MSE-205 Lecture 5

Crystal Systems and Bravais Lattices

In last lecture difference between primitive and non-primitive cell was discussed along with type of primitive cells. Now in this formation of unit cell will be explained.

For representing the type of distribution of lattice points in space, seven different co-ordinate systems are required. These co-ordinate systems are called crystal systems. The crystal systems are named on the basis of geometrical shape and symmetry. The seven crystal systems are: (1) Cubic (2) Tetragonal (3) Orthorhombic (4) Monoclinic (5) Triclinic (6) Trigonal (or Rhombohedral) and (7) Hexagonal.

Bravais Lattice

In a solid crystalline material, the atoms or molecules are arranged regularly and periodically in all three dimensions. The atomic arrangement in a crystal is called crystal structure. To explain crystal symmetries easily, it is convenient to represent an atom or a group of atoms that repeats in three dimensions in the crystal as a unit. If each such atom or unit of atoms in a crystal is replaced by a point in space, then the resultant points in space are called space lattice. Each point in a space lattice is called a lattice point and each atom or unit of atoms is called basis or pattern. A space lattice represents the geometrical pattern of crystal in which the surroundings of each lattice point is the same. If the surroundings of each lattice point is same or if the atom or all the atoms at lattice points are identical, then such a lattice is called Bravais lattice. On the other hand, if the atom or the atoms at lattice points are not same, then it is said to be a non-Bravais lattice.

Space lattices are classified according to their symmetry. In 1948, Bravais showed that 14 lattices are sufficient to describe all crystals. These 14 lattices are known as Bravais lattices and are classified into 7 crystal systems based on cell parameters. The Bravais lattices are categorized as primitive lattice (P); body-centered lattice (I); face-centered lattice (F) and base-centered lattice (C).

These seven crystal systems and Bravais lattices are described below.

1. Cubic crystal system: In this crystal system, all the unit cell edge lengths are equal and are at right angles to one another i.e., $a = b = c$ and $\alpha = \beta = \gamma = 90°$. In cubic system, there are three Bravais lattices; they are simple (primitive); body-

centered and face-centered as shown in Figure 1. Examples for cubic system are Au, Cu, Ag, NaCl, diamond, etc. In simple cubic lattice, lattice points or atoms are present at the corners of the cube. In body-centered cube, atoms are present at the corners and one atom is completely present at the center of the cube. In the case of face-centered cube, atoms are present at corners and at the centers of all faces of cube.

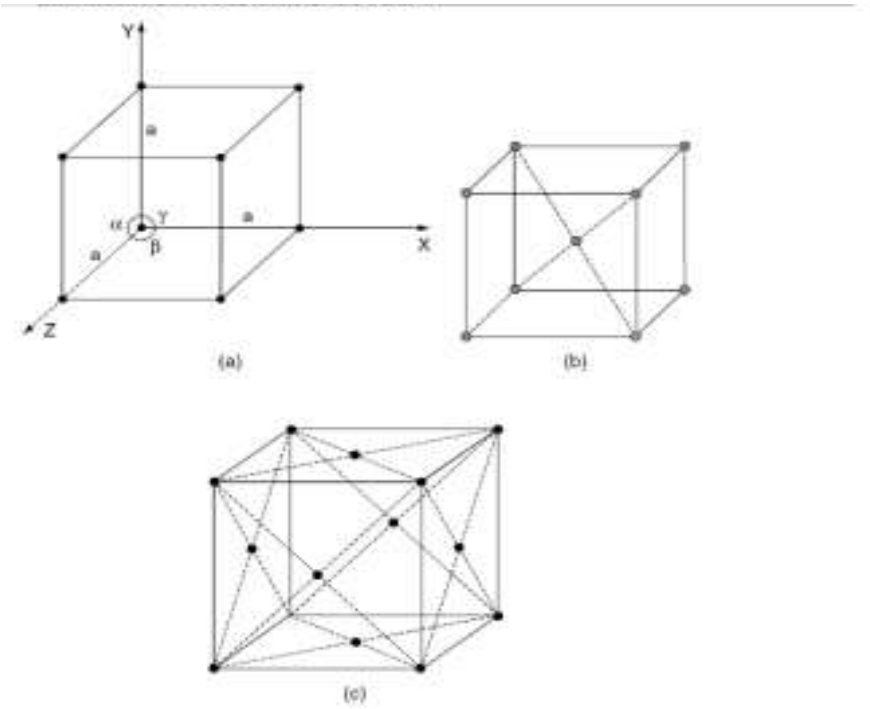

Figure 1 (a) Simple Cube (b) Body Centered Cube (c) Face Centered Cube

2.Tetragonal crystal system:

In this crystal system, two lengths of the unit cell edges are equal whereas the third length is different. The three edges are perpendicular to one another i.e., $a = b \neq c$ and $\alpha = \beta = \gamma = 90°$. In tetragonal system, there are two Bravais lattices; they are simple and body-centered. These are shown in Fig. 2. Examples for tetragonal crystal systems are TiO_2 , SnO_2 , etc.

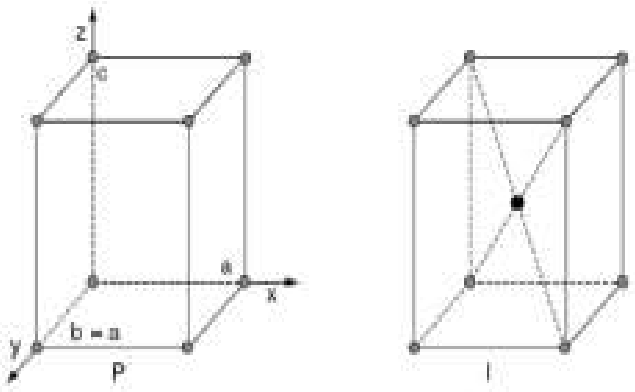

Figure 2 Tetragonal System

3.Orthorhombic crystal system:

In this crystal system, unit cell edge lengths are different and they are perpendicular to one another i.e., $a \neq b \neq c$ and $\alpha = \beta = \gamma = 90°$. There are four Bravais lattices in this system. They are simple, face centered, body centered and base centered. These are shown in Fig. 3. Examples for orthorhombic crystal system are $BaSO_4$, K_2SO_4, $SnSO_4$, etc.

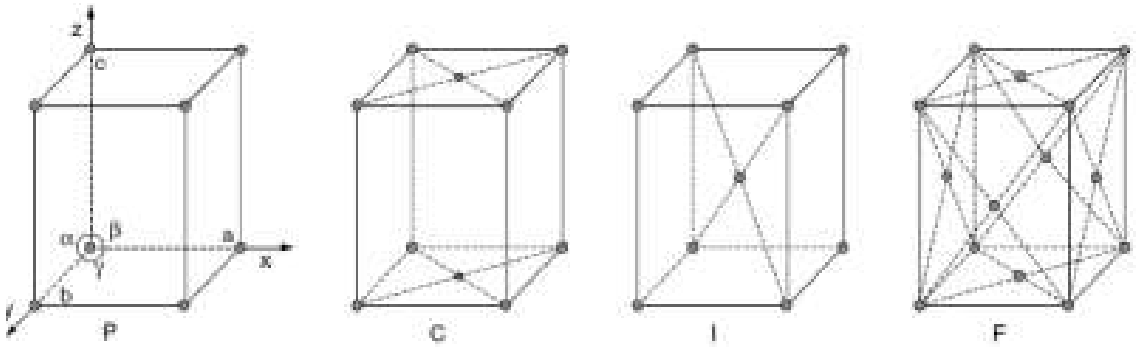

Figure 3 Orthorhombic crystal system

4.Monoclinic crystal system:

In this crystal system, the unit cell edge lengths are different. Two unit cell edges are not perpendicular, but they are perpendicular to the third edge i.e.,$a \neq b \neq c$; $\alpha = \gamma = 90° \neq \beta$. This crystal system has two Bravais lattices; they are simple and base centered. These are shown in Fig. 4. Examples for Monoclinic crystal system are $CaSO_4 .2H_2O$ (gypsum), Na_3AlF_6 (cryolite), etc.

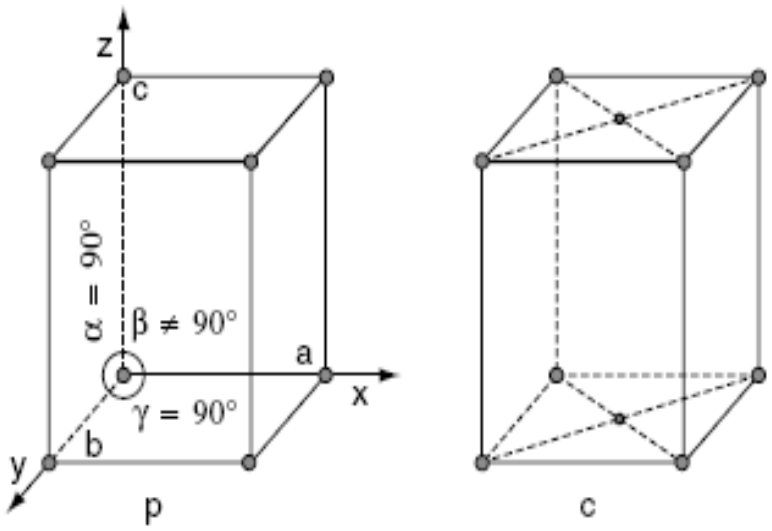

Figure 4. Monoclinic crystal system

5. Triclinic crystal system:

In this crystal system, the unit cell edge lengths are different and are not perpendicular i.e., $a \neq b \neq c$ and $\alpha \neq \beta \neq \gamma \neq 90°$ and all the angles are different. This crystal exists in primitive cell only. This is shown in Fig. 1.10. Examples for triclinic crystal system are $K_2Cr_2O_7$, $CuSO_4.5H_2O$, etc.

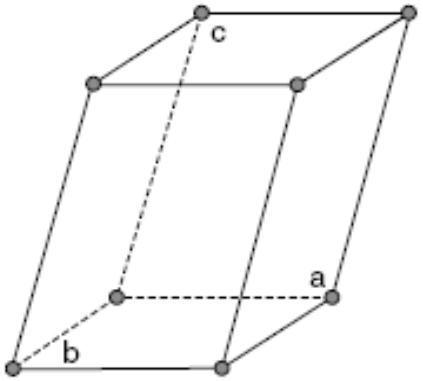

Figure 5 Triclinic Crystal System

6.Trigonal or Rhombohedral crystal system:

In this crystal system, all the lengths of unit cell edges are equal. The angles between the axes are equal but other than 90° i.e., a = b = c and α = β = γ ≠ 90°. The Bravais lattice is simple only as shown in Fig. 6. Examples for Rhombohedral crystal system are As, Bi, Sb, etc.

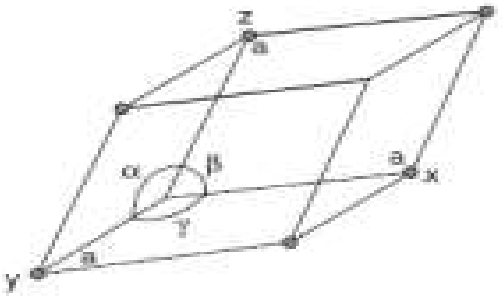

Figure 6 Trigonal or Rhombohedral crystal system

7.Hexagonal crystal system:

In this crystal system, two sides of the unit cell edge lengths are equal and the angle between these edges is 120°. These two edges are perpendicular to the third edge, and not equal in length i.e., a = b ≠ c and α = β = 90°; γ = 120°. The Bravais lattice is primitive only. This is shown in Fig. 7. The atoms in this crystal system are arranged in the form of a hexagonal close pack.

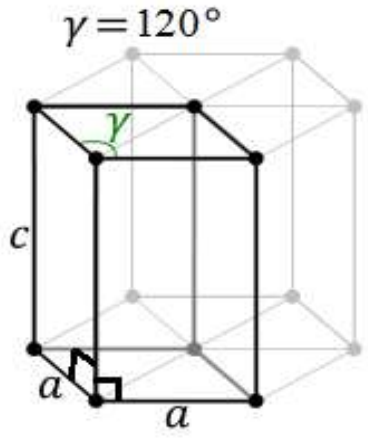

Figure 7 Hexagonal Crystal System

The table 1 given below can be used to summarize types of lattice formation.

Lattice	Types	Edge Length	Angles between faces	Examples
Cubic	Primitive, Body-centred, Face-centred	a = b = c	$\alpha = \beta = \gamma = 90^{\circ}$	NaCl, Copper and ZnS
Tetragonal	Primitive, Body-centred	a = b ≠ c	$\alpha = \beta = \gamma = 90^{\circ}$	White tin, SnO_2, TiO_2 and $CaSO_4$
Orthorhombic	Primitive, Body-centred, Face-centred , End-centred	a ≠ b ≠ c	$\alpha = \beta = \gamma = 90^{\circ}$	Rhombic Sulphur, $BaSO_4$ and KNO_3
Hexagonal	Primitive	a = b ≠ c	$\alpha = \beta = 90^{\circ}$ and $\gamma = 120^{\circ}$	Graphite, ZnO and CdS
Rhombohedral	Primitive	a = b = c	$\alpha = \beta = \gamma \neq 90^{\circ}$	$CaCO_3$ (Calcite) and HgS (cinnabar)
Monoclinic	Primitive, End-centred	a ≠ b ≠ c	$\alpha = \gamma = 90^{\circ}$ $\beta \neq 90^{\circ}$	Sulphur
Triclinic	Primitive	a ≠ b ≠ c	$\alpha \neq \beta \neq \gamma \neq 90^{0}$	H_3PO_3 , $CuSO_4.5\ H_2O$

Table 2: Seven Crystal systems and Fourteen Bravais Lattices

Sl. No	Crystal System	Types of Bravais Lattices	No. of Bravais Lattices	Relation between Lengths and Angles
1	Cubic	P, I, F	3	$a = b = c$ $\alpha = \beta = \gamma = 90°$
2	Tetragonal	P, I	2	$a = b \neq c$ $\alpha = \beta = \gamma = 90°$
3	Orthorhombic	P, I, F, C	4	$a \neq b \neq c$ $\alpha = \beta = \gamma = 90°$
4	Monoclinic	P, C	2	$a \neq b \neq c$ $\alpha = \gamma = 90° \neq \beta$
5	Triclinic	P	1	$a \neq b \neq c$ $\alpha \neq \beta \neq \gamma$
6	Rhombohedral (Trigonal)	P	1	$a = b = c$ $\alpha = \beta = \gamma \neq 90°$
7	Hexagonal	P	1	$a = b \neq c$ $\alpha = \beta = 90°$ $\gamma = 120°$