Electric field strength (Intensity) →

The region in which a charge experience a force is called electric field. The electric field strength at any point in an electric field is a vector quantity, whose magnitude is equal to force acting per unit positive test charge and whose direction is along the direction of force.

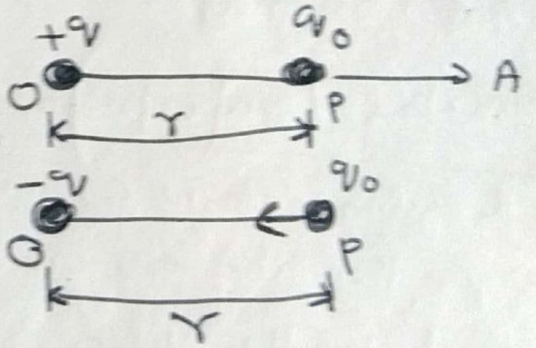
$$\vec{E} = \frac{\vec{F}}{q_0} \quad -(1)$$

Unit → Newton/Coulomb, Volt/meter

Dimensional formula → $[MLT^{-3}A^{-1}]$

[To find the electric field strength at a point in the electric field we place a very small (infinitesimal) positive charge (q_0) at that point. this charge is very small, so it called test charge, it is very small so it does not cause any change in initial electric field]

(A) $\vec{E}$ due to point charge →



According to Coulombs law, for +q charge

$$\vec{F} = \frac{1}{4\pi\epsilon_0}\frac{q q_0}{r^2}\hat{r} \quad [P \to A]$$

Putting in eq (1)

$$\vec{E} = \frac{\vec{F}}{q_0} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{r} \quad (P \to A) \quad -(2)$$

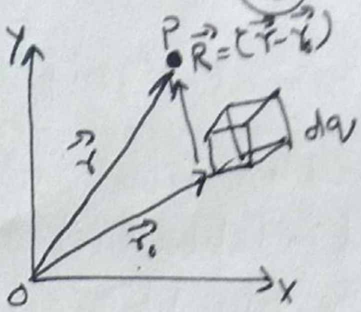
If charge is -q

$$\vec{E} = \frac{F}{q_0} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{r} \quad (P \to O) \quad -(3)$$

Here $\hat{r}$ is unit vector along vector $\vec{r}$

So
$$\boxed{\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{q}{|r|^3}\vec{r}} \quad -(4)$$

) Due to Continuous charge distribution →



A Continuous charge distribution may be supposed to be formed of a large number of very small charge element. Consider a small element of charge dq. if charge dq is located at position $\vec{r}_0$ and point P under Consideration is located at position $\vec{r}$ then the distance of point P from this charge element is R. The electric field strength at P, due to this charge element

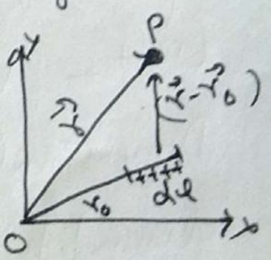
$$d\vec{E} = \frac{dq}{4\pi\epsilon_0 R^2} \quad — (1)$$

Total electric field at P is founded by integrating above expression for whole charge distribution.

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\int \frac{dq}{R^2}\hat{R} = \frac{1}{4\pi\epsilon_0}\int \frac{dq}{R^3}\vec{R} = \frac{1}{4\pi\epsilon_0}\int \frac{dq(\vec{r}-\vec{r}_0)}{|\vec{r}-\vec{r}_0|^3} \quad — (2)$$

(i) Due to line charge → if λ is charge per unit length then charge on element of length dl is $dq = \lambda dl$

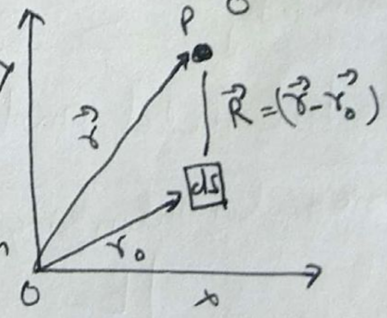
$$\vec{E} = \frac{1}{4\pi\epsilon_0}\int \frac{\lambda dl}{|\vec{r}-\vec{r}_0|^3}(\vec{r}-\vec{r}_0) \quad — (A)$$



(ii) Due to Surface charge distribution.

Let charge be distributed at the surface and σ is surface charge density. Then charge on surface element of area dS is $dq = \sigma dS$, so electric field strength at point P -

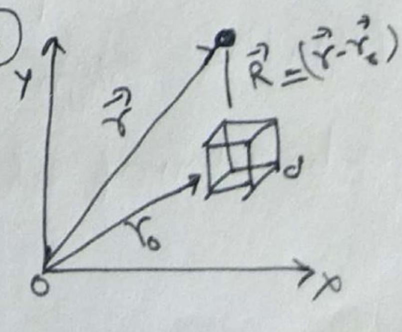
$$\vec{E} = \frac{1}{4\pi\epsilon_0}\int_S \frac{\sigma dS}{|\vec{r}-\vec{r}_0|^3}(\vec{r}-\vec{r}_0) \quad — (B)$$



(iii) Due to volume charge distribution →

ρ volume charge density $dq = \rho dV$

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\int_V \frac{\rho dV}{|\vec{r}-\vec{r}_0|^3}(\vec{r}-\vec{r}_0) \quad — (C)$$



→ **Physical significance of electric field-**

Electric field is the characteristic of charge of system if independent on test charge.
The true physical significance of electric field appears on when we kept in view that electrostatic intraction is only a part of fundamental force, named as electromagnetic intraction. Thus electric field aswell as magnetic field are detected by their intraction forces.

→ **Electrostatic Potential [V]** :→ The work done in bringing a unit positive charge from infinity to any point in the electric field is called potential at that point

$$\boxed{V = \frac{W}{q_0} = \frac{f \cdot r}{q_0} = \frac{1}{4\pi\epsilon_0}\frac{q}{r} \quad \text{Volt}}$$

Unit – Volt, Joule/Coulomb, Newton-meter/Coulomb
Dimensional formula → $[ML^2T^{-3}A^{-1}]$
Quantity → Scalar

→ **Electric field and potential difference** →
Electric field intensity is equal to negative potential gradient

$$\boxed{E = -\frac{dV}{dx}}$$

Negative sign signifies that potential decreases in the direction of electric field.

Electric field from potential :

Let V and V+dV is the values of electric potential at two neighbouring points A and B having co-ordinates (X, Y, Z) and (X+dX, Y+dY, Z+dZ) respectively.
The small displacement from A to B can be written as

$$d\vec{r} = dx\,\hat{i} + dy\,\hat{j} + dz\,\hat{k} \quad \text{— (1)}$$

As V is function of co-ordinates (X, Y, Z) so small change dV in the value of potential V corresponding to small displacement $d\vec{r}$ from A to B is given as

$$dV = \frac{\partial V}{\partial x}dx + \frac{\partial V}{\partial y}dy + \frac{\partial V}{\partial z}dz \quad \text{— (2)}$$

$$= \left[\hat{i}\frac{\partial V}{\partial x} + \hat{j}\frac{\partial V}{\partial y} + \hat{k}\frac{\partial V}{\partial z}\right]\cdot\left[dx\,\hat{i} + dy\,\hat{j} + dz\,\hat{k}\right]$$

$$= \left[\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right]V\cdot\left[dx\,\hat{i} + dy\,\hat{j} + dz\,\hat{k}\right]$$

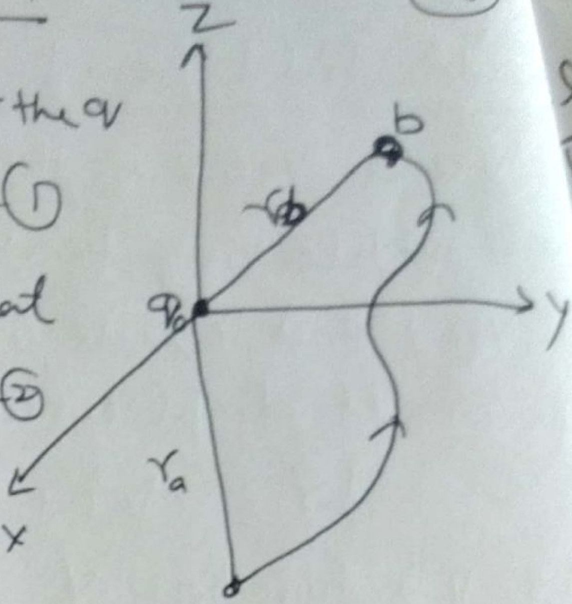
$$dV = \vec{\nabla}V\cdot d\vec{r} \quad (3)$$

But, we know $dV = -\vec{E}\cdot d\vec{r}$ — (4) &

So from (3) and (4)

$$-\vec{E}\cdot d\vec{r} = \vec{\nabla}V\cdot d\vec{r} \quad \underline{\text{or}} \quad \vec{E}\cdot d\vec{r} = (-\vec{\nabla}V)\cdot d\vec{r}$$

$$\boxed{\vec{E} = -\vec{\nabla}V} \text{ or } \boxed{\vec{E} = -\text{grad}\,V} \quad \text{— (5)}$$



Curl of $\vec{E}$ -

The electric field for the q charge is $\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{r}$ —(1)

So from fig, for spherical Co-ordinat

$$d\vec{r} = dr\hat{r} + r d\theta\hat{\theta} + r\sin\theta d\phi\hat{\phi} \quad (2)$$

So $\vec{E}\cdot d\vec{l} = \vec{E}\cdot d\vec{r}$ —(3)

$$\oint_{r_a}^{r_b}\vec{E}\cdot d\vec{l} = \oint_{r_a}^{r_b}\frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{r}\,(dr\hat{r} + r d\theta\hat{\theta} + r\sin\theta\, d\phi\hat{\phi})$$

$$\oint_{r_a}^{r_b}\vec{E}\cdot d\vec{l} = \frac{q}{4\pi\epsilon_0}\int_{r_a}^{r_b}\frac{dr}{r^2} = \frac{q}{4\pi\epsilon_0}\left[\frac{1}{r}\right]_{r_a}^{r_b} = \frac{q}{4\pi\epsilon_0}\left[\frac{1}{r_b} - \frac{1}{r_a}\right] \quad (4)$$

for closed path $r_a = r_b$ so that

$$\oint_{r_a}^{r_b}\vec{E}\cdot d\vec{l} = 0 \quad (5)$$

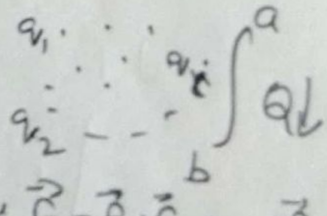
By Stokes theorem we know

$$\oint\vec{E}\cdot d\vec{l} = \iint(\vec{\nabla}\times\vec{E})\,dS \quad (6)$$

So from 5 and 6

$$\boxed{\vec{\nabla}\times\vec{E} = 0 \quad \text{or} \quad \text{Curl}\,\vec{E} = 0} \quad (7)$$

Work and energy in Electrostatics →



Let a test charge Q is displaced from a to b in the presence of $q_1, q_2 \ldots q_i$, the resultant force $\vec{F} = \vec{F}_1 + \vec{F}_2 + \cdots + \vec{F}_i$ —(1)

The work done by the resultant force for displacement a to b

$$W = \int_a^b \vec{F}\cdot d\vec{l} = \int_a^b Q\vec{E}\cdot d\vec{l} \qquad [\because \vec{F} = Q\vec{E}]$$

$$= Q\int_a^b E\cdot dl = Q\left[\int_a^{\infty}\vec{E}\cdot d\vec{l} + \int_{\infty}^{b}\vec{E}\cdot d\vec{l}\right] \qquad \text{[in this we get work done]}$$

$$= Q[V(a) - V(b)] = -Q[V(b) - V(a)] \Rightarrow V(b) - V(a) = \frac{-W}{Q} \quad (2)$$

If $b \to 0$ and $a \to r$ then $V(r) = \frac{W}{Q}$ or $\boxed{W = QV(a)}$ (3)

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