

★ Inverse function (or inverse mapping) :-

Let $f: X \rightarrow Y$ be one-one onto mapping

Let $y \in Y$, since f is onto, there exist $x \in X$ such that $f(x) = y$.

Again since f is one-one this is the only element of X such that $f(x) = y$.

Thus, we have seen that for each $y \in Y$, there is unique $x \in X$ such that $f(x) = y$.

This mapping from Y to X is called inverse of f .

It is denoted by $f^{-1}: Y \rightarrow X$.

Thus $f(x) = y \Rightarrow x = f^{-1}(y)$.

Ques - Let $f: \mathbb{Q} \rightarrow \mathbb{Q}$ be defined by $f(x) = 3x + 5$ ~~find~~ ~~also~~ ~~the~~ ~~formula~~ ~~which~~ ~~define~~ ~~the~~ ~~inverse~~ ~~function~~ f^{-1} .
Prove that f is one-one onto. Find also the formula which define the inverse function f^{-1} .

Soln - Let $x_1, x_2 \in \mathbb{Q}$, then $f(x_1) = f(x_2)$

$$3x_1 + 5 = 3x_2 + 5$$

$$x_1 = x_2$$

$\therefore f$ is one-one.

$$\text{Let } y \in \mathbb{Q} \quad f(x) = y$$

$$3x + 5 = y$$

$$x = \frac{y-5}{3} \in \mathbb{Q} \text{ be such that}$$

$$f(x) = f\left(\frac{y-5}{3}\right) = 3\left(\frac{y-5}{3}\right) + 5 = y$$

$\Rightarrow f$ is onto.

$$f^{-1}: \mathbb{Q} \rightarrow \mathbb{Q}$$

$$x = f^{-1}(y) = \frac{y-5}{3}$$

$\Rightarrow f$ is inverse.

Ques - If \mathbb{Q} is a set of rational numbers and $f: \mathbb{Q} \rightarrow \mathbb{Q}$ is defined by $f(x) = 3x + 2$, $x \in \mathbb{Q}$ then prove that f is one-one onto. Find also f^{-1} .

Solu - Let $x_1, x_2 \in \mathbb{Q}$, then $f(x_1) = f(x_2)$

$$3x_1 + 2 = 3x_2 + 2$$

$$x_1 = x_2$$

$\therefore f$ is one-one.

Let $y \in \mathbb{Q}$ $f(x) = y$

$$3x + 2 = y$$

$$x = \frac{y-2}{3} \in \mathbb{Q} \text{ be such that}$$

$$f(x) = f\left(\frac{y-2}{3}\right) = 3\left(\frac{y-2}{3}\right) + 2 = y$$

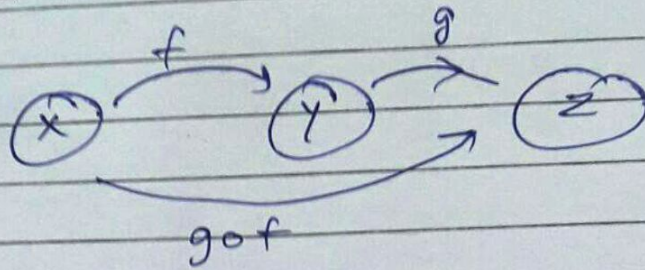
$\therefore f$ is onto.

$$f^{-1}: \mathbb{Q} \rightarrow \mathbb{Q}$$

$$x = f^{-1}(y) = \frac{y-2}{3}$$

$\therefore f$ is inverse

Composite function -



Let x, y, z be three non-empty sets $f: x \rightarrow y$ & $g: y \rightarrow z$ be two mappings, then the composition of f and g is denoted by $g \circ f$ and defined by $g \circ f: x \rightarrow z$

$$(g \circ f)(x) = g(f(x)), \forall x \in x$$

ex - Let f and g be two functions defined by $f(x) = \sqrt{x-1}$ and $g(x) = \sqrt{4-x^2}$ find $g \circ f$.

$$(g \circ f)(x) = g(f(x)) \Leftrightarrow g(\sqrt{x-1})$$

$$= g(\sqrt{x-1})$$

$$= g(\sqrt{4 - (\sqrt{x-1})^2}) = \sqrt{5-x}$$