Motion of charge in electric and magnetic field.

All magnetic effect are associated with the motion of electric charges or current elements. Since moving charge and current element have magnitude and direction, so they are vector quantities where as a static charge has only scalar properties -

→ stationary charge produce only electric field $\vec{E}$ in the space around it

$$\vec{V}=0,\ \vec{a}=0 \longrightarrow \vec{E}$$

→ A Moving charge Produce magnetic field $\vec{B}$

$$\vec{V}\neq 0,\ \vec{a}=0 \longrightarrow \vec{B}$$

→ A moving charge with acceleration produce E.M.F. [Electromagnetic field]

$$\vec{V}\neq 0,\ \vec{a}\neq 0 \longrightarrow \vec{E}, \vec{B}, \text{energy}.$$

Current - per second flow of charge is called current

$$i=\frac{q}{t}=\pm\frac{ne}{t} \quad \text{coulomb/sec} \ \text{or Ampere}$$

Current is classified with the shape and size of the conductor

1- **linear current density** ($\vec{I}$) The current produce due to the linear charge density is called linear current density $\vec{I}$.

$$I=\frac{dq}{dt}=\frac{d(\lambda\vec{l})}{dt}=\lambda\frac{d\vec{l}}{dt}=\lambda\vec{V} \quad \text{or} \quad \boxed{\vec{I}=\lambda\vec{V}}$$

and

$$\vec{F}_m=\int(\vec{V}\times\vec{B})\,dq \quad \text{(The force due to small charge } dq\text{)}$$

$$=\int(\vec{V}\times\vec{B})\lambda\,d\vec{l}=\int(\lambda\vec{V}\times\vec{B})\,d\vec{l}=\int(I\times\vec{B})\,dl$$

Here $I=\lambda\vec{V}$ shows the current in scalar quantity because current (I) does not hold vector addition rule. but current density is vector dependent.

$$\text{so } \vec{F}_m=\int I\,(d\vec{l}\times\vec{B})=I\int(d\vec{l}\times\vec{B})$$

2- **Surface current density** ($\vec{K}$) The current flows on the surface with perpendicular distance $dl_\perp$ and charge density (σ) is known as surface current density.

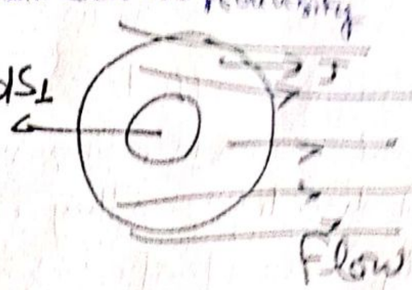
$$\vec{K}=\frac{dI}{dl_\perp}\hat{n} \Rightarrow I\int\vec{K}\cdot d\vec{l_\perp} \quad \text{or} \quad dI=\frac{dq}{dt} \ \text{then} \ \vec{K}=\frac{dq}{dt}\frac{\hat{n}}{dl_\perp}=\frac{\sigma ds}{dt}\frac{\hat{n}}{dl_\perp}$$

$$\vec{K}=\frac{\sigma\,dl\,dl_\perp}{dt}\frac{\hat{n}}{dl_\perp}=\sigma\frac{dl}{dt}\hat{n}=\sigma\vec{V}$$

and

$$\vec{F}_m=\iint(\vec{V}\times\vec{B})\,dq=\iint(\vec{V}\times\vec{B})\,\sigma\,ds=\iint(\vec{K}\times\vec{B})\,ds$$

3. Volume current density ($\vec{J}$) The current due to volume charge density on the perpendicular surface area ($dS_\perp$) is known as current density ($\vec{J}$).



$$\vec{J} = \frac{dI}{dS_\perp}\hat{n} = \frac{dq}{dt}\frac{\hat{n}}{dS_\perp} = \frac{\rho\, dV}{dt}\frac{\hat{n}}{dS_\perp}$$

$$= \frac{\rho\, dS_\perp\, dl}{dt}\cdot\frac{\hat{n}}{dS_\perp} = \frac{\rho\, dl\,\hat{n}}{dt} = \rho\vec{v}$$

$$\boxed{\vec{J} = \rho\vec{v}}$$

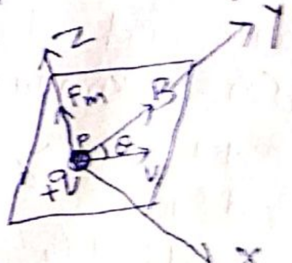
and $\vec{F}_m = \iiint(\vec{v}\times\vec{B})\,\rho\, dV = \iiint(\vec{J}\times\vec{B})\, dV$

Magnetic field → The No. of magnetic line perpendicular to surface per unit area is called ~~magnet~~ magnetic field. It is denoted by $\vec{B}$.
or the space around a magnet or a current carrying conductor in which its magnetic effect present is known as magnetic field $\vec{B}$.
It is a vector quantity. Its magnitude and direction at any point are specified by $\vec{B}$ called magnetic flux density or magnetic induction.

$$B = \frac{\phi}{S} = \frac{\text{magnetic line perpendicular to surface}}{\text{Surface area}}$$

Unit → in SI system, Tesla, weber/m^2 and in CGS, Gauss $= 10^{-4}$ T

Dimensional formula → $[M A^{-1} T^{-2}]$



Lorentz force → (Magnetic Lorentz force)

A moving charge particle having charge q enters in a magnetic field ($\vec{B}$), with a velocity $\vec{v}$, it experiences a force $\vec{F}_m$. This magnetic force is directly proportional to (1) charge q (2) velocity v (3) magnetic field $\vec{B}$ and (4) $\sin\theta$, where θ is angle between $\vec{v}$ and $\vec{B}$.

→

so

$$\vec{F}_m \propto qvB\sin\theta \Rightarrow \vec{F}_m = K\, qvB\sin\theta$$

$$\boxed{\vec{F}_m = qvB\sin\theta = q(\vec{v}\times\vec{B})}$$

Here $\vec{F}_m$ is magnetic Lorentz force, experienced by a moving charged particle in the magnetic field. The direction of $\vec{F}_m$ can be determined by using right hand rule for cross product.

Lorentz force → The total force $\vec{F}$ experienced by a charged particle in a region where both electric and magnetic field exist is called Lorentz force.

Lorentz force due to electric field $\boxed{\vec{F}_e = q\vec{E}}$ — (1)

" " " " Magnetic field $\boxed{\vec{F}_m = q(\vec{v}\times\vec{B})}$ — (2)

When the charged particles enters in a region where both electric and magnetic field are present then total force is

$$\boxed{\vec{F} = \vec{F}_e + \vec{F}_m = q\vec{E} + q(\vec{v}\times\vec{B}) = q[\vec{E} + (\vec{v}\times\vec{B})]} \quad (3)$$

$\vec{F}$ is Lorentz force. This equation is universal and valid for constant ~~and~~ as well as varying electric field and magnetic field.

Magnetic flux (ϕ_B) → Total no. of magnetic lines of induction passing through the surface.

$$\phi_B = BS\cos\theta = \vec{B}\cdot\hat{n}S = \vec{B}\cdot\vec{S}$$

Unit → SI - weber or Tesla m² $\qquad$ 1 weber = 10^8 Maxwell

CGS - Maxwell or Gauss cm²

Dimensional formula → $[M L^2 A^{-1} T^{-2}]$

Gauss law in magnetostatics → The isolated ~~mag~~ magnetic poles (Monopoles) do not exist in nature, but always occurs in pairs of equal and opposite poles. So magnetic flux density B is given as, $\oint \vec{B}\cdot d\vec{S} = 0$ — (1)

It shows that total magnetic flux through a closed surface is always zero.

But according to divergence theorem, $\oint(\nabla\cdot\vec{B})\,dV = 0$ — (2)

sin

$$\oint_S \vec{B}\cdot d\vec{S} = \int_V (\mathrm{div}\,\vec{B})\cdot dV \quad — (2)$$

from eq (1) and (2)

$$\int_V (\mathrm{div}\,\vec{B})\cdot dV = 0 \quad — (3)$$

Since the volume is arbitrary so we have

$$\boxed{\begin{aligned}\mathrm{div}\,\vec{B} &= 0\\ \text{or}\quad \nabla\cdot\vec{B} &= 0\end{aligned}} \quad — (4) \;!$$

Torque on a current coil ~~or Bar magnet~~ in magnetic field

Torque experienced by a current carrying coil, which placed in a magnetic field of one tesla making an angle 90° with to normal to plane of coil.

Torque on a current loop is $\tau = NIB\sin\theta$

$$= (NiA)\,B\sin\theta$$

Here NiA is defined as magnetic dipole moment "M" of the loop. Therefore -

$$\vec{\tau} = MB\sin\theta \quad \cancel{= \vec{M}\times\vec{B}}$$

$$\vec{\tau} = MB\sin\theta$$

unit → Ampere-m², Joule/Tesla

Magnetic Dipole ~~and Magnetic dipole Moment~~ → A magnetic dipole consist of two unlike poles of equal strengths, separated by a small distance. Bar Magnet and Compass needles are magnetic dipole. They consist of two poles called north and South poles, of equal strength but opposite in nature. Magnetic poles always exist in pair.

Scanned with CamScanner

The distance between two poles of magnet is called magnetic length of the magnet. The product of the magnitude q_m of the poles strength and magnetic length $2l$ of the magnet is called magnetic dipole moment denoted by m, So

$$m = q_m \times 2l$$

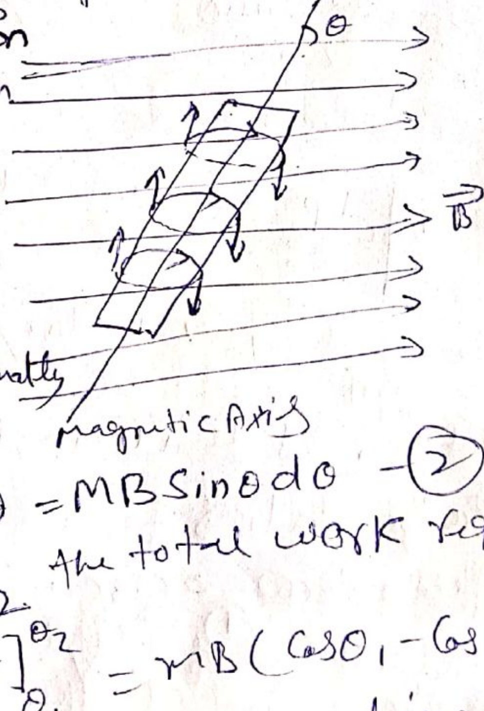
magnetic dipole moment is vector quantity, directed from South pole towards the north pole of the magnet.

SI unit — Ampere-m²

Potential energy of a magnetic dipole (Magnet) in a Magnetic field

A bar magnet placed in an external magnetic field is acted upon by a torque which tends to align the magnet in the direction of field. Therefore work must be done to change the orientation of magnet against to torque. This means that the magnet has magnetic P.E. depending on its orientation in the magnetic field.

→ Suppose a magnet of magnetic moment $\vec{m}$ is held at an angle θ with the direction of a uniform magnetic field $\vec{B}$. The magnitude of the torque acting on the magnet is $\boxed{\tau = MB\sin\theta}$ —(1)



Suppose magnet rotated through an infinitesimally small angle $d\theta$ against the torque.

The work required is $dw = \tau d\theta = MB\sin\theta\, d\theta$ —(2) the total work required will be if the Magnet rotated from θ_1 to θ_2

$$W = \int_{\theta_1}^{\theta_2} mB\sin\theta\, d\theta = mB[-\cos\theta]_{\theta_1}^{\theta_2} = mB(\cos\theta_1 - \cos\theta_2) \quad (3)$$

This work stored as Potential energy U of the magnet in new orientation θ_2, $\boxed{U = mB(\cos\theta_1 - \cos\theta_2)}$ —(4)

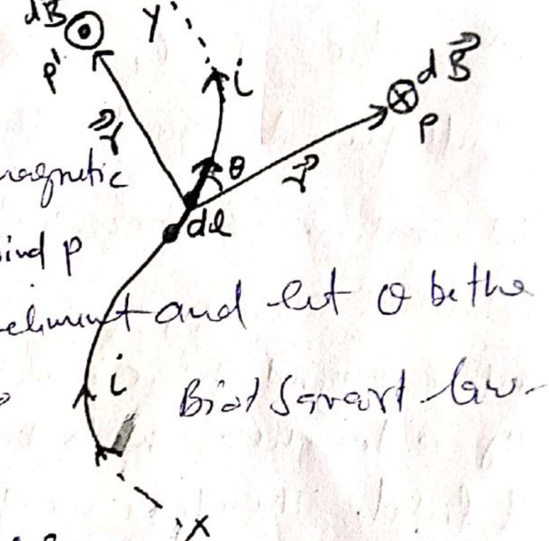
we can assume the P.E. of magnet to be zero for any arbitrary orientation suppose $U = 0$ when $\theta = 90°$. Then the P.E. of magnet in any orientation θ can be obtained by putting $\theta_1 = 90°$ and $\theta_2 = \theta$ in the last expression, thus

$$U_\theta = mB(\cos 90 - \cos\theta) = -mB\cos\theta = -\vec{m}\cdot\vec{B} \quad (5)$$

So a magnet has minimum P.E. when $\vec{m}$ and $\vec{B}$ are parallel ($\theta = 0$, $U_0 = -mB$) and maximum P.E. when $\vec{m}$ and $\vec{B}$ are antiparallel ($\theta = 180$ $U_{180} = mB$), the difference between these two orientation is

$$\Delta U = U_{180} - U_0 = mB - (-mB) = 2mB$$

Biot Savart law



Consider a small element of length dl of conductor through which a current is flowing. Let $d\vec{B}$ be the small magnetic induction due to small element dl at a point P having position vector $\vec{r}$ from the given element and let θ be the angle between dl and $\vec{r}$. Then according to Biot Savart law

$$dB \propto i$$
$$\propto dl$$
$$\propto \sin\theta$$
$$\propto \frac{1}{r^2}$$

$$\Rightarrow dB \propto \frac{i\,dl\sin\theta}{r^2} \quad \text{or}\quad dB = \frac{\mu_0}{4\pi}\frac{i\,dl\sin\theta}{r^2}$$

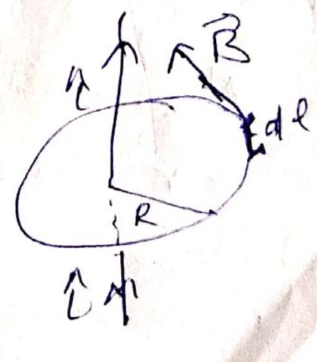
or $$\boxed{dB = \frac{\mu_0}{4\pi}\frac{i\,d\vec{l}\times\vec{r}}{r^3}}$$

Here $\frac{\mu_0}{4\pi} = 10^{-7}$ weber/A-m, μ_0 is permeability of free space.

Biot Savart law analogous to Coulomb's law in electrostatics.

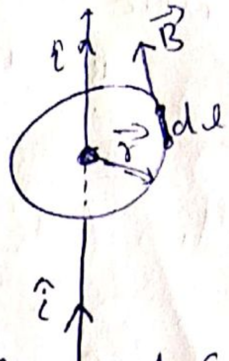
Ampere's circuital law ⇒ It is analogous to Gauss's law in electrostatics, It state that.

"The line integral of the magnetic field $\vec{B}$ around a closed curve is equal to μ_0 times the net current i through the area bounded by the curve that is



$$\boxed{\oint \vec{B}\cdot d\vec{l} = \mu_0 i}$$

Proof → Let us consider an infinitely long straight wire carrying a current i. The magnitude of the magnetic field due to the current carrying wire at a distance r from it is given by Biot Savart law.



$$\vec{B} = \frac{\mu_0}{2\pi}\frac{i}{r} \quad -(1)$$

The lines of this magnetic field are in the form of concentric circles around the wire and its direction is along the tangent of a circle of radius $\vec{r}$.

All the line elements $d\vec{l}$ of this circle are directed along $\vec{B}$.

Therefore line integral of this magnetic field around any circle of radius r.

$$\therefore \vec{B}\cdot d\vec{l} = B\,dl\cos 0^\circ = B\,dl \quad -(2)$$

As magnitude of $\vec{B}$ is same all around the circle

$$\int \vec{B}\cdot d\vec{l} = \oint B\,dl = B\oint dl = \frac{\mu_0 i}{2\pi r}\oint dl = \frac{\mu_0 i \cdot 2\pi r}{2\pi r} = \mu_0 i$$

Here $\oint dl = 2\pi r$ is circumference of the circle

$$\therefore \boxed{\oint \vec{B}\cdot d\vec{l} = \mu_0 i} \quad -(3)$$

This is Ampere's Circuital law.

Application – (1) Magnetic field due to a long straight current carrying wire –

$\oint \vec{B}\cdot d\vec{l} = \mu_0 i$ But $\oint dl = 2\pi r$ (circumference)

So $B(2\pi r) = \mu_0 i$

$$\boxed{B = \frac{\mu_0 i}{2\pi r}}$$

② Magnetic field Induction of Solenoid

$$B = \mu_0 i = \mu_0 n i_0$$

Here, n is no. of turn in Solenoid

i_0 = Current in each turn

Properties of $\vec{B}$.

① $\int \vec{B} \cdot d\vec{l} = \mu_0 i$

② $\vec{\nabla} \times \vec{B} = \mu_0 \vec{J}$ or Curl $\vec{B} = \mu_0 \vec{J}$

③ $\vec{\nabla} \cdot \vec{B} = 0$ or div $\vec{B} = 0$

Divergence of Magnetic field or → (Gauss law in magnetostatics)

Magnetic field $\vec{B}$ is Continuous, for discribe this nature of magnetic field line, it is said that magnetic field $\vec{B}$ is Solenoidal. due to this property of magnetic field ~~line~~, (Continuous nature)

The magnetic flux entering any region is equal to the flux leaving it. Hence

$$\oint_s \vec{B} \cdot d\vec{s} = 0 \quad (1)$$

According to Gauss divergence theorem

$$\oint_s \vec{B} \cdot d\vec{s} = \iiint_V (\vec{\nabla} \cdot \vec{B})\, dV \quad -(2)$$

So from (1) and (2) [Since flux entering any volume equal to flux leaving, the net flux over the volume is zero.

$$\iiint_V (\vec{\nabla} \cdot \vec{B})\, dV = 0 \quad \text{or} \quad \boxed{\vec{\nabla} \cdot \vec{B} = 0 \text{ or div } \vec{B} = 0} = (3)$$

Volume is arbitrary So —

Curl of Magnetic field -or

According to Ampere's law, the line Integral of magnetic Induction $\vec{B}$ along a closed path in a magnetic field due to an electric current is equal to μ_0 times total current inclosed by the closed path, $\oint \vec{B}\cdot d\vec{l} = \mu_0 I = \mu_0 \int_S \vec{J}\cdot d\vec{S}$ —(1)

According to stoke's theorem
$$\oint \vec{B}\cdot d\vec{l} = \int_S (\text{Curl } \vec{B})\cdot dS \quad -(2)$$

Hence from eq (1) and (2)
$$\int_S (\text{Curl } \vec{B})\cdot d\vec{S} = \mu_0 I = \mu_0 \int_S \vec{J}\cdot d\vec{S} \quad (3)$$

or
$$\text{Curl } \vec{B} = \mu_0 \vec{J}$$
$$\boxed{\vec{\nabla}\times\vec{B} = \mu_0 \vec{J}} \quad -(4)$$

This ~~eqn~~ equation valid for steady current for varying electric field.