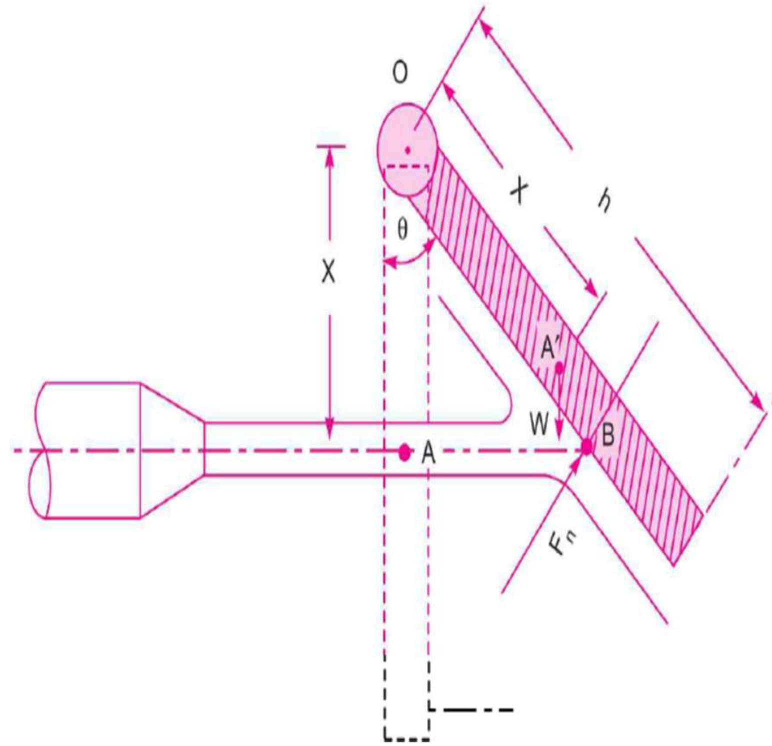
Force exerted by the jet on a hinged plate

$$F_n = \rho A V^2 cos\theta$$

$$sin\theta = \frac{\rho A V^2}{W}$$



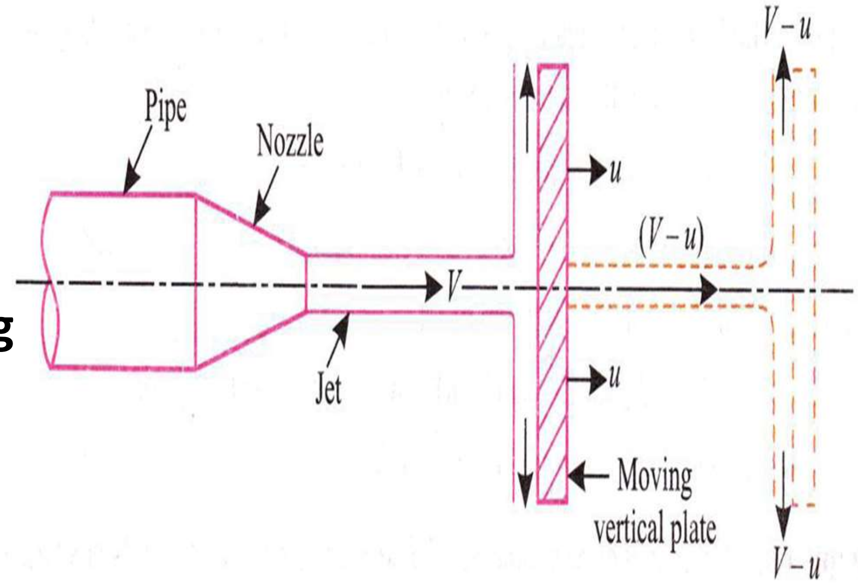
x = distance of the centre of jet from hinge O,

θ = angle of swing about hinge,

W = weight of plate acting at C.G. of the plate.

Force exerted by the jet on a vertical flat plate moving in direction of jet

$$F = \rho A(V^2 - U^2)$$



Work done by the jet on the flat moving plate

W= Force x Distance in the direction of force

W = $\rho a(V^2 - u^2) \times u$

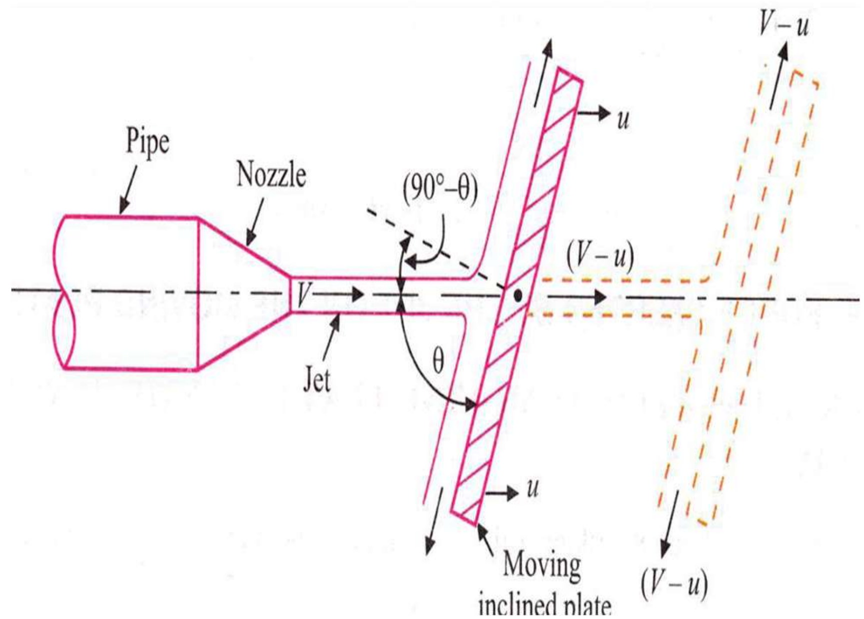
V = velocity of jet
U = Velocity of the plate

Force exerted by the jet on a vertical inclined plate moving in direction of jet

$F_n = \rho A\,(V - u)^2 \sin\theta$

$F_x = F_n \sin\theta$

$F_y = F_n \cos\theta$



$W/Sec = \rho A\,(V - u)^2 \sin^2\theta$

Nm/sec

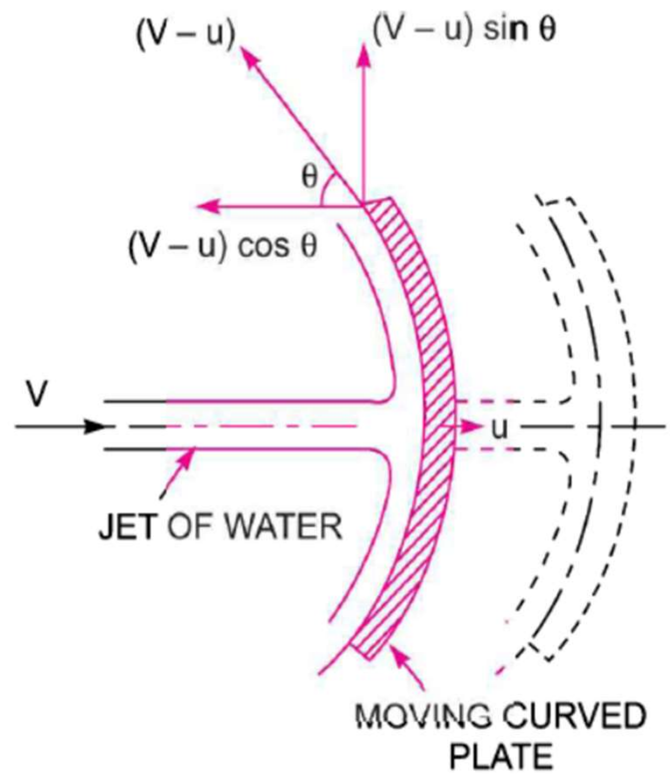
Force exerted by the jet on a vertical inclined plate moving in direction of jet

a. Jet strikes at center

$F_x = \rho A\,(V - u)^2[1+\cos\theta]$

$F_y = -\rho A\,(V - u)^2 \sin\theta$

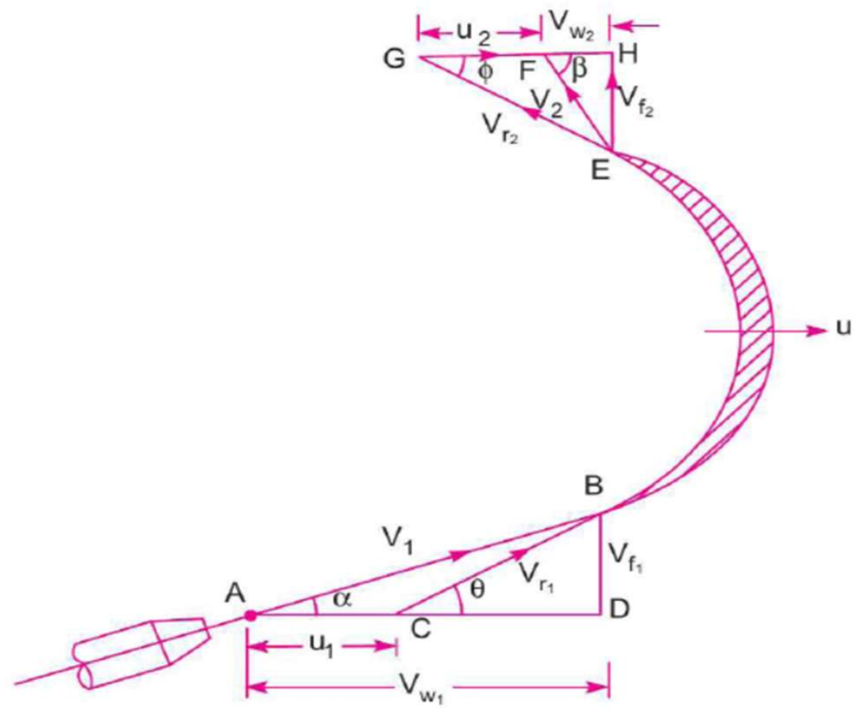
$W/sec = \rho A\,(V - u)^2[1+\cos\theta].\,u$



b. Jet strikes the curved Plate (unsymmetrical) at one end tangentially

$\mathbf{F_x} = \rho\mathbf{A}\,(\mathbf{V} - \mathbf{u})[V_{w1} + V_{w2}]$

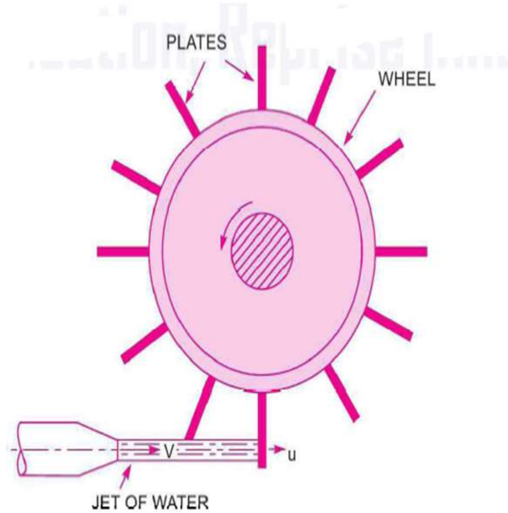
W/sec = $\rho\mathbf{A}\,(\mathbf{V} - \mathbf{u})[V_{w1} \pm V_{w2}]$ u



Force Exerted by a Jet of Water on a Series of Vanes

The force exerted by a jet of water on a *single* moving plate (which may be flat or curved) is not practically feasible.

large number of plates are mounted on the circumference of a wheel at a fixed distance apart as shown in Fig



Jet striking a series of vanes.

V = Velocity of jet,

d = Diamcter of jet,

a = Cross-sectional area of jet,

$= \dfrac{\pi}{4} d^2$

u = Velocity of vane.

$$F_x = \text{Mass per second [Initial velocity – Final velocity]}$$
$$= \rho aV[(V-u)-0] = \rho aV[V-u]$$

Work done by the jet on the series of plates per second

$$= \text{Force} \times \text{Distance per second in the direction of force}$$
$$= F_x \times u = \rho aV[V-u] \times u$$

Efficiency, $$\eta = \frac{\text{Work done per second}}{\text{Kinetic energy per second}} = \frac{\rho aV[V-u]\times u}{\frac{1}{2}\rho aV^3} = \frac{2u[V-u]}{V^2}$$

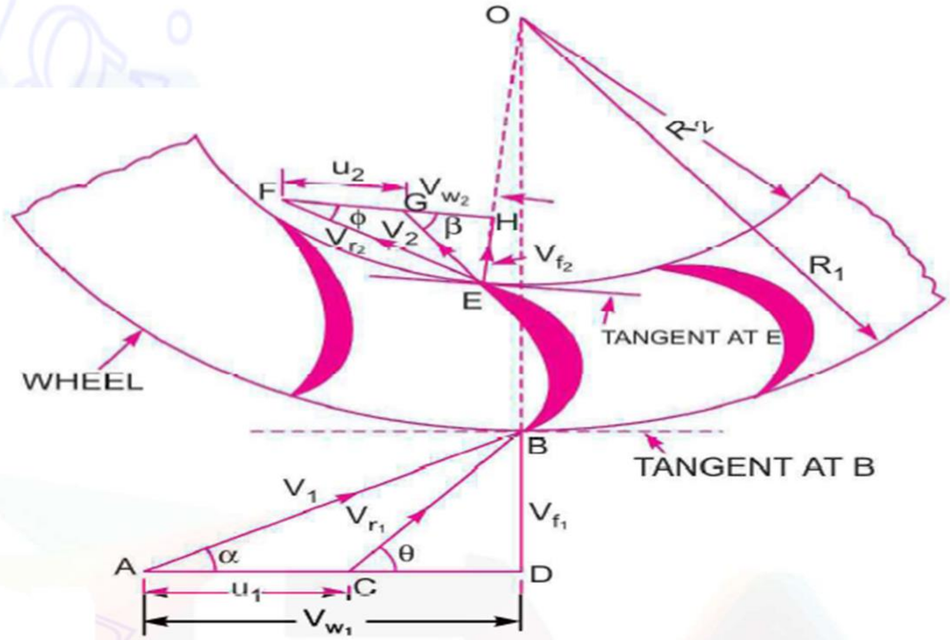
Force Exerted on a Series of Radial Curved Vanes

R_1 = Radius of wheel at inlet of the vane,

R_2 = Radius of the wheel at the outlet of the vane,

ω = Angular speed of the wheel.

$u_1 = \omega R_1$ and $u_2 = \omega R_2$



Series of radial curved vanes mounted on a wheel.

Torque exerted by the water on the wheel,

$$T = \text{Rate of change of angular momentum}$$

$$= [\text{Initial angular momentum per second} - \text{Final angular momentum per second}]$$

$$= \rho a V_1 \times V_{w_1} \times R_1 - (-\rho a V_1 \times V_{w_2} \times R_2) = \rho a V_1 [V_{w_1} \times R_1 + V_{w_2} R_2]$$

Work done per second on the wheel

$$= \text{Torque} \times \text{Angular velocity} = T \times \omega$$

$$= \rho a V_1 [V_{w_1} \times R_1 + V_{w_2} R_2] \times \omega = \rho a V_1 [V_{w_1} \times R_1 \times \omega + V_{w_2} R_2 \times \omega]$$

$$= \rho a V_1 [V_{w_1} u_1 + V_{w_2} \times u_2] \qquad (\because \; u_1 = \omega R_1 \text{ and } u_2 = \omega R_2)$$

If the angle β is an obtuse angle then work done per second will be given as

$$= \rho a V_1 [V_{w_1} u_1 - V_{w_2} u_2]$$

The general expression for the work done per second on the wheel

$$= \rho a V_1 [V_{w_1} u_1 \pm V_{w_2} u_2]$$

If the discharge is radial at outlet, then $\beta = 90°$ and work done becomes as

$$= \rho a V_1 [V_{w_1} u_1] \qquad (\because \; V_{w_2} = 0)$$

$$\eta = \frac{\text{Work done per second}}{\text{Kinetic energy per second}}$$

$$= \frac{\rho a V_1 \left[V_{w_1} \ u_1 \pm V_{w_2} \ u_2 \right]}{\frac{1}{2} \left(\rho a V_1 \right) \times V_1^2} = \frac{2 \left[V_{w_1} \ u_1 \pm V_{w_2} \ u_2 \right]}{V_1^2}$$