Antisymmetric relation -

A relation R is said to be antisymmetric if aRb and $bRa \Rightarrow a=b$

Ex -

In the set of natural number, the relation a divides b is antisymmetric since a divides b and b divides a is possible only when $a=b$.

i.e. $(a,b) \in R$ and $(b,a) \in R \Rightarrow a=b$.

$(2,2)$

Composite relation -

Let A, B and C be three non-empty sets and R be a relation from A to B and S be a relation B to C. then the composite relation of the two relations are R and S is a relation from A to C and denoted by SOR (S composition R) and defined as

$SOR = \{(a,c) : \exists$ an element $b \in B$ such that $(a,b) \in R$ and $(b,c) \in S\}$

Hence we can say that

$(a,b) \in R, (b,c) \in S \Rightarrow (a,c) \in SOR$

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ans -

Let $A = \{1,2,3\}$, $B = \{p,q,r\}$ and $C = \{x,y,z\}$
and let $R = \{(1,p),(1,r),(2,q),(3,q)\}$
and $S = \{(p,y),(q,x),(r,z),\}$ compute ROS.

$ROS = \{(1,y),(1,z),(2,x),(3,x)\}$

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* Theorem: Let R be a relation from the set A to the set B and S be a relation from the set B to set C then,

$(SOR)^{-1} = R^{-1} o S^{-1}$.

Proof: Let $(c,a) \in (SOR)^{-1} \Rightarrow (a,c) \in SOR \; \forall a \in A, c \in C$

$\therefore$ There exist an element $b \in B$ with $(a,b) \in R$ and $(b,c) \in S$

$(a,b) \in R$ and $(b,c) \in S \Rightarrow (b,a) \in R^{-1}$ and $(c,b) \in S^{-1}$.

$= (c,a) \in R^{-1} o S^{-1}$

$\therefore (c,a) \in (SOR)^{-1} \Rightarrow (c,a) \in R^{-1} o S^{-1}$

Thus $(SOR)^{-1} = R^{-1} o S^{-1}$