★ Inverse function (or inverse mapping) :-

Let $f: X \to Y$ be one-one onto mapping

Let $y \in Y$, since f is onto, there exist $x \in X$ such that $f(x) = y$.

Again since f is one-one this is the only element of X such that $f(x) = y$.

Thus, we have seen that for each $y \in Y$, there is unique $x \in X$ such that $f(x) = y$.

This mapping from Y to X is called inverse of f.

It is denoted by $f^{-1}: Y \to X$.

Thus $f(x) = y \Rightarrow x = f^{-1}(y)$.

Ques - Let $f: Q \to Q$ be defined by $f(x) = 3x + 5$ then prove that f is one-one onto. Find also the formula which define the inverse function f^{-1}.

Soln - Let $x_1, x_2 \in Q$, then $f(x_1) = f(x_2)$

$$3x_1 + 5 = 3x_2 + 5$$

$$x_1 = x_2$$

$\therefore f$ is one-one.

Let $y \in Q$ $\quad f(x) = y$

$$3x + 5 = y$$

$$x = \frac{y-5}{3} \in Q \text{ be such that}$$

$$f(x) = f\left(\frac{y-5}{3}\right) = 3\left(\frac{y-5}{3}\right) + 5 = y$$

$\Rightarrow f$ is onto.

$f^{-1}: Q \to Q$

$x = f^{-1}(y) = \frac{y-5}{3}$

$\Rightarrow$ f is inverse.

Ans – If Q is a set of rational numbers and $f: Q \to Q$ is defined by $f(x) = 3x+2$, $x \in Q$ then prove that f is one-one onto. Find also f^{-1}.

Soln – Let $x_1, x_2 \in Q$, then $f(x_1) = f(x_2)$

$3x_1 + 2 = 3x_2 + 2$

$x_1 = x_2$

$\therefore$ f is one-one.

Let $y \in Q$ $f(x) = y$

$3x + 2 = y$

$x = \frac{y-2}{3} \in Q$ be such that

$f(x) = f\left(\frac{y-2}{3}\right) = \cancel{3}\left(\frac{y-\cancel{2}}{\cancel{3}}\right) + \cancel{2} = y$

$\therefore$ f is onto.

$f^{-1}: Q \to Q$

$x = f^{-1}(y) = \frac{y-2}{3}$

$\therefore$ f is inverse

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Composite function –

$$\begin{tikzcd} X \arrow[r, "f"] \arrow[rr, bend right, "g\circ f"'] & Y \arrow[r, "g"] & Z \end{tikzcd}$$

Let X, Y, Z be three non-empty sets $f: X \to Y$ & $g: Y \to Z$ be two mappings, then the composition of f and g is denoted by $g \circ f$ and defined by $g\circ f : X \to Z$

$$(g\circ f)(x) = g(f(x)), \ \forall \ x \in X$$

us – Let f and g be two functions defined by $f(x) = \sqrt{x-1}$ and $g(x) = \sqrt{4-x^2}$ find $g\circ f$.

$$(g\circ f)(x) = g(f(\sqrt{x-1}) \Leftrightarrow g(f(x))$$

$$= g(\sqrt{x-1})$$

$$= g(\sqrt{4-(\sqrt{x-1})^2}) = \sqrt{5-x}$$