UNIT - III

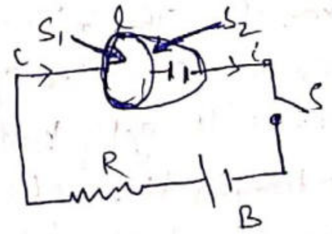
① **DISPLACEMENT CURRENT** 184 Frequently asked: According to Maxwell's postulate A changing electric field in a vaccum, or in a dielectric also Produces a magnetic field.

It is Counterpart (equivalent) of the Phenomenon which observed experimentally by Faraday, According to ~~this~~ That a changing magnetic field Produces an electric field (Induced e.m.f.).

Thus changing electric field is equivalent to a Current which flows as long as the electric field is changing and produces the Same magnetic effect as an ordinary Conduction current. This is known as displacement current. This Concept is responsible for the Success of the E.M. theory.

Maxwell Modified Ampere's Law: He Consider a plane Surface S_1 and a hemispherical Surface S_2, both bounded by the Same Closed path l. During the process of charging the Capacitor Current will flow through S_1 but not through S_2. Therefore if Ampere's Law is applied to the Path l and the Surface S_1, we find.

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 i \quad -①$$



But if Applied to the Path l and Surface S_2 then

$$\oint \vec{B}\cdot d\vec{l} = 0 \quad . ⑪$$

Evidently (Doubtless (नि:संदेह)), both of these equation Can not be Correct. For Removing this Contradiction Maxwell added one more term on the R.H.S of eq ①. He regarded the changing electric field in the gap between the Capacitor Plates (during-charging) as equal to displacement Current i_d to be equal to $\epsilon_0 \frac{d\phi}{dt}$, So Modified Ampere's Law is

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 (i + i_d) = \mu_0 \left(i + \epsilon_0 \frac{d\phi}{dt}\right) \quad -③$$

Imp. 61 <u>Prove that</u> the the displacement current i_d in the gap is equal to the Conduction Current i in the lead wires.

If q is the charge on Capacitor plates at an instant, then the instantaneous electric field between the plates is:-

$E = \frac{q}{\epsilon_0 A}$. then A is the area of each plate, on differentiating it gives,

$$\frac{dE}{dt} = \frac{1}{\epsilon_0 A}\frac{dq}{dt} = \frac{1}{\epsilon_0 A} i \quad \text{or} \quad i = \epsilon_0 A \frac{dE}{dt} \quad \text{--- (iii)}$$

Now the displacement current i_d, by definition

$$i_d = \epsilon_0 \frac{d\phi}{dt} \qquad \text{But } \phi = EA$$

So

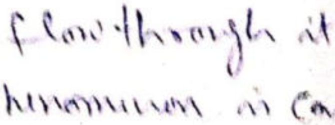
$$i_d = \epsilon_0 \frac{d}{dt}(EA) = \epsilon_0 A \frac{dE}{dt} \quad \text{--- (iv)}$$

Comparing (iii) & (iv) we get $\boxed{i_d = i}$.

So the concept of displacement current makes the current continuous through the entire circuit including the capacitor.

→ Significance of Displacement Current ⇒

- Displacement current is not linked with the motion of charges but it is called current because it produces a magnetic field. There are no charges in vacuum but still displacement current has a finite value.
- Displacement current makes the current continuous across the discontinuity in a conduction current.
  Example- The conduction current given by a battery is not continuous across a capacitor gap [because there is no actual flow of charge across the gap] but displacement current makes the current continuous through entire circuit including the capacitor.
- Magnitude of displacement current is equal to rate of change of electric displacement vector. ($\vec{J}_d = \frac{\partial \vec{D}}{\partial t}$)
- In a good conductor, displacement current is negligible as compared to conduction current.



Electromagnetic Induction → A change in magnetic flux linked with a closed circuit, an e.m.f. is induced (उत्पन्न) in the circuit that lasts (रहता है) only so long as (जब तक) the change in the flux lasts (रहता).

- If the circuit is closed a current flow through it, which is called induced current. This phenomenon is called Electromagnetic Induction. The electromotive force responsible for the induced current is called the induced e.m.f. The direction of induced current depends upon the motion of magnet to coil and vice versa.

Faraday's law of Electromagnetic Induction →

1- Whenever there is a change in the magnetic flux linked with a circuit (or Conductor) an E.m.f. is induced in it.

2- Induced E.m.f. is equal to the negative rate of change of Magnetic flux

$$e = -\frac{d\phi}{dt} \quad -(1)$$

- If the change in magnetic flux in time Δt is $\phi_2 - \phi_1$ then $e = \frac{\phi_2 - \phi_1}{\Delta t} = -\frac{\Delta\phi}{t}$

- If Conductor is in the form of Coil of N turns then

$$e = -N\frac{d\phi}{dt} \quad \text{or} \quad e = -N\frac{\Delta\phi}{\Delta t} \quad -(3)$$

- If Conductor is the part of closed circuit having resistance R, an induced current will flow in circuit, given by

$$i = \frac{e}{R} = -\frac{1}{R}\frac{d\phi}{dt} \quad - - (4)$$

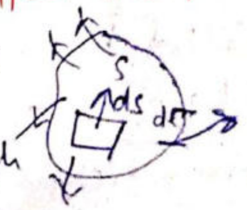
- The charge induced in the circuit in the time Δt will be

$$q = i\Delta t = \frac{1}{R}\frac{\Delta\phi}{\Delta t}\cdot\Delta t = \frac{\Delta\phi}{R} \quad -(5)$$

Lenz's Law. The direction of induced E.M.F. or induced current in the Electromagnetic induction is obtained by using Lenz's Law.
According to this Law " Direction of induced current is such that it always oppose the cause of electromagnetic induction".

Vector form of faraday's Law - [Integral and differential form]

Integral form → According to faradays law, $e = -\frac{d\phi}{dt}$ (1)



Let us consider a closed circuit enclosing a surface S in a magnetic field $\vec{B}$. Then magnetic flux through a small area $d\vec{S}$ will be $\vec{B}\cdot d\vec{S}$ and flux through the whole circuit, $\phi = \int_S \vec{B}\cdot d\vec{S} \quad -(2)$

Due to change in magnetic flux an electric field induced around the circuit. By definition the line integral of the electric field gives the induced e.m.f. in the closed circuit, i.e.

$$e = \oint \vec{E}\cdot d\vec{l} \quad -(3)$$

Here dl is the current element of the circuit. Combining (1), (2) and (3)

$$\oint E\cdot dl = -\frac{d}{dt}\int_S \vec{B}\cdot d\vec{S} \quad -(4)$$

This is integral form of the faraday's law.

Differential form. According to Stoke's law, $\oint \vec{E}\cdot d\vec{l} = \int_S (\text{Curl}\,\vec{E})\cdot d\vec{s}$ -(5)

Then from (4) and (5) $\int_S (\text{Curl}\,\vec{E})\cdot d\vec{s} = -\int_S \frac{dB}{dt}\cdot ds$

$$\boxed{\text{Curl}\,\vec{E} = -\frac{dB}{dt}} \quad -(6)$$

This is differential form of Faraday's Law of E.m.f.

Prob. (1) Find the e.m.f. Induced in a Conducting Rod Moving through a uniform magnetic field. $(e = Bvl)$

(2) A vertical Copper disc of diameter 20 c.m. makes 10 revolutions per sec about a horizontal axis. A uniform magnetic field of 100 gauss acts perpendicular to the plane of disc. Calculate the potential difference between its Centre and rim in volts $(e = 3.14\times10^{-3}$ volt$)$

(3) How many volts are generated in a wire 10 c.m. long which cuts directly across a Magnetic field of flux density 1.4 weber/m² If it moves at a speed of 2 meter/sec $(e = .28$ volt$)$

(4) 

In fig the magnetic flux through the loop is Perpendicular to the Plane of the Coil and directed into the paper $\phi = 3t^2 + 4t + 1$, where ϕ is in milli webers. What is the magnitude of e.m.f. Induced in the loop at $t = 2$ sec? What is the direction through R. $[e = 1.6\times10^{-2}$ volt (outward)$]$

Self Induction

The Phenomenon of the Production of an Induced e.m.f. in a Circuit itself due to change in current through it is Called Self Induction and Induced e.m.f. is Called back e.m.f.

Coefficient of Self Induction or Self Inductance -

Magnetic field at any Point due to Current Carrying Coil is directly Proportional to the Current, therefore magnetic flux (ϕ) Passing through the Coil will be Proportional to the Current I

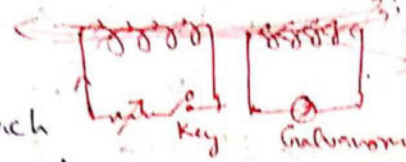
$$\phi \propto I \rightarrow \phi = LI \rightarrow L = \frac{\phi}{I} \quad -(1)$$

According to Faraday's Law, $e = -\frac{d\phi}{dt} = -L\frac{dI}{dt}$ -(2)

Unit → 1 Henry = 1 Wb/A = 1 Volt-Sec/A

For Long Solenoid. $L = \mu_0 n^2 A l$ -(3)

Mutual Inductance



if two Coils are Placed close to each other and Current in one of them is changed. Current is Induced in the other this Phenomenon is called "Mutual Inductance".

The Coil in which Current is changed is called the Primary coil and the other one in which current is Induced is called Secondary Coil.

→ Let the current flowing through the Primary Coil P is I_1 And Magnetic flux linked with the Secondary Coil 'S' is ϕ_2 Per turn and there are N_2 turn in the Secondary. Then the net magnetic flux linked with S will proportional to I

$$N_2\phi_2 \propto I_1 \quad \text{or} \quad N_2\phi_2 = MI_1 \quad (1)$$

$$\boxed{M = \frac{N_2\phi_2}{I_1}}$$

'M' is proportionality Constant and known as Mutual Inductance or Coefficient of Mutual Induction.

If e.m.f. Induced in Coil S due to change in Current in Coil P be e_2 then

$$e_2 = -\frac{d}{dt}(N_2\phi_2) \quad (2)$$

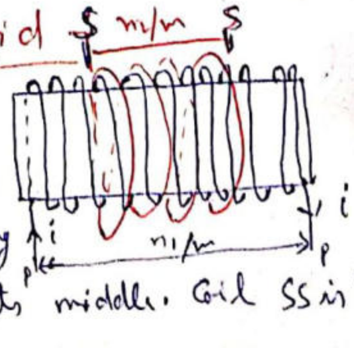
from (1) and (2)

$$e_2 = -\frac{d}{dt}(MI_1) = -M\frac{dI_1}{dt} \quad (3)$$

Similarly If Coil P and S are Interchanged, then e.m.f. Induced in P, due to the change in current I_2 in S_2 will be,

$$e_1 = -M\frac{dI_2}{dt} \quad (4)$$

Mutual Inductance between two Solenoid



Let a long Solenoid of radius r have n_1 Turn Per unit length and a current i is flowing through it. Let another Coil SS having n_2 turn Per unit length be wound over PP in its middle. Coil SS is Small and has radius r. "Due to current i in PP magnetic field B Produced in it is, $\boxed{B = \mu_0 n i}$ -(1)

Therefore magnetic flux (ϕ_1) linked with each turn of SS will be.

$$\phi_1 = BA = B\pi r^2 = \mu_0 n_1 \pi r^2 \quad -(2)$$

If length of SS is l it will have $n_2 l$ Turns in it and hence magnetic flux linked with it will be.

$$\phi = \phi_1 n_2 I = (\mu_0 n_1 l \pi r^2) n_2 I = \mu_0 n_1 n_2 I \pi r^2 l \quad -(3)$$

But we know $\phi = MI$ then

$$\boxed{M = \mu_0 n_1 n_2 \pi r^2 l} \quad -(4)$$

(Q1) A 1m long wire is moving with velocity $10 ms^{-1}$ perpendicular to its length and magnetic field of 1000 Gauss. Find e.m.f. Induced in the wire.

$$e = -\frac{d\phi}{dt} = -\frac{d}{dt}(BA)$$

Since B is constant,

$$e = -B\frac{dA}{dt} = -Blv$$

Given $B = 10^3$ Gauss, $L = 1m$, $V = 10 ms^{-1}$

$$|e| = .1 \times 1 \times 10 = 1 \text{ volt}$$

(Q2) If Current in primary coil of a transformer change from .5A to 0 in 1 mili second, the emf Induced in the secondary is 500V, Calculate Mutual Inductance between the Coil.

Induced e.m.f. in Secondary coil $e_2 = -M\frac{\Delta i}{\Delta t}$

$$M = -\frac{e_2}{\Delta i/\Delta t}$$

Given $|e_2| = 500V$, $\Delta i = 0 - .5 = -.5A$, $\Delta t = 1ms = 1 \times 10^{-3}$ sec

$$\boxed{M = -\frac{500 \times 10^{-3}}{-.5} = 1H}$$

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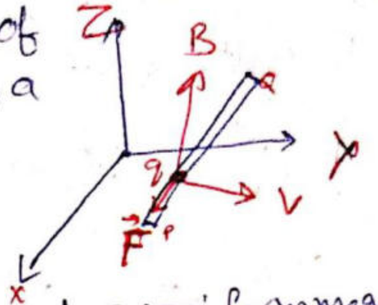
(3) Solution → · Consider a thin conducting rod of length l moving with constant velocity $\vec{v}$ in a direction perpendicular to its length.



→ [The length of rod along x-axis and it is moving parallel to y-axis. There exist a uniform magnetic field $\vec{B}$ along z-axis]

→ [Since the rod is conducting it contains charged particles. These charges will also move with the rod with the velocity ($\vec{v}$) of the rod]

Then a particle of charge q will experience a force

$$\vec{F} = q(\vec{v} \times \vec{B}) \quad \text{—(1)}$$

- [This force $\vec{F}$ pushes electron towards one end of the rod. Consequently the other end develops a positive charge ~~and negative charge~~. This process of separation of positive and negative charges continues until an electric field $\vec{E}$ is developed inside the rod. The force which balance the magnetic force ($\vec{F}$) which given by eq (1)]

Thus in steady state,

$$q\vec{E} + q(\vec{v} \times \vec{B}) = 0$$

$$\vec{E} = -(\vec{v} \times \vec{B})$$

So the emf induced between the ends of the rod

$$\boxed{e = \int_0^l \vec{E} \cdot d\vec{l} = \left| -\int_0^l (v \times B)\, dl \right| = Bvl}$$

This expression is a special case of the general expression of $e = -\frac{d\phi}{dt}$

(4) Solution The magnetic flux linked with the disc is $\phi = BA$

The induced e.m.f between rim and centre of disc, $e = -\frac{d\phi}{dt} = -\frac{d(BA)}{dt}$

Given $B = 100$ gauss $= 100 \times 10^{-4} = 10^{-2}$ weber/m², $r = 10$ cm $= .1$ m

Area swept by disc in unit time, $\frac{dA}{dt} = \pi r^2 \times$ number of revolutions per second

$= 3.14 \times (.1)^2 \times 10 = .314$

So $e = -B\left(\frac{dA}{dt}\right) = 10^{-2} \times .314 = 3.14$ milivolt

(5) Solution →

emf generated in wire $e = Bvl = 1.4 \times 2 \times .10$

$= .28$ volt

(6) The rate of change of magnetic flux ($\phi = 3t^2 + 4t + 1$)

$\frac{d\phi}{dt} = 6t + 4$, At $t = 2$ sec, $\frac{d\phi}{dt} = 6 \times 2 + 4$

$= 16$ milli weber/sec

Since $e = \frac{d\phi}{dt} = 1.6 \times 10^{-2}$ volt

The direction of Current through R according to Lenz's law must be such as to oppose the Increasing flux, therefore The Current in R must create a flux directed outward from the paper. This will be so when the current in R is from left to right according to right hand screw rule.

Maxwell's Equation's

The divergen ($\vec{\nabla}\cdot$–) and Curl ($\vec{\nabla}\times$–) relations of electromagnetic fields are called Maxwell's equation.

① **First equation** - (It is differential form of Gauss Law in electrostatic)

$$\boxed{\text{div}\,\vec{D} = \rho \text{ or } \vec{\nabla}\cdot\vec{D} = \rho} \quad \text{— (A)}$$

Derivation - According to Gauss's theorem for closed surface.

$$\int_S \vec{E}\cdot d\vec{S} = \frac{1}{\epsilon_0} q \qquad (1)$$

If density of a uniform charge distribution at any point is ρ then

$$q = \int_V \rho\, dV \quad \text{— (2)}$$

from eq (1) and (2)

$$\int_S \vec{E}\cdot d\vec{S} = \frac{1}{\epsilon_0}\int_V \rho\, dV \quad \text{— (3)}$$

we know ~~from~~ Gauss divergen theorem, $\int_S \vec{E}\cdot d\vec{S} = \int_V (\vec{\nabla}\cdot\vec{E})\, dV$ — (4)

So from (3) & (4)

$$\int_V (\text{div}\,\vec{E})\, dV = \frac{1}{\epsilon_0}\int_V \rho\, dV$$

$$\boxed{\text{div}\,\vec{E} = \frac{\rho}{\epsilon_0} \text{ or } \text{div}\,\vec{D} = \rho} \quad \text{— (5)}$$

② **Second equation** → (It is differential form of Gauss law in Magneto-statics)

$$\boxed{\text{div}\,\vec{B} = 0 \text{ or } \vec{\nabla}\cdot\vec{B} = 0} \quad \text{— (B)}$$

Magnetic lines of force are either closed curves or they extend up to infinity. The number of lines of forces entering any closed surface is exactly equal to that leaving the surface, i.e. "The net magnetic flux through a closed surface is always zero.

$$\boxed{\int_S \vec{B}\cdot d\vec{S} = 0} \quad \text{— (1)}$$

we know Gauss divergen theorem $\int_S \vec{B}\cdot d\vec{S} = \int_V (\vec{\nabla}\cdot\vec{B})\, dV$ (2)

This equation valid for any arbitrary volume, so the integration must be zero, hence from (1) & (2)

$$\boxed{\vec{\nabla}\cdot\vec{B} = 0 \text{ or } \text{div}\,\vec{B} = 0} \quad \text{— (3)}$$

(3) Third equation. It is Faraday's law of electromagnetic Induction

$$\boxed{\text{Curl } \vec{E} = -\frac{\partial \vec{B}}{\partial t}} \quad \text{— (C)}$$

According to faraday's Law, the Induced e.m.f. ($e = \oint E \cdot dl$) Produced in a closed Circuit is equal to the negative of the rate of change of magnetic flux linked with that circuit i.e

$$e = -\frac{d\phi}{dt} = -\frac{d}{dt}\int_s \vec{B} \cdot d\vec{s} \quad \text{— (1)}$$

and $e = \oint \vec{E} \cdot d\vec{l}$ — (2)

from (1) and (2)

$$\oint \vec{E} \cdot d\vec{l} = -\frac{d}{dt}\int_s \vec{B} \cdot d\vec{s} \quad \text{— (3)}$$

or

$$\oint E \cdot dl = -\int_s \frac{\partial \vec{B}}{\partial t} \cdot d\vec{s} \quad (3)$$

According stokes theorem

$$\oint E \cdot dl = \int_s (\text{Curl } \vec{E}) \cdot ds \quad \text{— (4)}$$

So from (3) & (4) we get $\int_s (\text{Curl } \vec{E}) \cdot ds = -\int_s \frac{\partial \vec{B}}{\partial t} \cdot \vec{ds}$ (5)

This equation valid for any arbitrary Surface, hence both the vectors in the Integral must be equal at each point

⇒

$$\boxed{\text{Curl } \vec{E} = -\frac{\partial \vec{B}}{\partial t}} \quad \text{— (6)}$$

(4) Fourth equation → [It is Modified Ampere's law which Modified by Maxwell]

$$\boxed{\text{Curl } \vec{B} = \mu_0 \left[\vec{J} + \frac{\partial \vec{D}}{\partial t}\right]} \quad \text{— D}$$

According to Ampere's Law, $\oint \vec{B} \cdot d\vec{l} = \mu_0 I = \mu_0 \int_s \vec{J} \cdot d\vec{s}$ (1)

According Stoke's theorem $\oint \vec{B} \cdot d\vec{l} = \int_s (\text{Curl } \vec{B}) \cdot d\vec{s}$

Hence $\int_s (\text{Curl } \vec{B}) \cdot d\vec{s} = \mu_0 \int \vec{J} \cdot d\vec{s}$

So that $\text{Curl } \vec{B} = \mu_0 \vec{J}$ — (2)

This equation valid for steady current for varying electric fields,

$$\text{div } \vec{J} + \frac{\partial \rho}{\partial t} = 0 \quad (\text{from eq. of Continuity})$$

Taking div of eq (2) we get. $\text{div Curl } \vec{B} = \mu_0 \text{ div } \vec{J}$

But $\vec{\nabla} \cdot (\vec{\nabla} \times \vec{B}) = 0$ so $\text{div } \vec{J} = 0$

- eq (1) obtain from Ampere's Law is not in accordance with the equation of Continuity, Hence it needs corrections

According to Maxwell, eq (2) is incomplete for the definition of total current density. For this Maxwell suggested that we must add some vector J_d to it. Then total current density must be solenoid i.e.

$$\text{Curl } \vec{B} = \mu_0 (\vec{J} + \vec{J}_d) \quad — (3)$$

$$\therefore \quad 0 = \text{div } (J + J_d)$$

or

$$\text{div} \cdot J_d = - \text{div} \vec{J} = \frac{\partial \rho}{\partial t}$$

(From 1st eq)

But $\vec{\nabla} \cdot \vec{D} = \rho \Rightarrow \text{div } \vec{J}_d = \frac{\partial}{\partial t} (\vec{\nabla} \cdot \vec{D})$

where $\vec{D}$ is electric displacement vector, thus

$$\vec{\nabla} \cdot \vec{D} = \frac{\partial}{\partial t} (\vec{\nabla} \cdot \vec{D}) = \nabla \cdot \frac{\partial \vec{D}}{\partial t} = \text{div} \left(\frac{\partial D}{\partial t} \right)$$

$$\text{div } \vec{J}_d = \text{div } \frac{\partial D}{\partial t} \quad \text{so that} \quad \vec{J}_d = \frac{\partial \vec{D}}{\partial t} \quad -(4)$$

Hence eq (3) will be

$$\boxed{\text{Curl } \vec{B} = \mu_0 \left(\vec{J} + \frac{\partial \vec{D}}{\partial t} \right)} \quad — (5)$$

or $\vec{D} = \epsilon_0 \vec{E}$ then

$$\boxed{\text{Curl } \vec{B} = \mu_0 \left(\vec{J} + \epsilon_0 \frac{\partial \vec{E}}{\partial t} \right)} \quad - (6)$$

Maxwell's Equation in Integral form

(1) **First**, $\vec{\nabla} \cdot \vec{D} = \rho$ —(1) Integrating this equation over volume, we get $\int \vec{\nabla} \cdot \vec{D} \, dV = \int_V \rho \, dV = q$ —(2)

But from Gauss divergence theorem

$$\int_V (\vec{\nabla} \cdot \vec{D}) \, dV = \int_S \vec{D} \cdot d\vec{s} \quad —(3)$$

so from eq (2) and (3)

$$\boxed{\int_S \vec{D} \cdot d\vec{s} = \int_V \rho \, dV = q} \quad \text{or} \quad \boxed{\int_S \vec{E} \cdot d\vec{s} = \frac{q}{\epsilon_0}} \quad —(4)$$

(2) **Second** → $\vec{\nabla} \cdot \vec{B} = 0$ —(1) Integrating this equation over all volume V then,

$$\int_V (\vec{\nabla} \cdot \vec{B}) \, dV = 0 \quad —(2)$$

we know Gauss divergence theorem.

$$\int_V (\vec{\nabla} \cdot \vec{B}) \, dV = \int_S \vec{B} \cdot d\vec{s} \quad —(3)$$

so from (2) and (3)

$$\boxed{\int_S \vec{B} \cdot d\vec{s} = 0} \quad —(4)$$

③ Third. $\vec{\nabla}\times\vec{E} = -\frac{\partial B}{\partial t}$ —(1) Integrating over a surface "S" bounded by a curve we get, $\int_S (\vec{\nabla}\times\vec{E})\cdot d\vec{S} = -\int \frac{\partial \vec{B}}{\partial t}\cdot d\vec{S} = -\frac{\partial}{\partial t}\int \vec{B}\cdot d\vec{S}$ —(2)

from Stokes theorem we get. $\oint \vec{E}\cdot d\vec{l} = -\frac{\partial}{\partial t}\int_S \vec{B}\cdot d\vec{S}$ —(3)

So electromotive force ($e = \oint E\cdot dl$) around a closed path is equal to negative rate of change of magnetic flux bounded by the path.

[Stokes theorem = $\boxed{\int_S (\vec{\nabla}\times\vec{E})d\vec{S} = \oint \vec{E}\cdot d\vec{l}}$

④ Fourth Curl $B = \mu_0 (\vec{J} + \frac{\partial \vec{D}}{\partial t})$ —(1)

Integrating over the surface S we get

$$\int_S (\text{Curl}\,\vec{B})\cdot d\vec{S} = \mu_0 \int_S (\vec{J} + \frac{\partial \vec{D}}{\partial t})\cdot d\vec{S} \quad -(2)$$

using Stokes theorem $\int_S (\text{Curl}\,\vec{B})\cdot dS = \oint \vec{B}\cdot d\vec{l}$ (3)

So from (2) and (3)

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 \int_S (J + \frac{\partial D}{\partial t}) dS \quad -(4)$$

The magnetomotive force around a closed path is equal to the conduction current plus the time derivative of the electric displace and through any surface bounded by the path.

Maxwell's equation for free space

In a free medium - devoid (without)(खाली),(शून्य) of free charge and current ($\rho = 0$ and $\vec{J} = 0$)

Then

(1) $\vec{\nabla}\cdot\vec{E} = 0$ (2) $\vec{\nabla}\cdot\vec{B} = 0$

(3) $\vec{\nabla}\times\vec{E} = -\frac{\partial \vec{B}}{\partial t}$ (4) $\vec{\nabla}\times\vec{B} = \mu\epsilon\frac{\partial E}{\partial t}$

Electromagnetic wave equation

Let us consider a uniform linear medium having permittivity (ϵ), permeability (μ) and conductivity (σ), but not any charge or current other than that determined by ohm's law.

Then $D = \epsilon E$, $B = \mu H$, $J = \sigma E$ and $\rho = 0$ —(1)

and Maxwell's equations

div $D = 0$ div $B = 0$ curl $E = -\frac{\partial B}{\partial t}$ Curl $H = J + \frac{\partial D}{\partial t}$ (2)

In this case takes the form, $\text{div}\,\vec{E}=0$ —③ $\text{div}\,\vec{H}=0$ ④

$\text{Curl}\,\vec{E} = -\mu\dfrac{\partial \vec{H}}{\partial t}$ ⑤ $\text{Curl}\,\vec{H} = \sigma\vec{E} + \varepsilon\dfrac{\partial \vec{E}}{\partial t}$ ⑥

→ Taking Curl of eq ⑤ $\text{Curl Curl } E = -\mu\dfrac{\partial}{\partial t}(\text{Curl } H)$ —⑥

Putting the value of Curl H from ⑥ then we get

$$\text{Curl Curl }\vec{E} = -\mu\frac{\partial}{\partial t}\left(\sigma\vec{E} + \varepsilon\frac{\partial \vec{E}}{\partial t}\right) = -\sigma\mu\frac{\partial \vec{E}}{\partial t} - \varepsilon\mu\frac{\partial^2 \vec{E}}{\partial t^2} \quad (7)$$

→ Similarly taking the Curl of eq ⑥ and Putting the value of Curl $\vec{E}$ from eq ⑤, we get

$$\text{Curl Curl }\vec{H} = -\sigma\mu\frac{\partial \vec{H}}{\partial t} - \varepsilon\mu\frac{\partial^2 \vec{H}}{\partial t^2} \quad (8)$$

Now using vector identity $\boxed{\text{Curl Curl }\vec{A} = \text{grad div }\vec{A} - \nabla^2 A}$.

and Keeping in view eq ③ and ④ ($\text{div}\,\vec{E}=0$ and $\text{div}\,\vec{H}=0$)

Then eq ⑦ and ⑧ Take the form

$$\nabla^2 E - \sigma\mu\frac{\partial \vec{E}}{\partial t} - \varepsilon\mu\frac{\partial^2 \vec{E}}{\partial t^2} = 0 \quad (9)$$

$$\nabla^2 H - \sigma\mu\frac{\partial \vec{H}}{\partial t} - \varepsilon\mu\frac{\partial^2 H}{\partial t^2} = 0 \quad (10)$$

Equation ⑨ and ⑩ represent wave equation which govern the e.m. field in a homogeneous (समांगी), linear medium In which the charge density is zero.

Plane electromagnetic wave in vacuum

For free space, $\rho=0$, $\sigma=0$, $\mu=\mu_0$ and $\varepsilon=\varepsilon_0$ —①

Then Maxwell's eq. will be, $\text{div}\,\vec{E}=0$, ⓐ $\text{div}\,\vec{H}=0$ ⓑ

$\text{Curl}\,\vec{E} = -\mu_0\dfrac{\partial H}{\partial t}$ ⓒ $\text{Curl}\,H = \epsilon_0\dfrac{\partial \vec{E}}{\partial t}$ ⓓ ②

Taking Curl of eq ②ⓒ, $\text{curl Curl }\vec{E} = -\mu_0\dfrac{\partial}{\partial t}(\text{curl }\vec{H})$

and use ②ⓓ in this eq. we get

$$\vec{E} = -\mu_0\epsilon_0\frac{\partial^2 E}{\partial t^2} \quad (3)$$

Similarly taking curl of ②ⓓ, and use ②ⓒ in it

we get. $\text{Curl curl }\vec{H} = -\epsilon_0\dfrac{\partial}{\partial t}(\text{Curl }\vec{E}) = -\epsilon_0\dfrac{\partial}{\partial t}\left(\mu_0\dfrac{\partial H}{\partial t}\right)$

$$= -\mu_0\epsilon_0\frac{\partial^2 H}{\partial t^2} \quad (4)$$

Now using vector identity, $\boxed{\text{Curl Curl }\vec{A} = \text{grad div }\vec{A} - \nabla^2 A}$

and keeping in view ②ⓐ and ②ⓑ ($\text{div}\,\vec{E}=0$ and $\text{div}\,\vec{H}=0$)

Then (3) and (4) will be

$$\nabla^2 \vec{E} - \mu_0 \epsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2} = 0 \quad (5) \text{ and } \nabla^2 \vec{H} - \mu_0 \epsilon_0 \frac{\partial^2 \vec{H}}{\partial t^2} = 0 \quad (6)$$

eq. (5) and (6) represent wave equation governing electromagnet field $\vec{E}$ and $\vec{H}$ in free space. eq (5) and (6) are similar scalar wave equation

$$\boxed{\nabla^2 u - \mu_0 \epsilon_0 \frac{\partial^2 u}{\partial t^2} = 0} \quad -(7)$$

Here u is scalar and can stand for one of the component of $\vec{E}$ and $\vec{H}$. ~~this~~ is eq (7) resemble with the general wave equation

$$\nabla^2 u = \frac{1}{v^2} \cdot \frac{\partial^2 u}{\partial t^2} \quad -(8)$$

where v is velocity of wave. Comparing eq (7) and (8) we use field vector $\vec{E}$ and $\vec{H}$ are propagated in free space as waves at a speed equal to

$$v = \frac{1}{\sqrt{\mu_0 \epsilon_0}} = \sqrt{\frac{4\pi}{\mu_0 4\pi\epsilon_0}} = \sqrt{\frac{4\pi \times 9 \times 10^9}{4\pi \times 10^{-7}}} = 3 \times 10^8 \text{ m/s}$$

which is velocity of light c.

Therefore, It is reasonable to write c the speed of light in place of $\frac{1}{\mu_0 \epsilon_0}$ so eq. 5, 6 and 7

$$\left.\begin{aligned} \nabla^2 \vec{E} - \frac{1}{c^2} \frac{\partial^2 \vec{E}}{\partial t^2} &= 0 \\ \nabla^2 \vec{H} - \frac{1}{c^2} \frac{\partial^2 \vec{H}}{\partial t^2} &= 0 \\ \nabla^2 u - \frac{1}{c^2} \frac{\partial^2 u}{\partial t^2} &= 0 \end{aligned}\right| \quad (9)$$

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