A machine element is subjected to the following stresses $\sigma_x = 60$ MPa, $\sigma_y = 45$ MPa, $\tau_{xy} = 30$ MPa. Find the factor of safety if it is made of C45 steal having yield stress as 353 MPa, using the following theories of failure.

(i) Maximum principal stress theory,

(ii) Maximum shear stress theory,

(iii)Von Misses and Hencky ' s Theory

$\sigma_x = 60$ MPa, $\sigma_y = 45$ MPa, $\tau_{xy} = 30$ MPa yield stress, $\sigma_{ys} = 353$ MPa
Poisson ratio $\upsilon = 0.3$.

(i) According to maximum principal stress or Rankine's theory of equivalent stress

$$\sigma_{1,2} = \frac{1}{2} \bullet \left[(\sigma_x + \sigma_y) \pm \sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}\right] \quad \text{...(5}$$

$$\sigma_{1,2} = \frac{1}{2} \bullet \left[(60 + 45) \pm \sqrt{(60 - 45)^2 + 4(30)^2}\right] = 83.42 \text{ MPa}$$

$\sigma_2 = 43.16$ MPa.

$$\therefore \quad \text{FOS} = \frac{\sigma_{ys}}{\sigma_e} = \frac{353}{83.42} = 4.23$$

(ii) According to max. shear stress theory or Guest's theory equivalent stress

$$\tau_{max} = \frac{1}{2}\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2} \quad \text{...(5-}$$

or

$$\sigma_e = \sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2} \quad \left(\because \tau_{max} = \frac{\sigma_e}{2}\right)$$

$$= \sqrt{(60 - 45)^2 + 4(30)^2} = 61.85 \text{ MPa}$$

$$\therefore \quad \text{FOS} = \frac{\sigma_{ys}}{\sigma_e} = 353/61.85 = 5.71$$

(iii) According to shear energy theory or Hencky-Von-Mises theory, equivalent stress

$$\frac{\sigma_{st}}{F.S.} = \sqrt{83.48^2 + 43.16^2 - 83.42 \times 43.16.}$$

$$6968.9 + 1862.7 - 3600.4$$

$$\frac{\sigma_{st}}{F.S} = 72.32 \Rightarrow F.S = \frac{353}{72.32}$$

$$F.S. = 4.8$$

Find the diameter of a rod subjected to a bending moment of 3 kNm and a twisting moment of 1.8 kNm according to the following theories of failure, taking normal yield stress as 420 MPa and factor of safety as 3.

(i) Normal stress theory, (ii) Shear stress theory.

Given data: Bending moment, $M_b = 3$ kNm $= 3 \times 10^6$ N-mm

Twisting moment, $M_t = 1.8$ kNm $= 1.8 \times 10^6$ N-mm

Yield stress, $\sigma_{ys} = 420$ MPa FOS = 3

$$\therefore \quad \text{Allowable stress, } \sigma = \sigma_e = \frac{\sigma_{ys}}{\text{FOS}} = \frac{420}{3} = 140 \text{ MPa}$$

$$\text{Bending stress, } \sigma = \frac{M_b \cdot C}{I} = \frac{3 \times 10^6 \times d/2}{(\pi d^4/64)} = \frac{30.56 \times 10^6}{d^3}$$

$$\sigma = \frac{30.56 \times 10^6}{d^3} = \sigma_x$$

$$\text{Shear stress, } \quad \tau = \frac{M_t r}{J} = \frac{1.8 \times 10^6 \times d/2}{(\pi d^4/32)} = \frac{9.167 \times 10^6}{d^3} = \tau_{xy}$$

(i) According to maximum normal stress theory,

$$\sigma_e = \frac{1}{2}\left[(\sigma_x + \sigma_y + \sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}\right]$$

(Here $\sigma_y = 0$, no stress in $\perp lr$ direction)

$$140 = \frac{1}{2}\left[\frac{30.56 \times 10^6}{d^3} + \sqrt{\left(\frac{30.56 \times 10^6}{d^3}\right)^2 + 4\left(\frac{9.167 \times 10^6}{d^3}\right)^2}\right]$$

$\therefore$ $d = 61.834$ mm

(ii) According to maximum shear stress theory

$$\sigma_e = \sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}$$

$$140 = \sqrt{\left(\frac{30.56 \times 10^6}{d^3}\right)^2 + 4\left(\frac{9.167 \times 10^6}{d^3}\right)^2}$$

$\therefore$ $d = 63.376$ mm

$\therefore$ Recommended diameter, $d = 63.376 \simeq 64$ mm. (Take bigger one always).

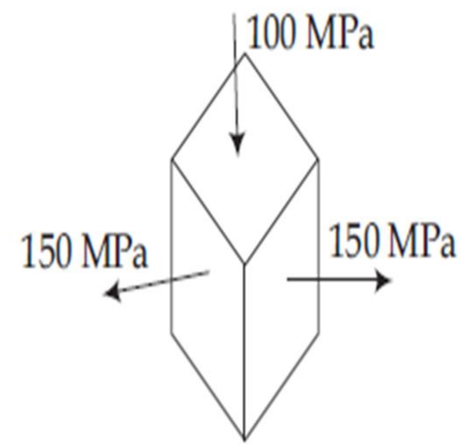
A stressed element is loaded as shown in Fig. Determine the following:

(i) Von-Mises stress,

(ii) Maximum shear stress,

(iii) Maximum normal stress,

(iv) Octahedral shear stress.



Arranging in descending order $150 \geq 150 > -100$

$\therefore$ $\sigma_1 = 150$ MPa,

$\sigma_2 = 150$ MPa and $\sigma_3 = -100$ MPa (compressive)

(i) Von-Mises stress

$$\tau_e = \sqrt{\frac{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2}{2}}$$

$$= \sqrt{\frac{(150 - 150)^2 + (150 + 100)^2 + (-100 - 150)^2}{2}} = 250 \text{ MPa}$$

(ii) Maximum shear stress

$$\tau_{12} = \frac{\sigma_1 - \sigma_2}{2} = \frac{150 - 150}{2} = 0$$

$$\tau_{23} = \frac{\sigma_2 - \sigma_3}{2} = \frac{150 - (-100)}{2} = 125 \text{ MPa}$$

$$\tau_{13} = \frac{\sigma_1 - \sigma_3}{2} = \frac{150 - (-100)}{2} = 125 \text{ MPa}$$

$\therefore$ $\tau_{max} = 125$ MPa (max of these 3 values)

(iii) Maximum normal stress

$\sigma_1 > \sigma_2 > \sigma_3$

then $\sigma_{max} = \sigma_1 = 150$ MPa.

(iv) Octahedral shear stress

$$\tau_e = \frac{1}{3}\sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2}$$

$$= \frac{1}{3}\sqrt{(150 - 150)^2 + (150 - 100)^2 + (-100 - 150)^2} = 117.85 \text{ MPa.}$$

max Strain theory

Saint Venant's theory.

the Failure or yielding occurs at a Point in a member when max Principle Strain in a bi axial Stress system reaches the limiting value of Strain a determined from a simple tensile test.

$$\varepsilon_{max} = \frac{\sigma_1}{E} - \frac{\mu \sigma_2}{E}$$

Strain at yield point.

$$\varepsilon_{max} = \frac{\sigma_1}{E} - \frac{\mu \sigma_2}{E} = \underset{\frac{\sigma_{yp}}{E}}{\overset{\parallel}{\varepsilon}} = \frac{S_{yt}}{E \times F.S.}$$

$S_{yt} \to \sigma_{yp}$ (According theory)

σ_1, σ_2 = max & min principal stress in a biaxial stress system.

ε = Strain at yield point

μ = Poission's ratio.

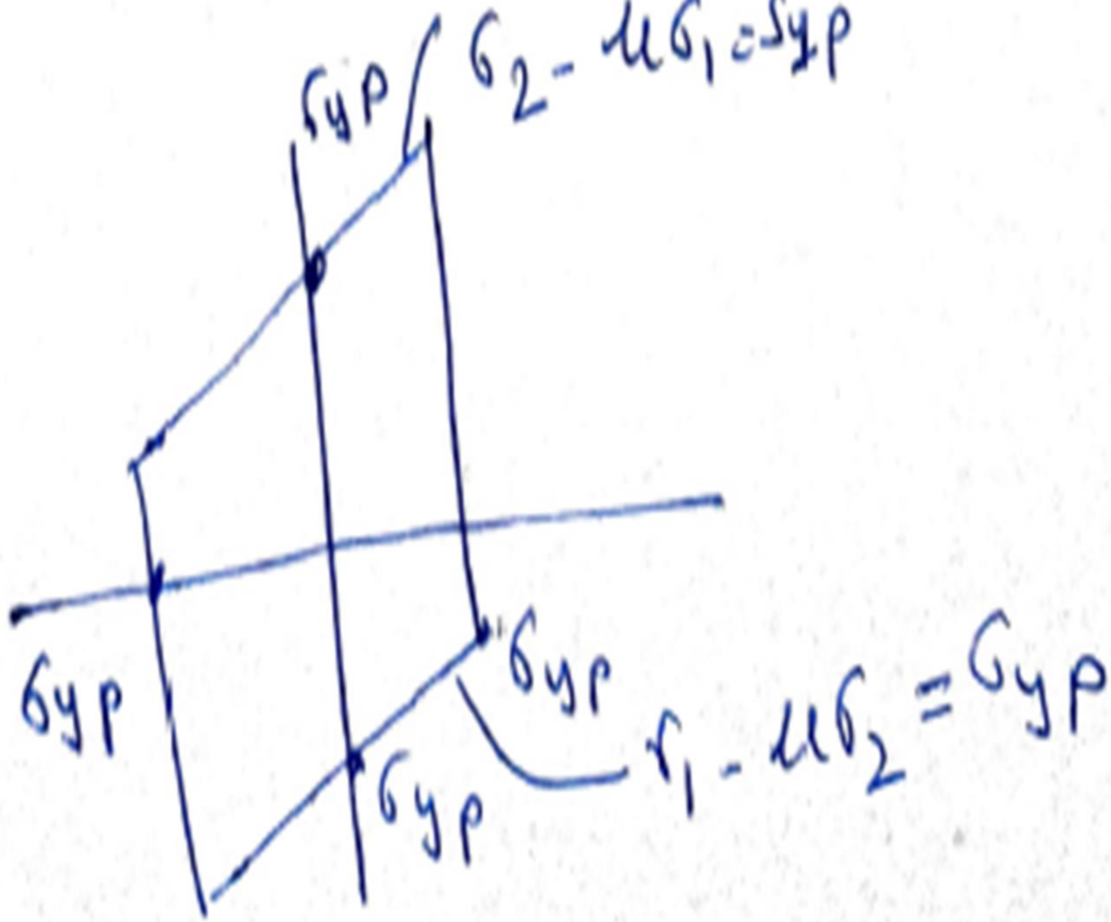
E = young's modulus.

$$\frac{\sigma_1}{E} - \frac{\mu \sigma_2}{E} = \frac{S_{yt}}{E\,F.S}$$

$$\boxed{\sigma_1 - \mu \sigma_2 = \frac{S_{yt}}{FS}}$$

theory is not used - it only gives reliable results in particular ~~cases~~ cases.

Rhombus.



Syp
$\sigma_2 - \mu\sigma_1 = S_{yp}$
σ_{yp}
σ_{yp}
σ_{yp}
$\sigma_1 - \mu\sigma_2 = \sigma_{yp}$

max Strain Energy theory

A Failure or yielding occurs at point in a member when the Strain Energy/unit ~~member~~ volume in a bi axial Stress system reaches the limiting Strain Energy per unit volume as determined from simple Tension test.

⇒ Strain Energy Per unit volume in a bi-axial stress system. $\sigma_3 = 0$

$$U_1 = \frac{1}{2E}\left[\sigma_1^2 + \sigma_2^2 - 2\sigma_1\sigma_2\mu\right]$$

Limiting Strain Energy / unit volume for yielding as determined from simple tension test

$$U_2 = \frac{1}{2E}\left(\frac{\sigma_{yp}}{F.S}\right)^2 = \frac{1}{2E}\sigma^2$$

According theory

$$U_1 = U_2$$

$$\frac{1}{2E}\left[\sigma_1^2 + \sigma_2^2 - 2\mu\sigma_1\sigma_2\right] = \frac{1}{2E}\left(\frac{\sigma_{yp}}{\cancel{F.S}}\right)^2$$

$$\sigma_1^2 + \sigma_2^2 - 2\mu\sigma_1\sigma_2 = \left(\frac{S_{yt}}{f.s}\right)^2 \qquad \sigma_{yp} = \frac{S_{yt}}{f.s}$$