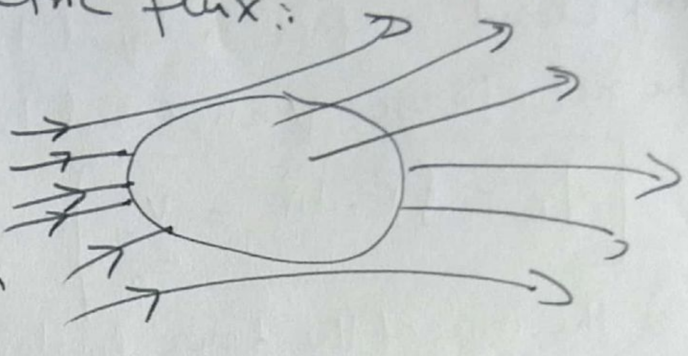
Electric flux → Total number of electric lines of force passing through any surface which placed in an electric field is called electric flux:

In case of a closed surface (like fig), that is a surface that completely encloses a volume, the net flux through the surface is given as



$$\boxed{\phi_E = \oint \vec{E} \cdot d\vec{A}} \quad -(1)$$

Here $d\vec{A}$ is the area vector of small element dA on the surface. Since $d\vec{A}$ is normal to the surface The angle between $\vec{E}$ and $d\vec{A}$ is θ and then eq (1)

$$\phi_E = \oint E\, dA \cos\theta$$

But $\oint dA = A$ so $\boxed{\phi_E = EA\cos\theta}$ $-(2)$

Electric flux density → The ratio of electric flux ϕ_E through a surface area A is called electric flux density at the location of surface

electric flux density $= \dfrac{\phi_E}{A} = \dfrac{E \cdot dA}{A} = \dfrac{EA\cos\theta}{A}$

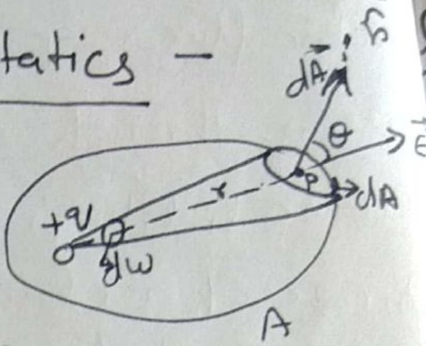
if plane surface normal to the electric field, then $\phi_E = EA$ and electric flux density $= \dfrac{EA}{A} = E$

Unit of electric flux → Newton m^2/Coulomb, Volt-m

Dimensional formula → $[ML^3T^{-3}A^{-1}]$

Quantity – Scalar

Gauss's Law or theorem in electrostatics –



It state that, "The electric flux ϕ_E through any closed surface is equal to $\frac{1}{\epsilon_0}$ times the net charge q enclosed by the surface. That is

$$\boxed{\phi_E = \oint \vec{E} \cdot d\vec{A} = \frac{q}{\epsilon_0}}$$

This is Integral form of Gauss theorem

It is the one of the four fundamental equation of electromagnetic theory (Maxwell's equations)

Proof – Let us consider a point charge $+q$ situated at O inside a closed surface A. Let dA is a small area element surrounding a point P on the surface. Let $OP = r$. The area element may be represented by a vector $d\vec{A}$ drawn outward along the normal to the element. The magnitude of intensity of electric field at P is

$$E = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \quad \text{— (1)} \quad \text{where } OP = r$$

The electric flux through the area element dA is

$$d\phi_E = \vec{E} \cdot d\vec{A} = E\, dA \cos\theta \quad \text{— (2)}$$

[Here θ is angle between $\vec{E}$ and $d\vec{A}$]

$$d\phi_E = \frac{q}{4\pi\epsilon_0} \cdot \frac{dA \cos\theta}{r^2} \quad \text{— (3)}$$

But $\frac{dA \cos\theta}{r^2}$ is the solid angle $d\omega$ subtended by the area dA at the point O. Therefore

$$d\phi_E = \frac{q}{4\pi\epsilon_0} d\omega \quad \text{— (4)}$$

The total flux ϕ_E through entire surface A is

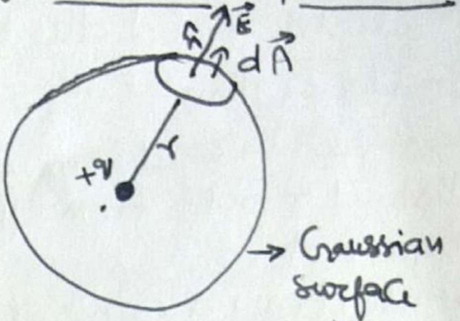
$$\phi_E = \frac{q}{4\pi\epsilon_0} \oint d\omega \quad \text{— (5)}$$

Now $\oint d\omega = 4\pi$ the solid angle subtended by the entire closed surface A at the point O. So

$$\boxed{\phi_E = \frac{q}{\epsilon_0}} \quad \text{— (6)}$$

Hence proved

electric field due to point charge [Deduction of Coulomb's law from Gauss's law]



From fig for any surface element dA, both the electric field vector $\vec{E}$ and the area vector $d\vec{A}$ are along the same direction (radially outward) that is, the angle between them is zero. therefore

$$\vec{E}\cdot d\vec{A} = E\,dA\cos\theta = E\,dA \quad -(1)$$

Hence the total electric flux leaving the Gaussian surface is given by -

$$\phi_E = \oint \vec{E}\cdot d\vec{A} = \oint E\,dA = E\oint dA \quad -(2)$$

because E is constant throughout the surface. But $\oint dA = 4\pi r^2$ (area of the sphere)

$$\phi_E = E(4\pi r^2) \quad -(3)$$

By Gauss's theorem $\phi_E = \frac{q}{\epsilon_0}$, where ϵ_0 is permittivity of free space

$$E(4\pi r^2) = \frac{q}{\epsilon_0}$$

$$E = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2} \quad -(4)$$

If we put a test charge q_0 at that point then the magnitude of force experienced by q_0 will be

$$\boxed{F = q_0 E = \frac{1}{4\pi\epsilon_0}\frac{q q_0}{r^2}} \quad -(5)$$

This is Coulomb's law, as derived from Gauss's law.

Differential form of Gauss's law ⇒

According to integral form of Gauss's law

$$\oint_S \vec{E}\cdot d\vec{A} = \frac{q}{\epsilon_0} \quad -(1)$$

Let ρ be the volume charge density then net charge $q = \int_V \rho\,dV$ -(2)

where V is volume charge density, by closed surface A

$$\oint_S \vec{E}\cdot d\vec{A} = \frac{1}{\epsilon_0}\int_V \rho\,dV \quad -(3)$$

According to divergence theorem

$$\oint_S \vec{E}\cdot d\vec{A} = \int_V \vec{\nabla}\cdot\vec{E}\,dV \quad -(4)$$

comparing (3) and (4) $\oint_V \vec{\nabla}\cdot\vec{E}\,dV = \frac{1}{\epsilon_0}\int_V \rho\,dV$

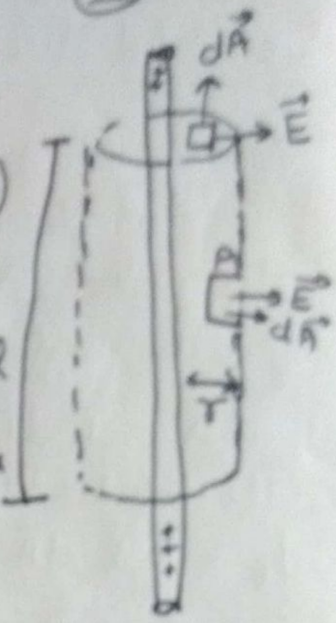
or $\int_V \vec{\nabla}\cdot\vec{E}\,dV = \int_V \left(\frac{\rho}{\epsilon_0}\right)dV$ -(5)

This can true for any arbitrary volume V only when

$$\vec{\nabla}\cdot\vec{E} = \frac{\rho}{\epsilon_0} \quad \text{or} \quad \boxed{\text{div}\,\vec{E} = \frac{\rho}{\epsilon_0}} \quad -(6)$$

Application of Gauss law :

① Electric field due to an infinite line of charge -



Consider a thin long charged rod or wire (line charge) having linear charge density $\lambda (C\,m^{-1})$. Let P is a point distant r metre from the wire at which the $\vec{E}$ is required.

Let us draw a co-axial Gaussian cylindrical surface of length l through the point P. The magnitude E of the electric field intensity will be the same at all points on this surface and directed radially outwards. So for any area element dA taken on the surface, both the electric field vector $\vec{E}$ and the area vector $d\vec{A}$ are along the same direction (radially outwards). Therefore

$$\vec{E}\cdot d\vec{A} = E\,dA\cos 0 = E\,dA$$

Hence the electric flux through the Gaussian surface is

$$\phi_E = \int_A \vec{E}\cdot d\vec{A} = \int_A E\,dA = E\int_A dA = E(2\pi r l) \quad \text{—①}$$

The flux through the plane ends of the surface is zero because $\vec{E}$ and $d\vec{A}$ are right angles everywhere on these faces $[\vec{E}\cdot d\vec{A} = 0]$. So total flux through Gaussian surface is

$$\phi_E = E(2\pi r l) \quad \text{—②}$$

According to Gauss law, $\phi_E = \frac{q}{\epsilon_0}$, where q is the net charge enclosed by Gaussian surface. Here, $q = \lambda l$ so that

$$\phi_E = \frac{\lambda l}{\epsilon_0} \quad \text{—③}$$

Comparing ② and ③ we get $E(2\pi r l) = \frac{\lambda l}{\epsilon_0}$

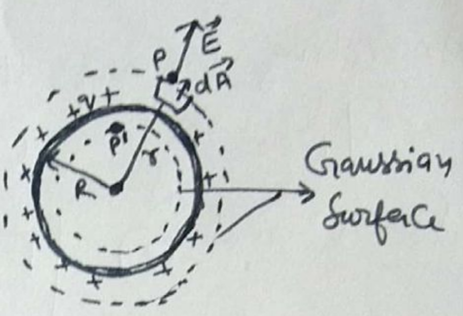
or $E = \frac{\lambda}{2\pi\epsilon_0 r}$ or $\boxed{\vec{E} = \frac{\lambda}{2\pi\epsilon_0 r}\hat{r}}$ —④

or $\boxed{\vec{E} = \frac{1}{4\pi\epsilon_0}\cdot\frac{2\lambda}{r}\hat{r}}$ —⑤

Application of Gauss law -

(2) Electric field due to a charged spherical shell

Let us consider a spherical shell of Radius R having surface density of charge σ then total charge on the shell, $\boxed{q = 4\pi R^2 \sigma}$ —(1)



Case-1 → At an external point (outside the charged shell) $(r > R)$

Let us draw a concentric spherical Gaussian surface of radius r through the point P. All points on this surface are equidistant from the surface of the charged shell. Because of the spherically symmetry, the magnitude E of the electric field intensity will be the same at all points on the Gaussian surface and directed radially outward.

Let us consider an area element dA around the point P. Both electric intensity vector $\vec{E}$ and area vector $d\vec{A}$ at point P are directed radially outward. That is the angle between them is zero therefore, the electric flux through the area element dA is, $d\Phi_E = \vec{E}\cdot d\vec{A} = E\,dA\cos 0 = E\,dA$ —(2)

The flux through the entire Gaussian surface is

$$\Phi_E = \oint E\,dA = E\oint dA = E(4\pi r^2) \quad —(3)$$

By Gauss theorem $\Phi_E = \frac{q}{\epsilon_0} = \frac{4\pi R^2 \sigma}{\epsilon_0}$ —(4) [use eq (1)]

we get from (3) and (4) $E(4\pi r^2) = \frac{4\pi R^2 \sigma}{\epsilon_0}$

$$\boxed{E = \frac{\sigma}{\epsilon_0}\frac{R^2}{r^2}} \quad —(5)$$

Case (2) At Surface $r = R \rightarrow$ $\boxed{E = \frac{\sigma}{\epsilon_0}}$ —(6)

Case (3) At Internal point (inside the shell) $r < R$

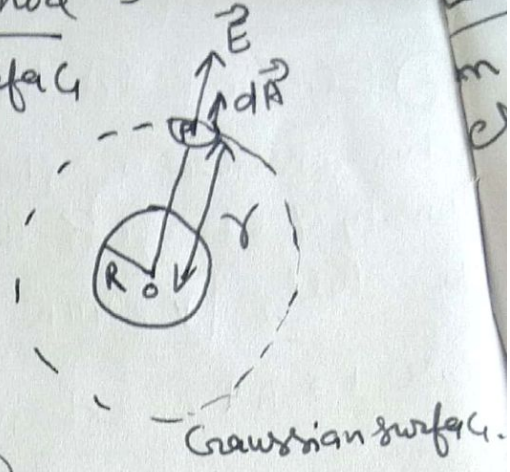
Gaussian surface through P' does not enclose any charge, so according to Gauss theorem $\Phi_E = E(4\pi r^2) = 0$ or $\boxed{E = 0}$ —(7)

(3) Electric field of a uniformly charged sphere →

Case-(1) $r > R$

Total Electric flux through the Gaussian surface of radius r

$$\phi_E = \oint \vec{E}\cdot d\vec{A} = \vec{E}\oint_S d\vec{A} = E\cdot 4\pi r^2 \quad (1)$$



According to Gauss theorem

$$\phi_E = \frac{q}{\epsilon_0} \quad -(2)$$

from (1) and (2) $\boxed{E = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}}$ -(3)

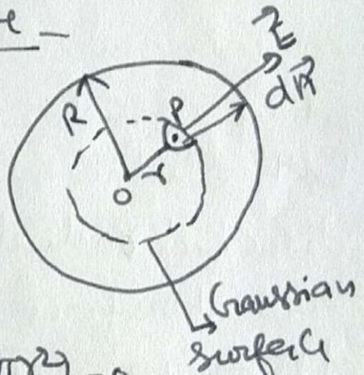
So electric field strength at any point outside a charged sphere is the same as if the whole charge were concentrated at the the centre

Case-(2) $r = R$ so eq (3) will be, $\boxed{E = \frac{1}{4\pi\epsilon_0}\frac{q}{R^2}}$ (4)

Case (3) If $r < R$ when point lies inside the sphere —

flux through Gaussian surface

$$\phi_E = E\,(4\pi r^2) \quad -(9)$$



There arises two cases —

(A) If sphere is Conducting ⇒ charge within Gaussian surface will be zero then $\phi_E = E(4\pi r^2) = 0$

or $\boxed{E = 0}$ — (4A)

(B) If sphere Non Conducting ⇒ There will be charge distributed through whole volume. if q' is the part of q which is inclosed within the Gaussian surface of radius r then volume charge density $\rho = \frac{q}{\frac{4}{3}\pi R^3} = \frac{q'}{\frac{4}{3}\pi r^3}$ So from this $q' = q\left(\frac{r}{R}\right)^3$ —(5)

and by Gauss law, flux through the Gaussian surface of radius r. $\phi_E = \frac{q'}{\epsilon_0}$ using (4) $E(4\pi r^2) = \frac{q'}{\epsilon_0}$ or $\boxed{E = \frac{1}{4\pi\epsilon_0}\frac{q'}{r^2}}$ -(6)

Putting the value of q' from (5) in (6) $\boxed{E = \frac{1}{4\pi\epsilon_0}\frac{q\,r}{R^3}}$ -(7)

So E due to a uniformly charged sphere of an internal point is proportional to distance r of the point from centre