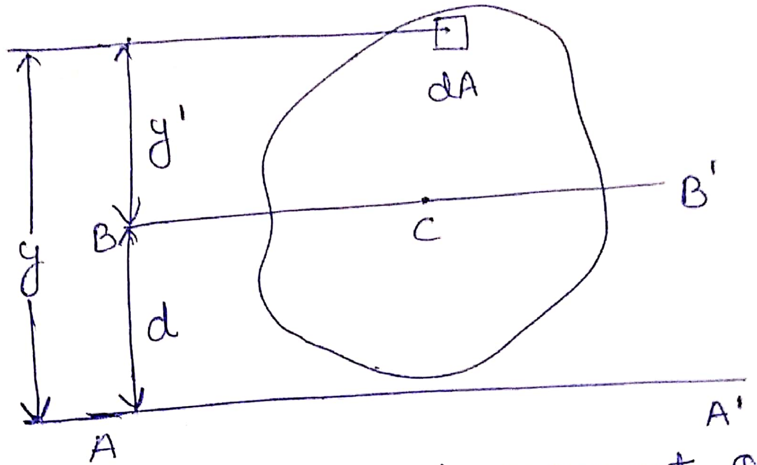
Parallel axis theorem (Another Method)

Here

$\boxed{A \cong M}$

Mass of the body.



Here we are considering that moment of inertia I_x of an area A with respect to an axis AA'. Denote by y, this is the distance from an element of area dA to AA'.

So $I_x = \int y^2 \, dA$

Now consider an axis BB' parallel to AA' through the centroid C of the area known as the centroidal axis. The equation of the moment of inertia becomes.

$$I_x = \int y^2 dA = \int (y' + d)^2 \, dA$$
$$= \int y'^2 dA + 2\int y' dA + d^2 \int dA$$

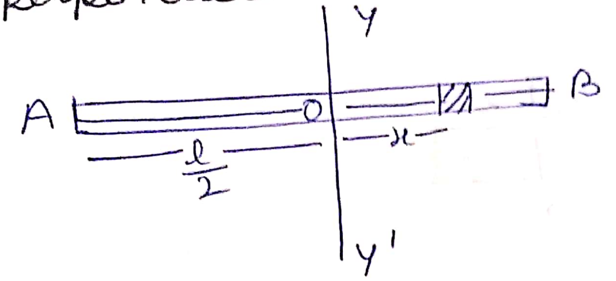
Now $2\int y' \, dA = 0$ because this axis is going through the centre of mass of a body ∴ Moment along centre of mass axis is zero.

$$I_x = \int y'^2 dA + d^2 \int dA$$

$$\boxed{I_x = \bar{I}_x + d^2 A}$$

Moment of inertia of a uniform rod

① about an axis through its centre of mass and perpendicular to its length.



Consider a small element of the rod of length dx at a distance x from the axis through O

we have mass element $= \left(\frac{M}{l}\right) \cdot dx$

Moment of inertia M.I about the axis YOY' through O $= \left(\frac{M}{l}\right) \cdot dx \cdot x^2$

the moment of inertia of the whole rod about the axis YOY' is given -

$$I = \int_{-l/2}^{l/2} \left(\frac{M}{l}\right) \cdot dx \, x^2$$

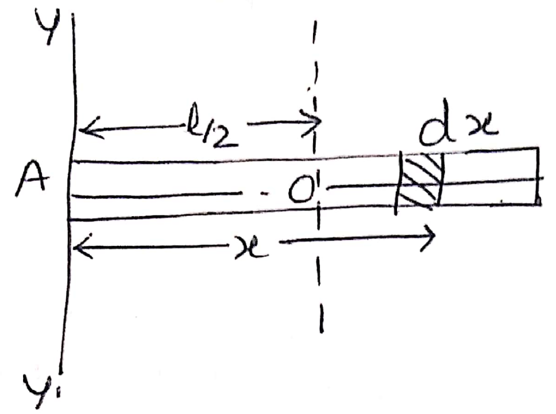
$$= 2 \int_0^{l/2} \left(\frac{2M}{l}\right) \left[\frac{x^3}{3}\right]_0 dx$$

$$= \left(\frac{2M}{l}\right) \left[\frac{x^3}{3}\right]_0^{l/2}$$

$$= \frac{2M}{l} \cdot \frac{l^3}{24}$$

$$\boxed{I = \frac{Ml^2}{12}}$$

(ii) about an axis through one end of the rod and perpendicular to its length.



we calculate the moment of inertia of the rod about the axis passing one end A of the rod ⇒

$$I = \int_0^l \frac{M}{l} (x^2)\, dx$$

$$I = \frac{M}{l} \left[\frac{x^3}{3}\right]_0^l$$

$$\boxed{I = \frac{Ml^2}{3}}$$

Alternate Method -

M.I. of the rod about the axis YAY' = its M.I. about a parallel axis through O

$$I = \frac{Ml^2}{12} + M\left(\frac{l}{2}\right)^2$$

$$I = \frac{Ml^2}{12} + \frac{Ml^2}{4} = \frac{Ml^2}{3}$$

Moment of inertia of a solid cylinder

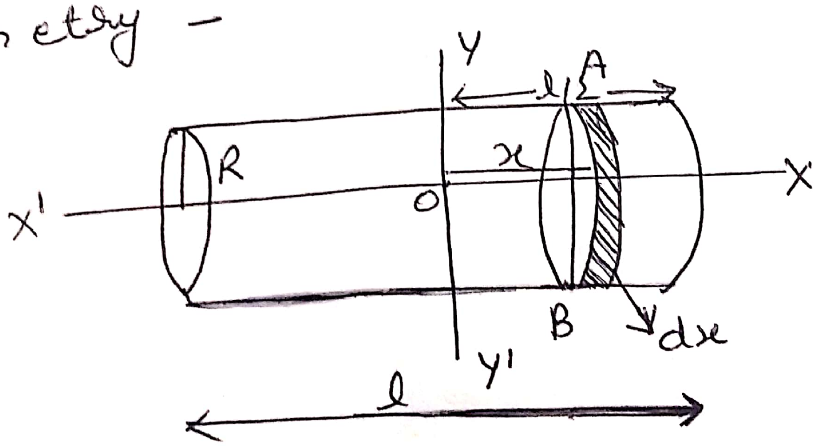
M.I. of the solid cylinder about its axis of cylindrical symmetry

I = M.I. of the thick disc of the same mass and radius about the axis through its centre and perpendicular to its plane

$$I = \frac{MR^2}{2}$$

Case-(I) [About its own axis of cylindrical symmetry]

Case (II) About the axis through its centre and perpendicular to its axis of cylindrical symmetry -



Mass element of the disc $\left(\frac{M}{l}\right) dx$

radius R

M.I. of the disc about its diameter.

$$AB = \left(\frac{M}{l}\right) dx \cdot \frac{R^2}{4}$$