_0}\frac{q q_i}{(r-r_i)^3}(\vec{r}-\vec{r}_i) = \frac{q}{4\pi\epsilon_0 k}\sum_{i=1}^{n}\frac{q_i(\vec{r}-\vec{r}_i)}{(r-r_i)^3}$$