University Institute of Engineering & Technology CSJMU KANPUR

Department of Electronics & Communication Engineering

Course Name- Basic Electrical & Electronics Engg. (ECE S 101)

Branch ECE

UNIT 2: Network Theory & Concept

Er. Atul Kumar Agnihotri

1 Introduction

This paper is a review of some of the important Concept of electric network theory. We deal only with passive, linear network elements.

A:- Network Concept

Electric network theory deals with two primitive quantities, which we will refer to as:

1. Potential(or voltage), and
2. Current.

Current is the actual flow of charged carriers, while difference in potential is the force that causes that flow. As we will see, potentials a single-valued function that maybe uniquely defined over the nodes of a network. Current, on the other hand, flows through the branches of the network. Figure 1 shows the basic notion of a branch, in which a voltage is defined across the branch and a current is defined to flow through the branch. A network is a collection of such elements, connected together by wires.

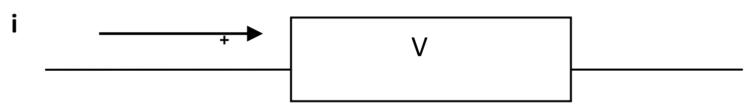


Figure 1: Basic Circuit Element

Network topology is the interconnection of its elements. That, plus the constraints on voltage and current imposed by the elements themselves, determines the performance of the network, described by the distribution of voltages and currents throughout the network.
Two important concepts must be described initially. These are of **"loop" and "node".**

1. **A loop** in the network is any closed path through two or more elements of the network. Any non-trivial network will have at least one such loop.

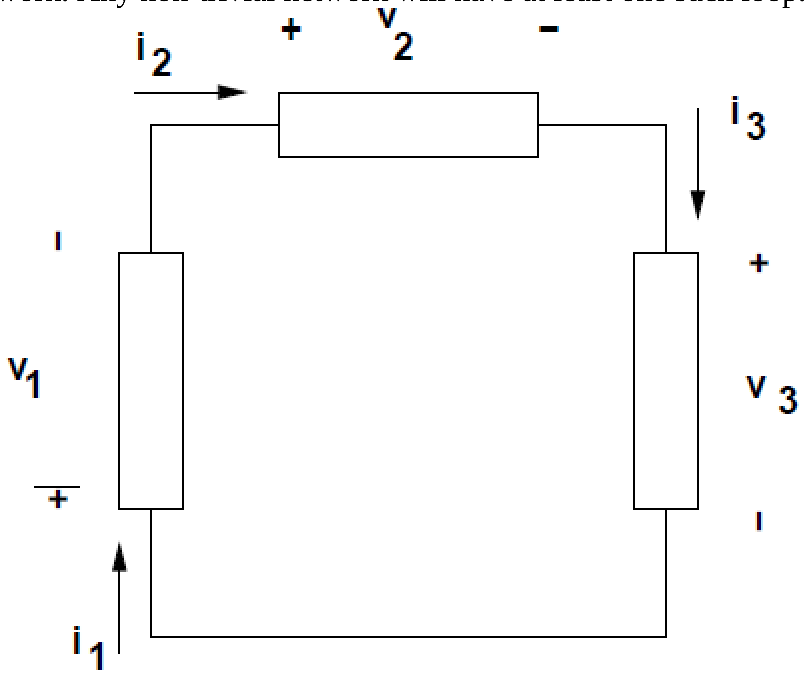


Figure 2: This is a loop

2. **A node** is a point at which two or more elements are interconnected.

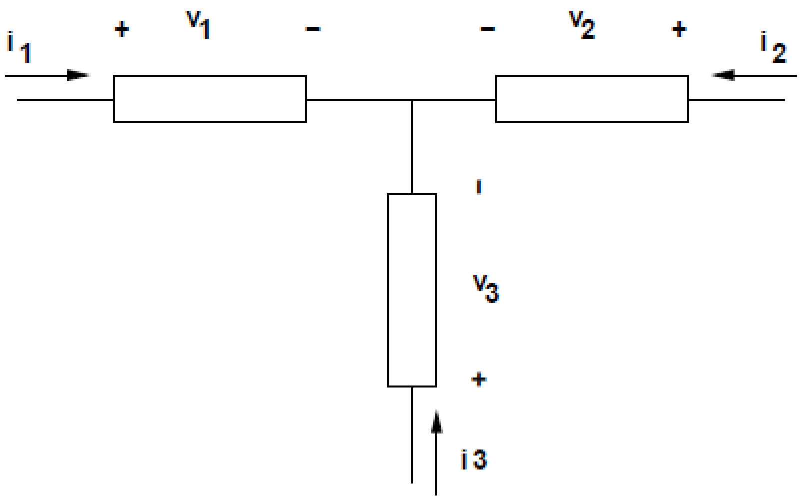


Figure 3: This is a node

3.The two fundamental laws of network theory are known as Kirchoff's Voltage Law (KVL), and Kirchoff's Current Law (KCL).These laws describe the topology of the network, . They are simply stated as:

- Kirchoff's Voltage Law states that, around any loop of a network, the sum of all voltages, taken in the same direction, is zero:

vk =0 (1) loop

- Kirchoff's Current Law states that, at any node of a network, the sum of all currents entering the node is zero:

ik =0 (2) node

1Note that KVLis a discrete version of Faraday's Law, valid to the extent that no time-varying flux links the loop. KCL is just conservation of current, allowing for no accumulation of charge at the node.

Network elements affect voltages and currents in one of three ways:

1. Voltage sources constrain the potential difference across their terminals to be of some fixed value (the value of the source).
2. Current sources constrain the current through the branch to be of some fixed value.
3. All other elements impose some sort of relationship, either linear or nonlinear, between voltage across and current through the branch.

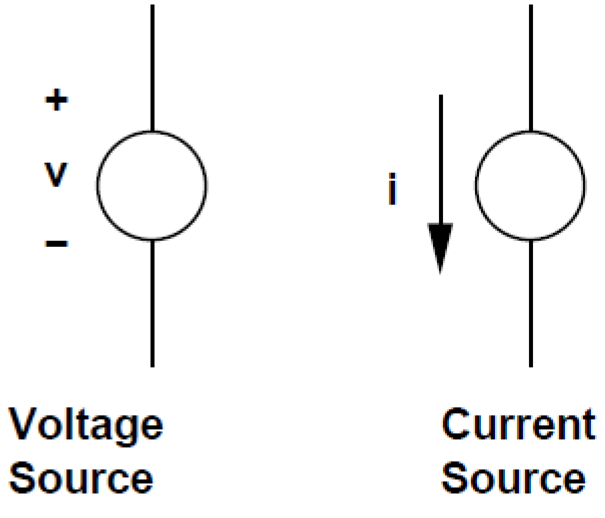


Figure 4: Notation for voltage and current sources

Voltage and current sources can be either independent or dependent. Independent sources have values which are, as the name implies, independent of other variables in a circuit. Dependent sources have values which depend on some other variable in a circuit. A common example of a dependent source is the equivalent current source used for modelling the collector junction in a transistor. Typically, this is modelled as a current dependent current source, in which collector current is taken to be directly dependent on emitter current. Such dependent sources must be handled with some care, for certain tricks we will be discussing below do not work with them.

For the present time, we will consider, in addition to voltage and current sources, only impedance elements, which impose a linear relationship between voltage and current. The most common of these is the resistance, which imposes the relationship which is often referred to as Ohm's law: $v_r = R\, i_r$ (3)

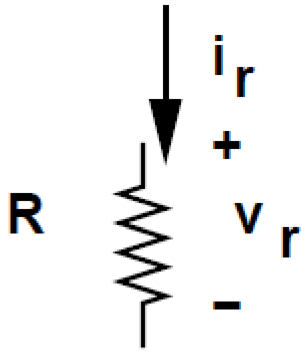


Figure 5: Resistance Circuit Element

4. Examples: Voltage and Current Dividers

Figure 6 may be used as an example to show how we use all of this. See that it has one loop and three nodes. Around the loop, KVL is: $V_s - v_1 - v_2 = 0$ At the upper right-hand node, we have, by KCL: $i_1 - i_2 = 0$ The constitutive relations imposed by the resistances are:

$$v_1 = R_1 i_1$$

$v_2 = R_2 i_2$ Combining these, we find that: $V_s = (R_1 + R_2) i_1$ We may solve for the voltage across, say, R2, to obtain the so-called voltage divider relationship:

$$v_2 = V_s \frac{R_2}{R_1 + R_2} \qquad (4)$$

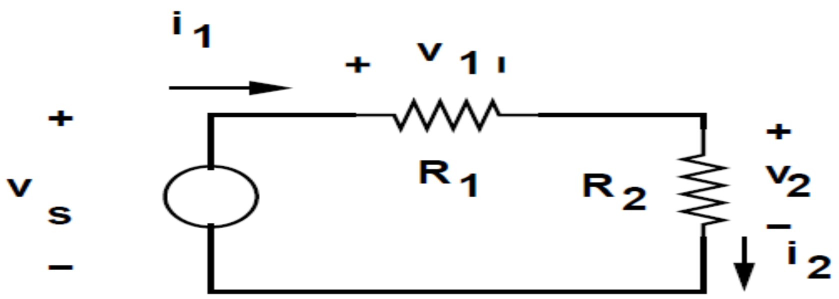


Figure 6: Voltage Divider

A second example is illustrated by Figure 7. Here, KCL at the top node yields: Is −i1 −i2 =0 And KVL, written around the loop that has the two resistances, is: R1i1 −R2i2 =0 Combining these together, we have the current divider relationship:

$$i_2 = I_s \frac{R_1}{R_1 + R_2}$$

Once we have derived the voltage and current divider relationships, we can use them as part of our "intellectual toolkit", because they will always be true

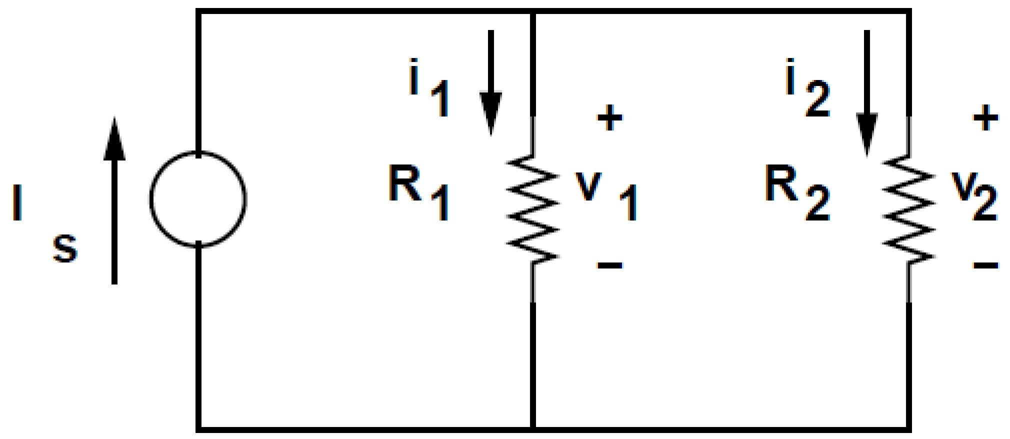


Figure 7: Current Divider

5. Loop and Node Equations

There are two well-developed formal ways of solving for the potentials and currents in networks, often referred to as loop and node equation methods. They are closely related, using KCL and KVL together with element constraints to build sets of equations which may then be solved for potentials and currents.

- In the node equation method, KCL is written at each node of the network, with currents expressed in terms of the node potentials. KVL is satisfied because the node potentials are unique.

- In the loop equation method, KVL is written about a collection of closed path in the network. "Loop currents" are defined, and made to satisfy KCL ,and the branch voltages are expressed in terms of them.

The two methods are equivalent and a choice between them is usually a matter of personal preference. The node equation method is probably more widely used, and lends itself well to computer analysis.

To illustrate how these methods work, consider the network of Figure 12.
This network has three nodes. We are going to write KCL for each of the nodes, but note that only two explicit equations are required. If KCL is satisfied at two of the nodes, it is

automatically satisfied at the third. Usually the datum node is the one for which we do not write the expression

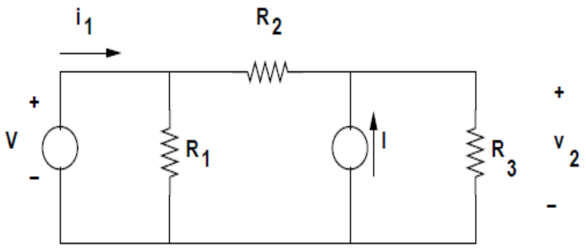


Figure 12: Sample Network

KCL written for the two upper nodes of the network is:

$$-i_1 + \frac{V}{R_1} + \frac{V - v_2}{R_2} = 0 \quad (9)$$

$$-I + \frac{v_2 - V}{R_2} + \frac{v_2}{R_3} = 0 \quad (10)$$

These two expressions are easily solved for the two unknowns, i_1 and v_2:

$$v_2 = \frac{R_3}{R_2 + R_3} V + \frac{R_2 R_3}{R_2 + R_3} I$$

$$i_1 = \frac{R_1 + R_2 + R_3}{R_1 (R_2 + R_3)} V - \frac{R_3}{R_2 + R_3} I$$

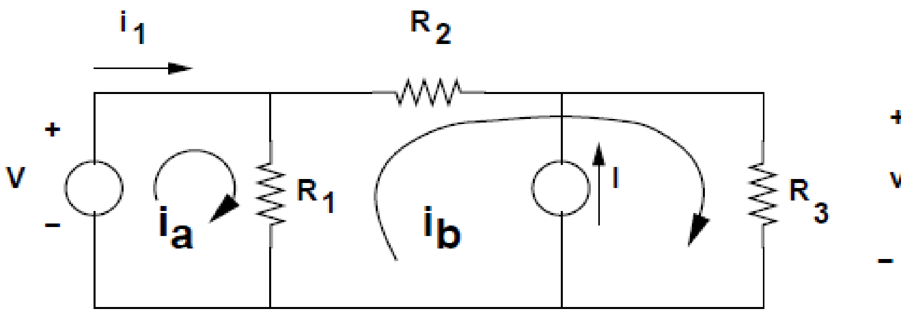


Figure 13: Sample Network Showing Loops

1. The loop that includes the voltage source and R1.

2. The loop that includes R1, R2, and R3. It is also necessary to define loop currents ,which we will denote as ia and ib. These are the currents circulating around the two loops. Note that where the loops intersect, the actual branch current will be the sum of or difference

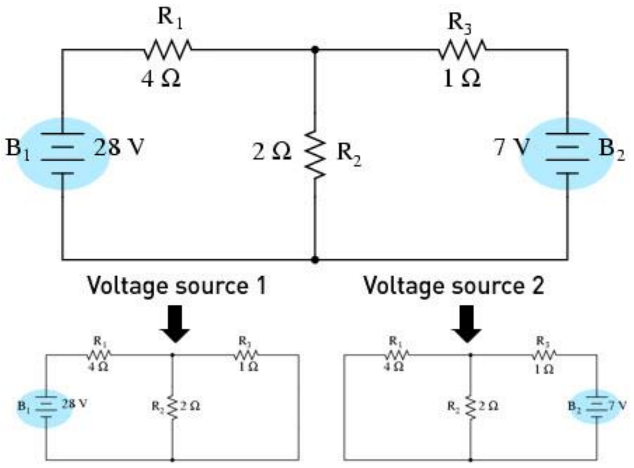
between loop currents. For this example, assume the loop currents are defined to be circulating counter-clockwise in the two loops. The two loop equations are: V + R1(ia −ib) = 0 (11)

R1(ib −ia)+R2ib + R3(ib −I) = 0 (12) These are equally easily solved for the two unknowns, in this case the two loop currents ia and ib.

B:- Network Theorem

1. Super Position Theorem

The Superposition theorem is a way to determine the currents and voltages present in a circuit that has multiple sources (considering one source at a time). The superposition theorem states that in a linear network having a number of voltage or current sources and resistances, the current through any branch of the network is the algebraic sum of the currents due to each of the sources when acting independently.



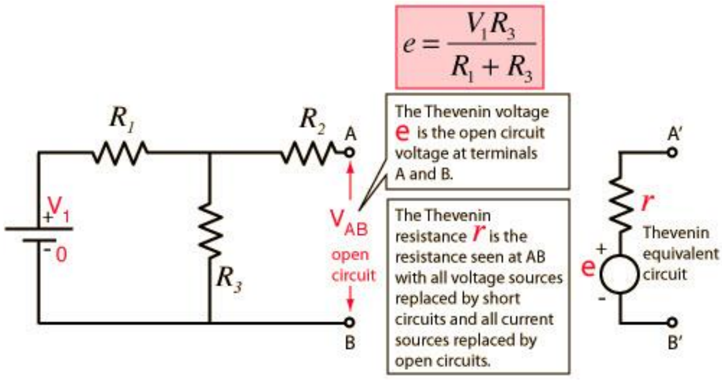
Super Position Theorem

Superposition theorem is used only in linear networks. This theorem is used in both AC and DC circuits wherein it helps to construct Thevenin and Norton equivalent circuit.

In the above figure, the circuit with two voltage sources is divided into two individual circuits according to this theorem's statement. The individual circuits here make the whole circuit look simpler in easier ways. And, by combining these two circuits again after individual simplification, one can easily find parameters like voltage drop at each resistance, node voltages, currents, etc.

2. Thevenin's Theorem

Statement: A linear network consisting of a number of voltage sources and resistances can be replaced by an equivalent network having a single voltage source called Thevenin's voltage (Vthv) and a single resistance called (Rthv).



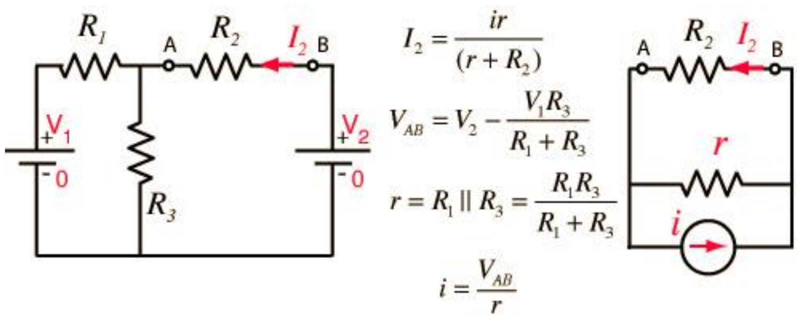
Thevenin's Theorem

The above figure explains how this theorem is applicable for circuit analysis. Thevinens voltage is calculated by the given formula between the terminals A and B by breaking the loop at the terminals A and B. Also, Thevinens resistance or equivalent resistance is calculated by shorting voltage sources and open circuiting current sources as shown in the figure.

This theorem can be applied to both linear and bilateral networks. It is mainly used for measuring the resistance with a Wheatstone bridge.

3. Norton's Theorem

This theorem states that any linear circuit containing several energy sources and resistances can be replaced by a single constant current generator in parallel with a single resistor.

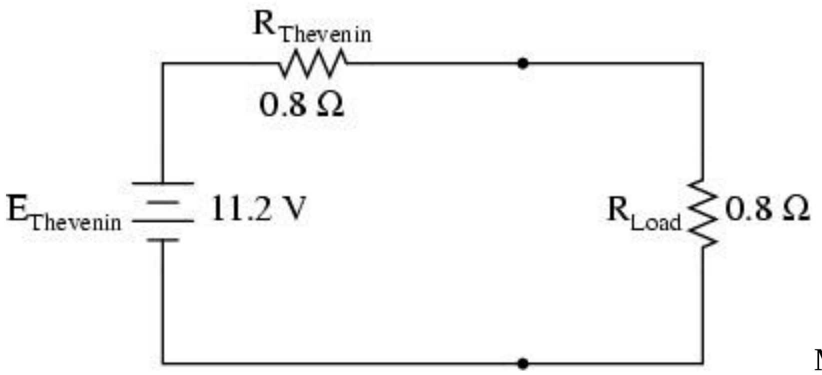


Norton's Theorem

This is also the same as that of the Thevinens theorem, in which we find Thevinens equivalent voltage and resistance values, but here current equivalent values are determined. The process of finding these values is shown as given in the example within the above figure.

4. Maximum Power Transfer Theorem

This theorem explains the condition for the maximum power transfer to load under various circuit conditions. The theorem states that the power transfer by a source to a load is maximum in a network when the load resistance is equal to the internal resistance of the source. For AC circuits load impedance should match with the source impedance for maximum power transfer even if the load is operating at different power factors.

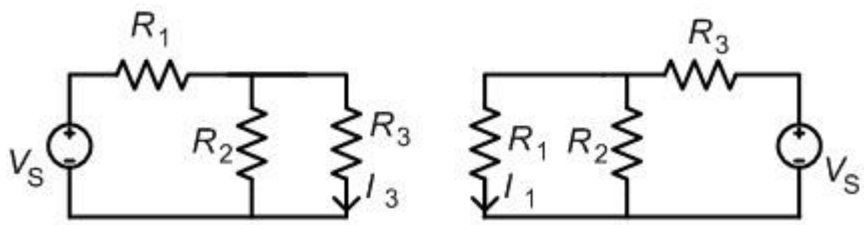


Maximum Power Transfer Theorem

For instance, the above figure depicts a circuit diagram wherein a circuit is simplified up to a level of source with internal resistance using Thevenin's theorem. The power transfer will be maximum when this Thevinens resistance is equal to the load resistance. The Practical application of this theorem includes an audio system wherein the resistance of the speaker must be matched to the audio power amplifier to obtain maximum output.

5. Reciprocity Theorem

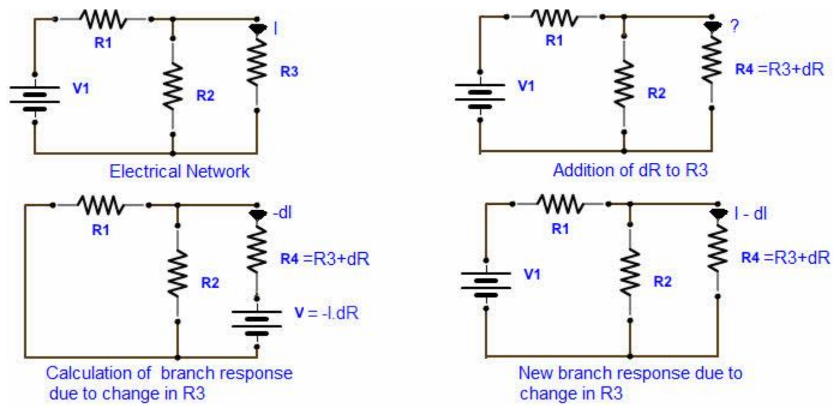
Reciprocity theorem helps to find the other corresponding solution even without further work, once the circuit is analyzed for one solution. The theorem states that in a linear passive bilateral network, the excitation source and its corresponding response can be interchanged.



Reciprocity Theorem

In the above figure, the current in the R3 branch is I3 with a single source Vs. If this source is replaced to the R3 branch and shorting the source at the original location, then the current flowing from the original location I1is the same as that of I3. This is how we can find corresponding solutions for the circuit once the circuit is analyzed with one solution.

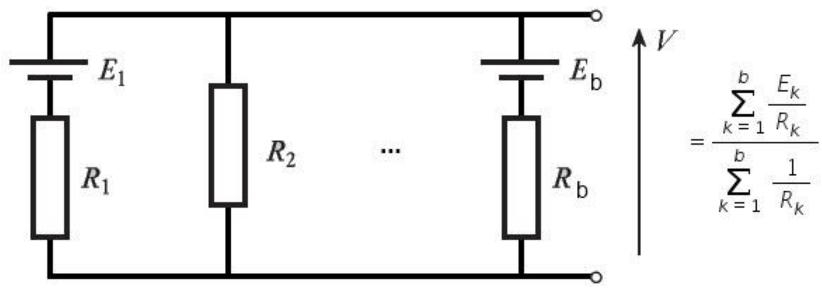
6. Compensation Theorem



Compensation Theorem

In any bilateral active network, if the amount of impedance is changed from the original value to some other value carrying a current of I, then the resulting changes that occur in other branches are same as those that would have been caused by the injection voltage source in the modified branch with a negative sign, i.e., minus of voltage current and changed impedance product. The four figures given above show how this compensation theorem is applicable in analyzing the circuits.

7. Millman's Theorem



Millman's Theorem

This theorem states that when any number of voltage sources with finite internal resistance is operating in parallel can be replaced with a single voltage source with series equivalent impedance. The Equivalent voltage for these parallel sources with internal sources in Millman's theorem is calculated by the below-given formula, which is shown in the above figure.

8. Tellegen's theorem

$$\sum_{k=1}^{n} P_k = V_k \times I_k = 0$$

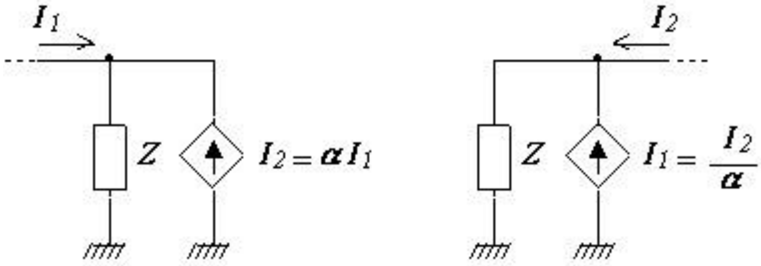
Tellegen's Theorem

This theorem is applicable for circuits with a linear or nonlinear, passive, or active and hysteric or non-hysteric networks. It states that the summation of instantaneous power in the circuit with n number of branches is zero.

9. Substitution Theorem

This theorem states that any branch in a network can be substituted by a different branch without disturbing the currents and voltages in the whole network provided the new branch has the same set of terminal voltages and current as of the original branch. The substitution theorem can be used in both linear and nonlinear circuits.

10. Miller’s Theorem



Miller’s Theorem

This theorem states that in a linear circuit if a branch exists with impedance Z connected between two nodes with nodal voltages, this branch can be replaced by two branches connecting the corresponding nodes to the ground by two impedances. The application of this theorem is not only an effective tool for creating an equivalent circuit but also a tool for designing modified additional electronic circuits by impedance.

These are all basic network theorems used widely in the electrical or electronic circuit analysis. We hope that you might have got some basic ideas about all these theorems.

The attention and interest with which you have read this article are really encouraging for us, and therefore, we anticipate your additional interests on any other topics, projects, and works. So you can write to us about your feedback, comments, and suggestions in the comments section given below.