

Ques - If $f(x) = [x]$ and $g(x) = |x|$. Find the value of $g \circ f \left(-\frac{5}{3} \right) - f \circ g \left(-\frac{5}{3} \right)$.

Solu - $(g \circ f)(x) = g(f(x)) = g([x]) = |[x]|$

$(f \circ g)(x) = f(g(x)) = f(|x|) = [|x|]$

$|[x]| - [x]$

$\left| \left[-\frac{5}{3} \right] \right| - \left[\left| -\frac{5}{3} \right| \right]$

$|[-1.66]| - [5/3]$

$|-2| - 1 = 1$

Ques - Find the domain and range of $f(x) = |\sin x|$.

Solu -

Range of $f = [0, 1]$

Domain of $f = \mathbb{R}$ (Real number)

Ques - Find the range of $f(x) = \frac{1}{2 - \cos 3x}$.

Solu - Let,

$$y = f(x) = \frac{1}{2 - \cos 3x}$$

$$y = \frac{1}{2 - \cos 3x}$$

$$2 - \cos 3x = \frac{1}{y}$$

$$2 - \frac{1}{y} \geq \cos 3x$$

$$\therefore \text{---} -1 \leq \cos 3x \leq 1$$

$$\therefore -1 \leq 2 - \frac{1}{y} \leq 1$$

$$\text{---} -3 \leq -\frac{1}{y} \leq 1$$

divide both side by 1,

$$\frac{1}{3} \leq y \leq 1$$

\Rightarrow Range of $f(x) = \frac{1}{2 - \cos 3x}$ is $[\frac{1}{3}, 1]$.

Ques - If $g(x) = \frac{x^2 + 2x + 3}{x}$ then find R_g (Range of g).

Solu - Let,

$$y = g(x) = \frac{x^2 + 2x + 3}{x}$$

$$y = \frac{x^2 + 2x + 3}{x}$$

$$x \cdot y = x^2 + 2x + 3$$

$$= x^2 + 2x - xy + 3$$

$$= x^2 + x(2-y) + 3$$

$$D = B^2 - 4AC$$

$$= (2-y)^2 - 4(1)(3)$$

$$= 4 + y^2 - 4y - 12 \geq y^2 - 4y - 8 \geq 0$$

$$\geq 0$$

~~$$y = \frac{4 \pm \sqrt{16 - 4(1)(-8)}}{2(1) + 3} = \frac{4 \pm \sqrt{48}}{5}$$~~

$$y^2 - 4y - 8 \geq 0$$

Range \rightarrow

$$y \in (-\infty, 2 - 2\sqrt{3}) \cup (2 + 2\sqrt{3}, \infty)$$

$$y \in \mathbb{R} - (2 - 2\sqrt{3}, 2 + 2\sqrt{3}) = R_g \text{ (Range of } g(x) \text{)}$$