CLOSED-LOOP TRANSFER FUNCTIONS

Chapter 11

STANDARD BLOCK-DIAGRAM SYMBOLS

- Block diagram was developed for the control of a stirred-tank heater
- some standard symbols for the variables and transfer functions, which are widely used in the control literature.

These symbols are defined as follows:

R set point or desired value

C controlled variable

ϵ error

B variable produced by measuring element

M manipulated variable

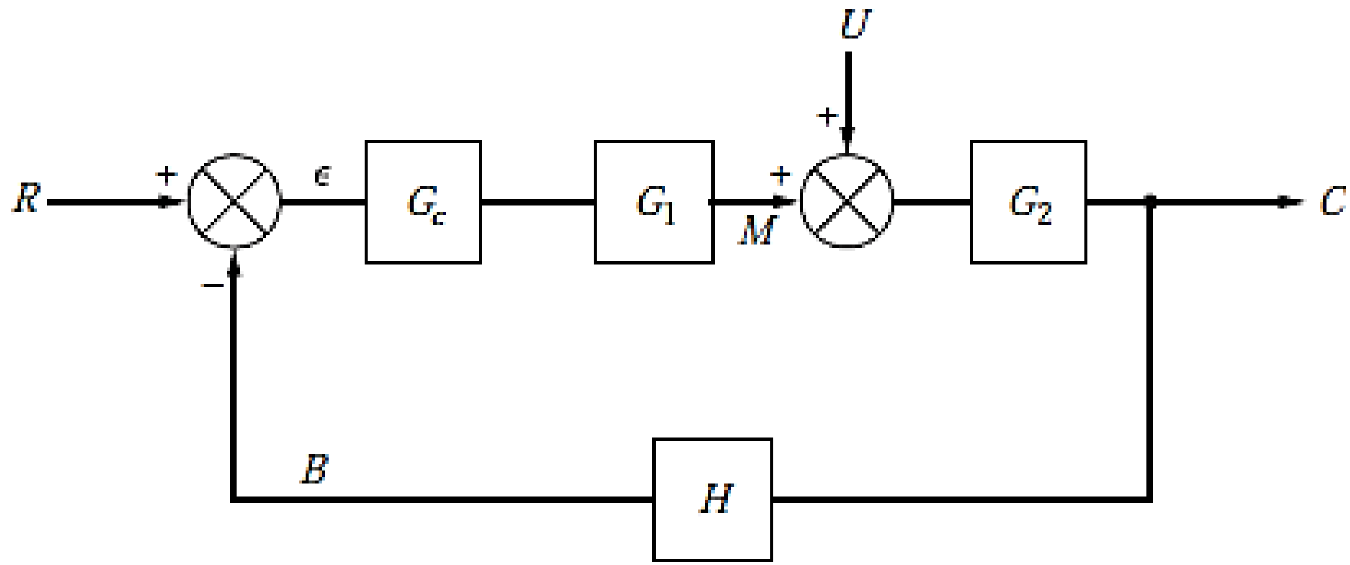
U load variable or disturbance

G_c transfer function of controller

G_1 transfer function of final control element

G_2 transfer function of process

H transfer function of measuring element



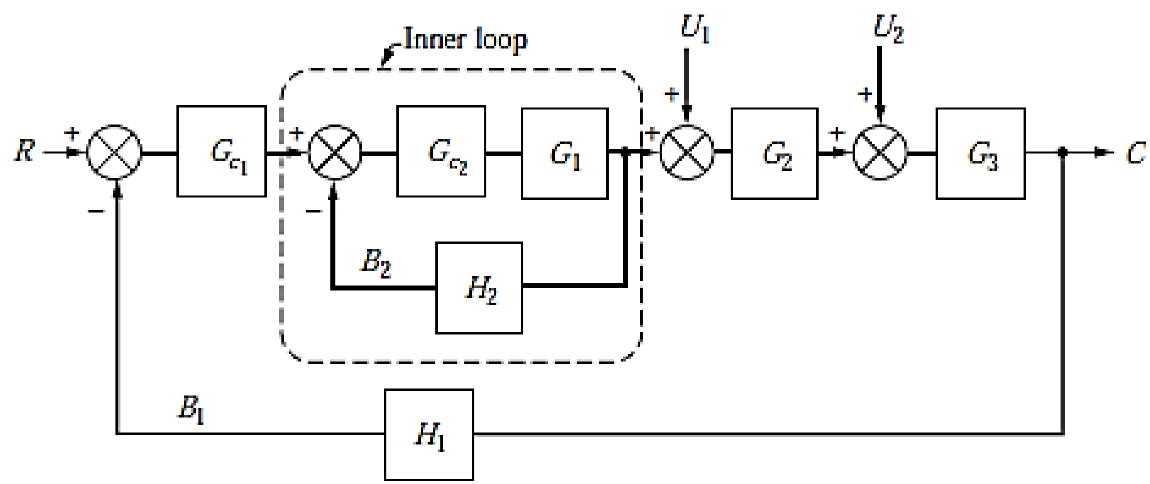
In some cases, the blocks labeled G_c and G_1 will be lumped together into a single block.

The series of blocks between the comparator and the controlled variable, which consist of G_c, G_1, and G_2, is referred to as the *forward path.*

The block *H* between the controlled variable and the comparator is called the *feedback path.*

The use of *G* for a transfer function in the forward path and *H* for one in the feedback path is a common convention.

- The product *GH,* which is the product of all transfer functions ($G_cG_1G_2H$) in the loop, is called the *open-loop transfer function.*
- We call *GH* the open-loop transfer function because it relates the measured variable *B* to the set point *R* if the feedback loop
- is disconnected (i.e., opened) from the comparator.
- The subject of this chapter is the closed-loop transfer function, which relates two variables when the loop is closed.
- In more complex systems, the block diagram may contain several feedback paths and several loads.
- An example of a multiloop system, is cascade control.

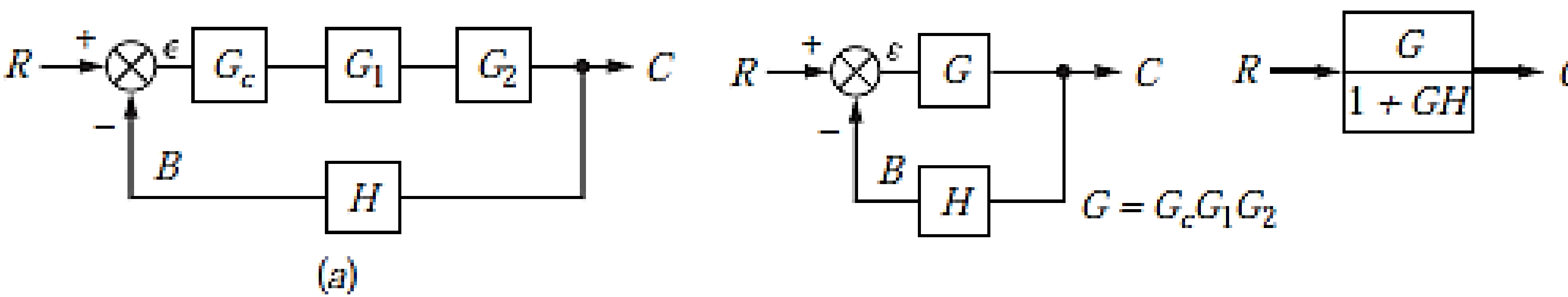


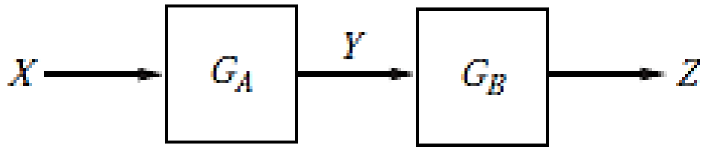
OVERALL TRANSFER FUNCTION FOR SINGLE-LOOP SYSTEMS

- Transfer function relating *C* to *R* or *C* to *U.*
- We refer to these transfer functions as *overall* transfer functions because they apply to the entire system.
- These overall transfer functions are used to obtain considerable information about the control system.
- The response to a change in set point *R,* obtained by setting *U*=0, represents the solution to the servo problem.
- The response to a change in load variable *U,* obtained by setting *R* = 0, is the solution to the regulator problem.
- A systematic approach for obtaining the overall transfer function for set point change and load change will now be presented.

Overall Transfer Function for Change in Set Point

- For this case, $U = 0$.
- In this reduction, we have made use of a simple rule of block diagram reduction
- Which states that a block diagram consisting of several transfer functions in series can be simplified to a single block containing a transfer function that is the product of the individual transfer functions.





$$\frac{Y}{X} = G_A \qquad \frac{Z}{Y} = G_B$$

$$\frac{Y}{X}\frac{Z}{Y} = G_A G_B$$

$$\frac{Z}{X} = G_A G_B$$

Thus, the intermediate variable *Y* has been eliminated, and we have shown the overall transfer function *Z/X* to be the product of the transfer functions $G_A G_B$.
This proof for two blocks can be easily extended to any number of blocks to give the rule for the general case.
This rule was developed for the specific case of several noninteracting, first-order systems in series.

With this simplification the following equations can be written directly from

$$C = G\varepsilon$$

$$B = HC$$

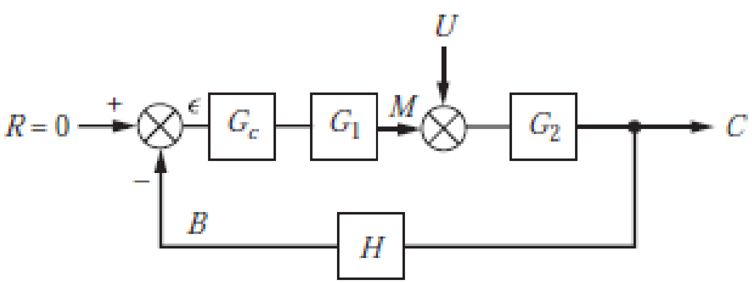
$$\varepsilon = R - B$$

$$C = G(R - B)$$

$$C = G(R - HC)$$

$$C = GR - GHC$$

$$\frac{C}{R} = \frac{G}{1 + GH}$$



(a)

$$C = G_2(U + M)$$

$$M = G_cG_1\varepsilon$$

$$\varepsilon = -B$$

$$B = HC$$

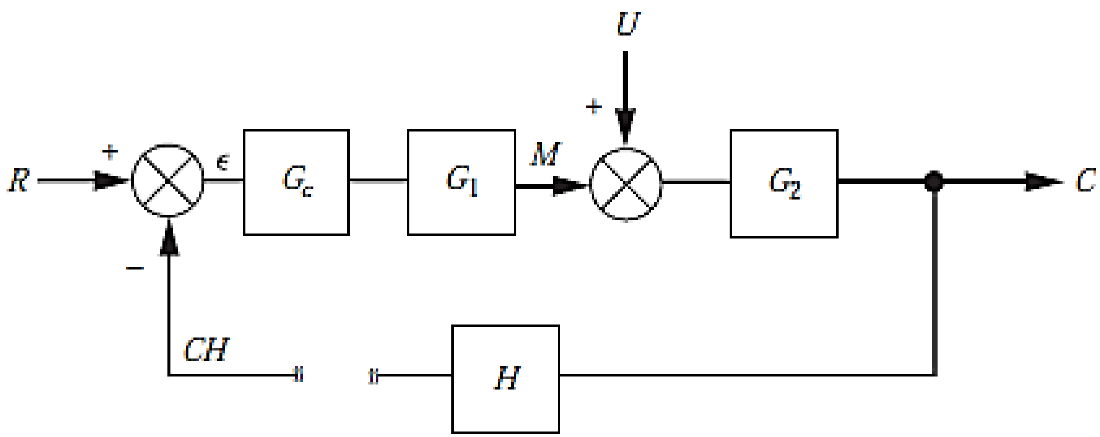
$$C = G_2(U + G_cG_1\varepsilon)$$

$$C = G_2\left[U + G_cG_1(-HC)\right]$$

$$\frac{C}{U} = \frac{G_2}{1 + GH}$$

where $G = G_cG_1G_2$.

- Notice that the transfer functions for load change or set point change have denominators that are identical, 1+ *GH.*
- Another approach to finding the closed-loop transfer functions from the block diagram is a “brute-force” technique that involves “breaking the loop” and working your way across the block diagram.



$$\left[(R - CH)G_cG_1 + U\right]G_2 = C$$

$$C = \frac{G_cG_1G_2}{1 + G_cG_1G_2H}R + \frac{G_2}{1 + G_cG_1G_2H}U$$

$$= \underbrace{\frac{G}{1 + GH}}_{\substack{\text{closed loop}\\ \text{transfer function}\\ \text{relating } C \text{ and } R}} R + \underbrace{\frac{G_2}{1 + GH}}_{\substack{\text{closed loop}\\ \text{transfer function}\\ \text{relating } C \text{ and } U}} U$$

- The following simple rule serves to generalize the results for the single-loop feedback system
- The transfer function relating any pair of variables *X, Y* is obtained by the relationship

$$\frac{Y}{X} = \frac{\pi_{\text{forward}}}{1 + \pi_{\text{loop}}} \quad \text{negative feedback}$$

where $_{\text{forward}}$ = Product of transfer functions in forward path betwπeen locations of *X* and *Y*

π_{loop} = Product of all transfer functions in,

$\pi_{\text{loop}} = G_c G_1 G_2 H$

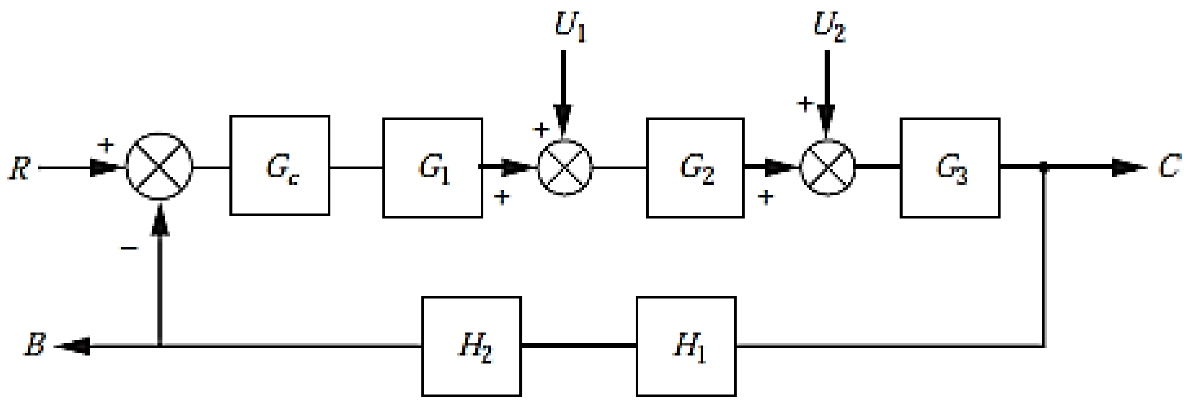
If this rule is applied to finding *C/R*

we obtain

$$\frac{C}{R} = \frac{G_c G_1 G_2}{1 + G_c G_1 G_2 H} = \frac{G}{1 + GH}$$

- For positive feedback, the reader should show that the following result is obtained:

$$\frac{Y}{X} = \frac{\pi_{\text{forward}}}{1 - \pi_{\text{loop}}} \quad \text{positive feedback}$$



$$\frac{C}{R} = \frac{G_c G_1 G_2 G_3}{1 + G}$$

$$C = \frac{G_c G_1 G_2 G_3}{1 + G} R$$

$$\frac{C}{U_1} = \frac{G_2 G_3}{1 + G}$$

$$C = \frac{G_2 G_3}{1 + G} U_1$$

$$\frac{B}{U_2} = \frac{G_3 H_1 H_2}{1 + G}$$

where $G = G_c G_1 G_2 G_3 H_1 H_2$

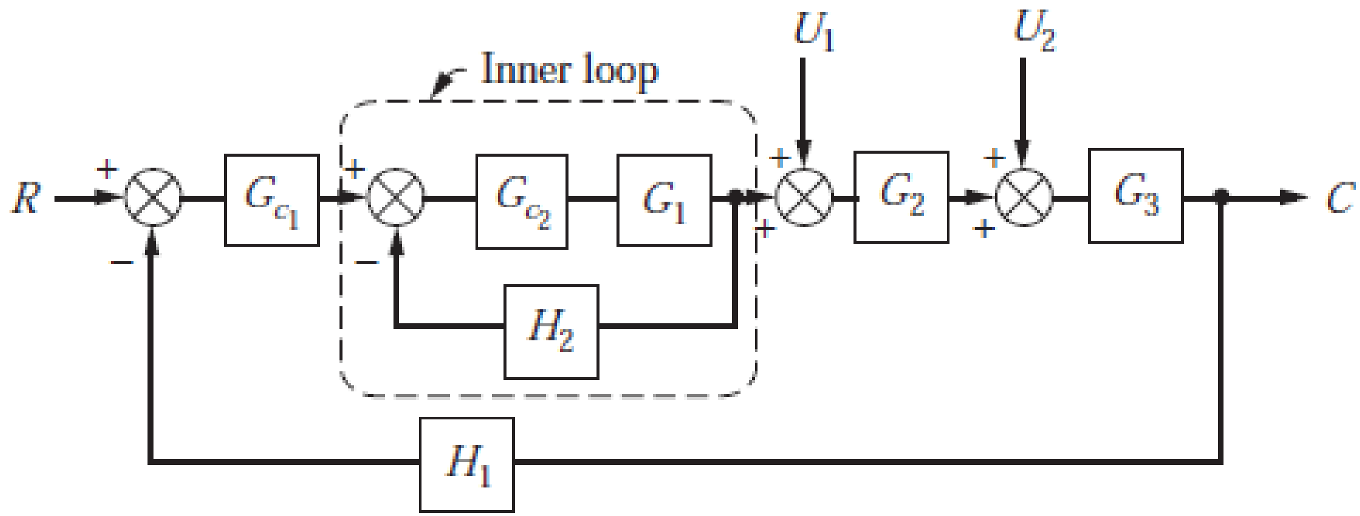
- If both *R* and U_1 occur simultaneously, the principle of superposition requires that the overall response be the sum of the individual responses; thus

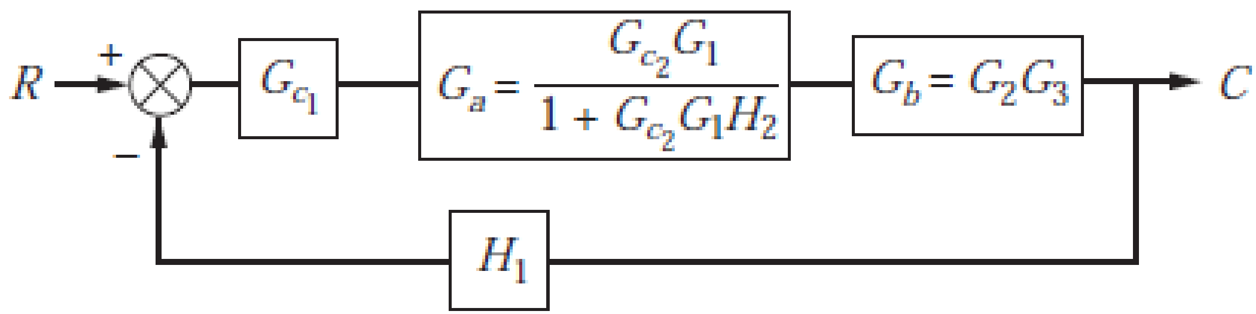
$$C = \frac{G_c G_1 G_2 G_3}{1 + G} R + \frac{G_2 G_3}{1 + G} U_1$$

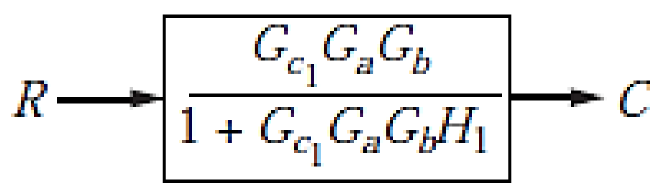
- It should be emphasized that regardless of the pair of variables selected, the denominator of the closed-loop transfer function will always contain the same term, 1+ *G,*

  where *G* is the open-loop transfer function of the single-loop control system.

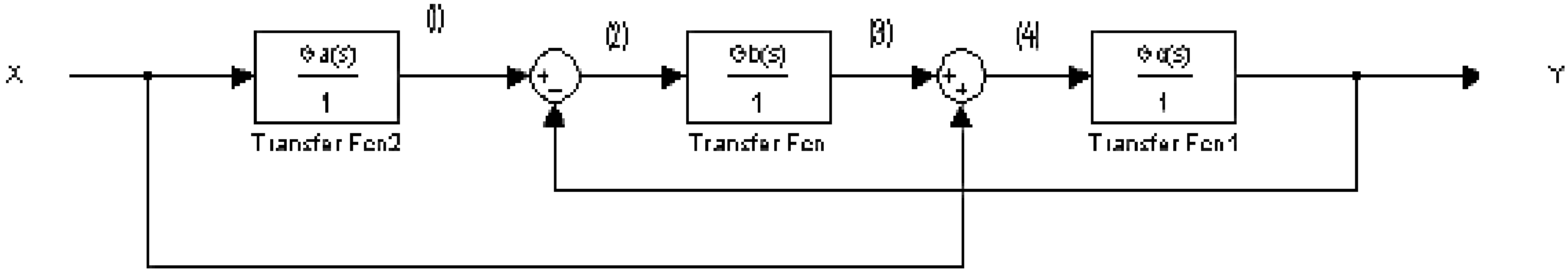
OVERALL TRANSFER FUNCTION FOR MULTILOOP CONTROL SYSTEM







Determine the transfer function Y(s)/X(s) for the block diagrams shown. express the results in terms of Ga, Gb and Gc



(a) Balances at each node

$(1) = GaX$

$(2) = (1) - Y = GaX - Y$

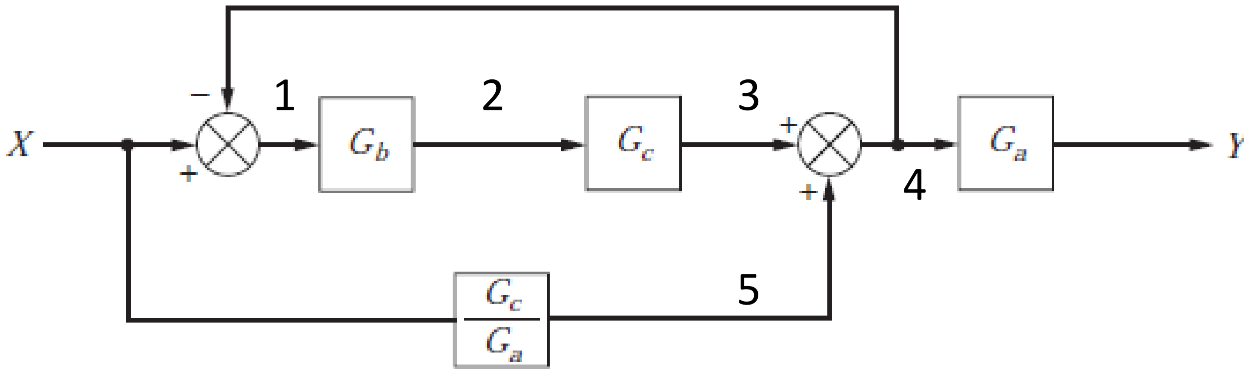
$(3) = Gb(2) = Gb(GaX - Y)$

$(4) = (3) + X = Gb(GaX - Y) + X$

$Y = Gc(4) = Gc\,(Gb(GaX - Y) + X)$

$= GaGbGcX - GbGcY + GcX$

$$\frac{Y}{X} = \frac{Gc(GaGb+1)}{1+GbGc}$$



(b) Balances at each node

$(1) = X - (4)$

$(2) = Gb(1) = Gb(X - (4))$

$(5) = GcX/Ga$

$(3) = Gc(2) = GbGc(X - (4))$

$(4) = (3) + (5)$ -------------

$= GbGc(X - (4)) + GcX/Ga$

$Y = Ga(4)$

From the fifth equation

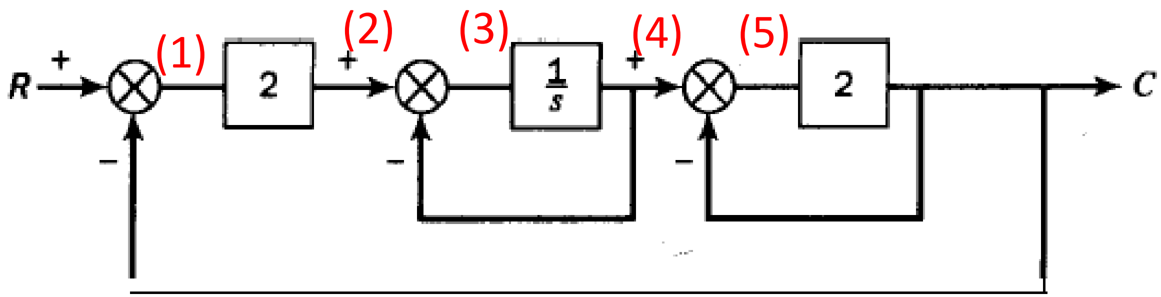
$$(4) = GbGcX - GbGc(4) + GcX/Ga$$

$$(4) = \frac{(GaGbGc + Gc)X}{(1 + GbGc)Ga}$$

From the sixth equation

$$\frac{Y}{X} = \frac{(GaGb + 1)Gc}{(1 + GbGc)}$$

For the control system shown in Fig. P12.3 determine the transfer function $C(s)/R(s)$.



Balances at each node

$(1) = R - C$

$(2) = 2\,(1) = 2(R - C)$

$(3) = (2) - (4) = 2(R - C) - (4)$

$(4) = (3)/s = (2(R - C) - (4))/s$

$(5) = (4) - C$

$C = 2(5)$

Solving for (4) using (d)

$s\,(4) = 2(R - C) - (4)$

$(4) = 2(R - C) / (s + 1)$

Using (e)

$(6) = 2(R - C) / (s + 1) - C$

$$C = 2\left[\frac{2}{s+1}(R - C) - C\right]$$

$$4R = C\big((s+1) + 4 + 2(s+1)\big)$$

$$\frac{C}{R} = \frac{4}{3s+7}$$