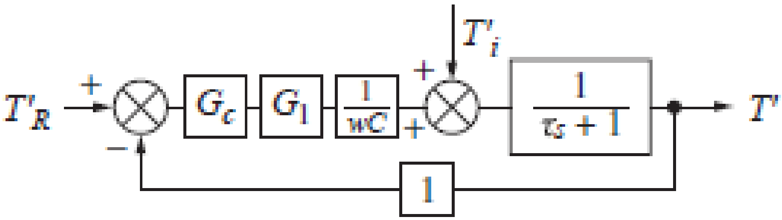
TRANSIENT RESPONSE OF SIMPLE CONTROL SYSTEMS

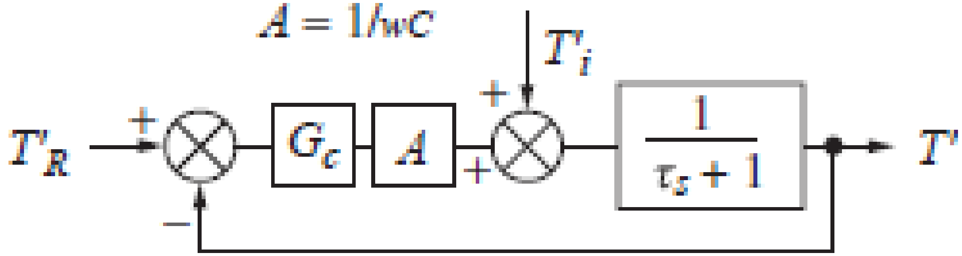
CHAPTER 12

- Determining the transient response of a simple control system to changes in set point and load.
- Measuring element (τ_m = 0) When the feedback transfer function is unity, the system is called a *unity-feedback* system.



PROPORTIONAL CONTROL FOR SET POINT CHANGE

(SERVO PROBLEM—SET POINT TRACKING)



For proportional control, $G_c = K_c$

$$\frac{T'}{T'_R} = \frac{K_c A/(\tau s + 1)}{1 + K_c A/(\tau s + 1)} = \frac{K_c A}{\tau s + (1 + K_c A)}$$

This may be rearranged in the form of a first-order lag to give

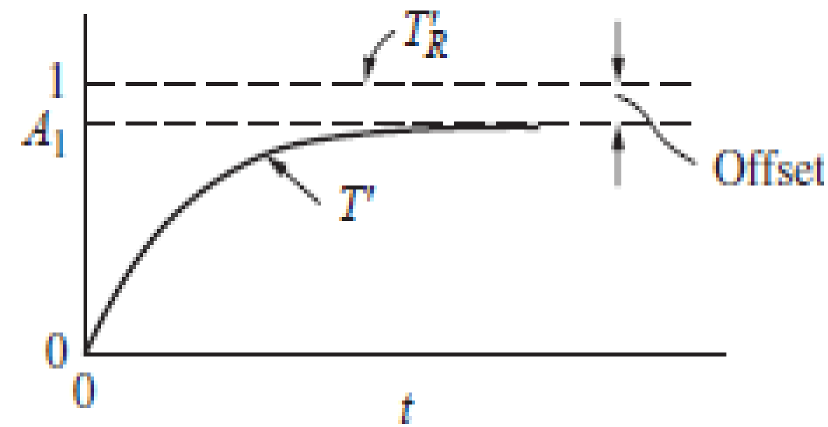
$$\frac{T'}{T'_R} = \frac{K_c A/(1 + K_c A)}{[\tau/(1 + K_c A)]s + 1} = \frac{A_1}{\tau_1 s + 1}$$

where $\tau_1 = \dfrac{\tau}{1 + K_c A}$

$$A_1 = \frac{K_c A}{1 + K_c A} = \frac{1}{1 + (1/K_c A)}$$

- The time constant for the closed-loop control system τ_1 is less than that of the stirred tank itself τ. This means that one of effects of feedback control is to speed up the response.

(We have selected a unit change in set point for convenience; responses to steps of other magnitudes are obtained by superposition.)



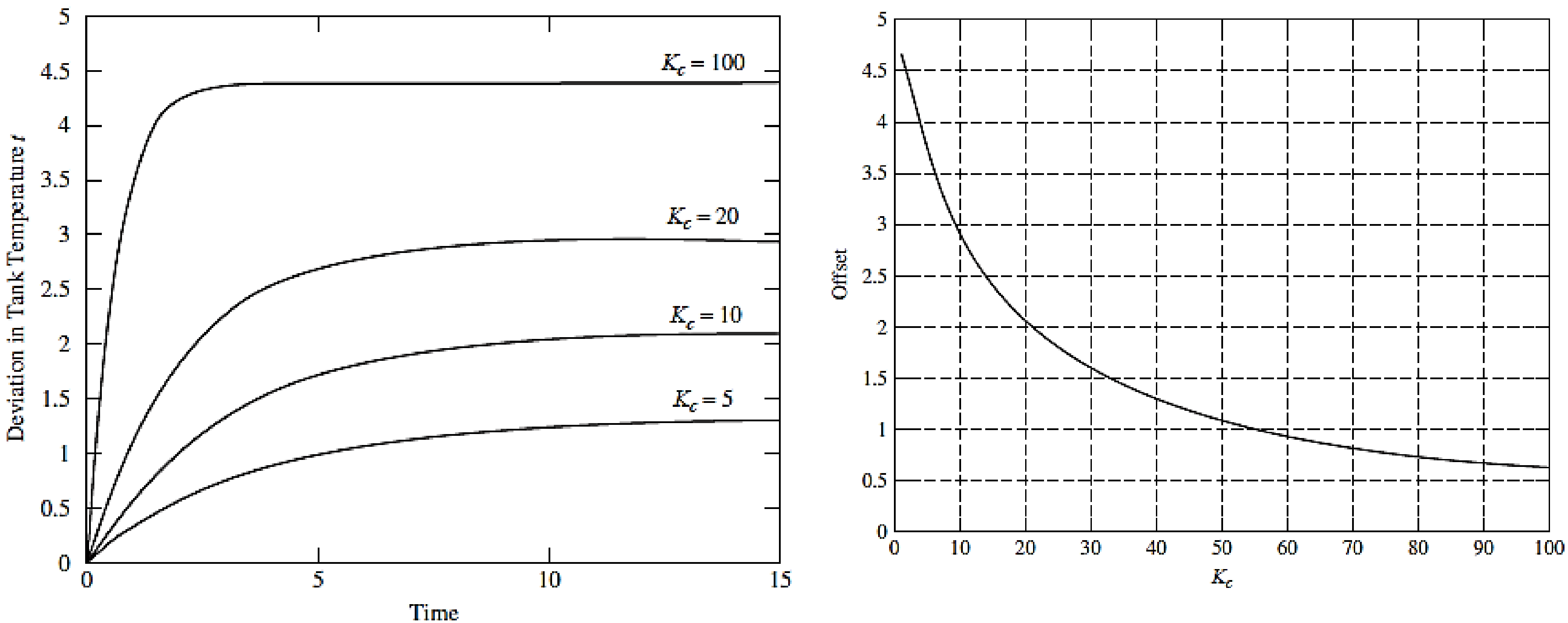
Remember, the goal of a control system is to force the system to track the set point.
Thus, the desired ultimate value of *T′*, which is *T′* (▯), is of course 1.
For the case of a unit-step change in set point, *T′* approaches
$A_1 = K_c A / (1 + K_c A)$, a fraction of unity.
The desired change is, of course, 1. Thus, the ultimate value of the temperature *T* (▯) does not match the desired change.

- Discrepancy is called *offset* and is defined as

$$\text{Offset} = T'_R(\infty) - T'(\infty) \qquad \text{Offset} = 1 - \frac{K_c A}{1 + K_c A} = \frac{1}{1 + K_c A}$$

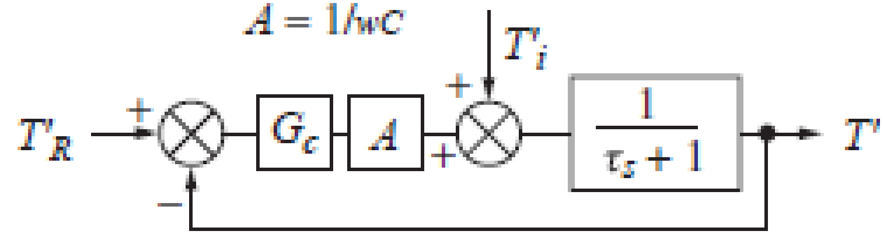
offset decreases as K_c increases, and in theory the offset could be made as small as desired by increasing K_c to a sufficiently large value.

Proportional control : As $K_c \uparrow$ Offset $\downarrow$



PROPORTIONAL CONTROL FOR LOAD CHANGE
(REGULATOR PROBLEM— DISTURBANCE REJECTION)

This time the set point remains fixed; that is, $T'_R = 0$



Remember that the goal of the control system in this case is to reject the effect of disturbances
(changes in the inlet temperature T'_i for this process)

Since the set point has not changed from its steady-state value for this case ($T'_R = 0$), we want T'(⍰)= 0.

the overall transfer function becomes

$$\frac{T'}{T'_i} = \frac{1/(\tau s + 1)}{1 + K_c A/(\tau s + 1)} = \frac{1}{\tau s + 1 + K_c A}$$

$$\frac{T'}{T_i'} = \frac{A_2}{\tau_1 s + 1} \qquad \text{where} \quad A_2 = \frac{1}{1 + K_c A}$$

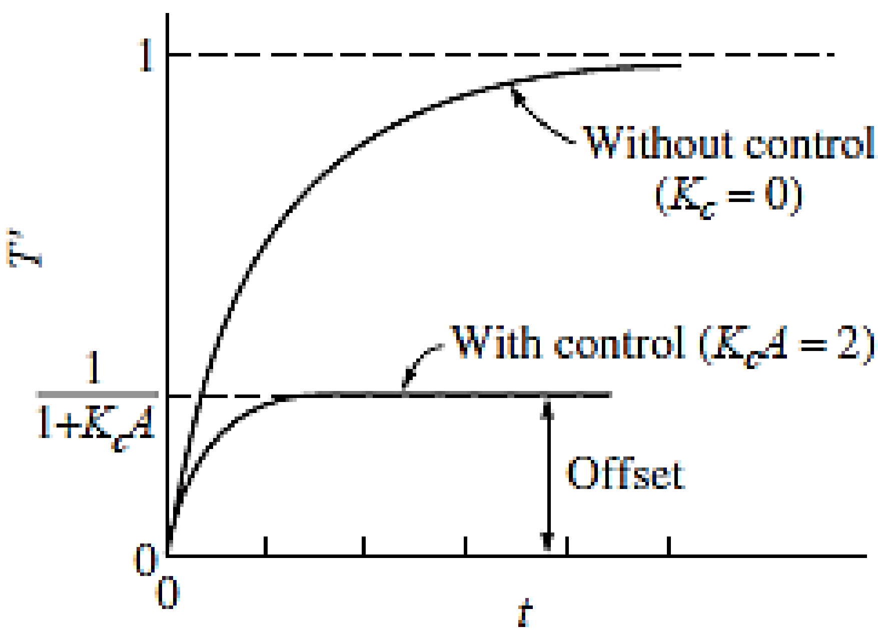
$$\tau_1 = \frac{\tau}{1 + K_c A}$$

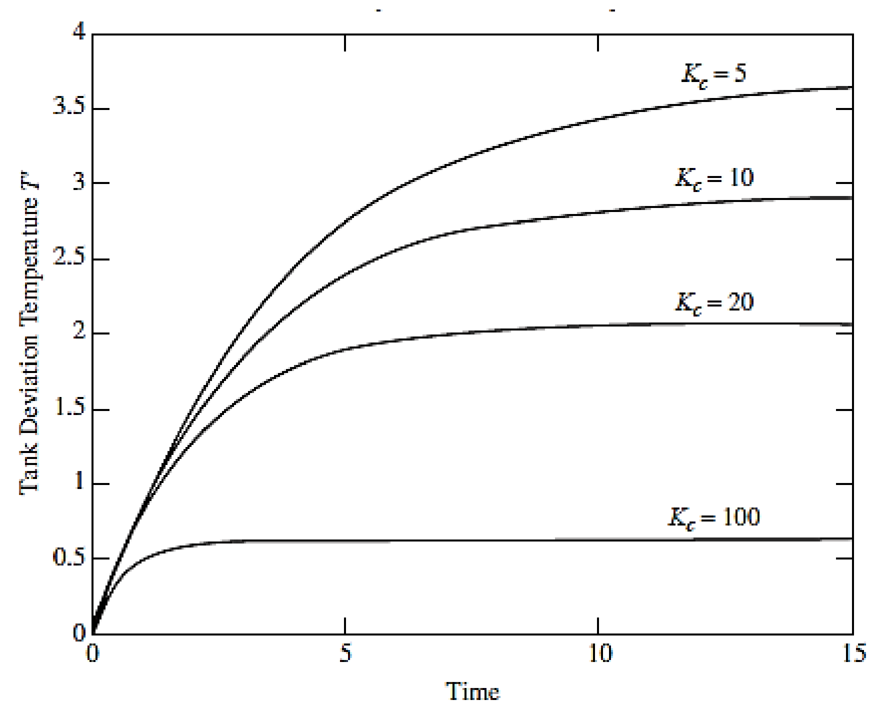
Thus for $T_i = 1/s$,

$$T' = \frac{1}{s}\frac{1/(1 + K_c A)}{[\tau/(1 + K_c A)]s + 1}$$

and the ultimate (steady-state) value of T' is

$$T'(\infty) \lim_{s \to 0} sT'(s) = \frac{1}{1 + K_c A}$$

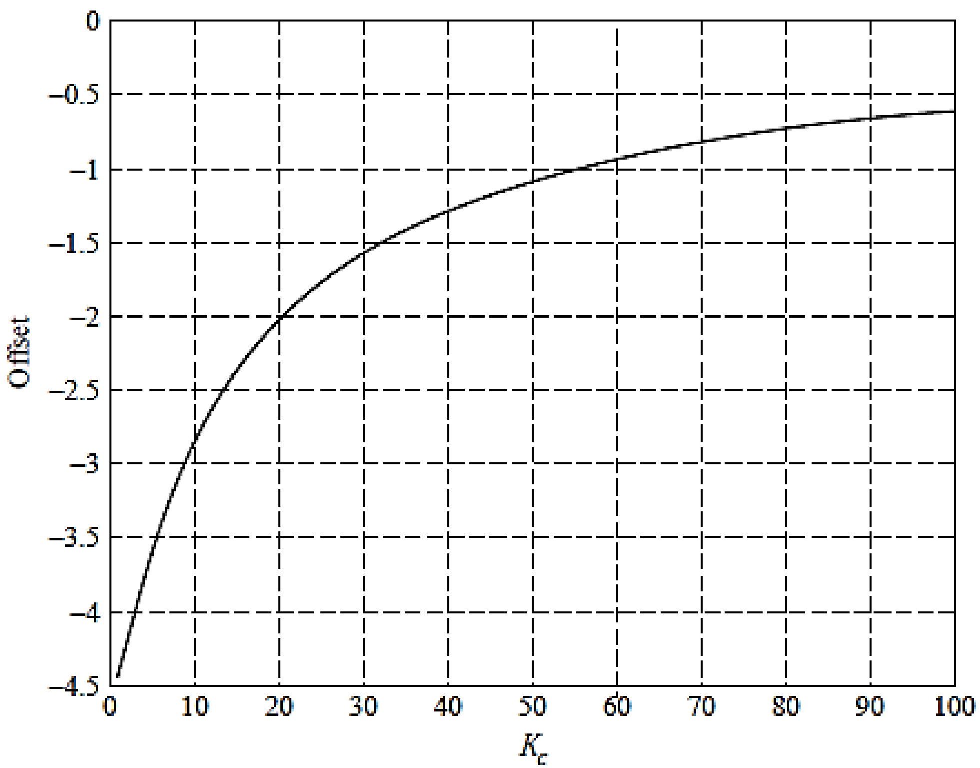




$$\text{Offset} = T_R'(\infty) - T'(\infty) = 0 - \frac{1}{1 + K_c A}$$

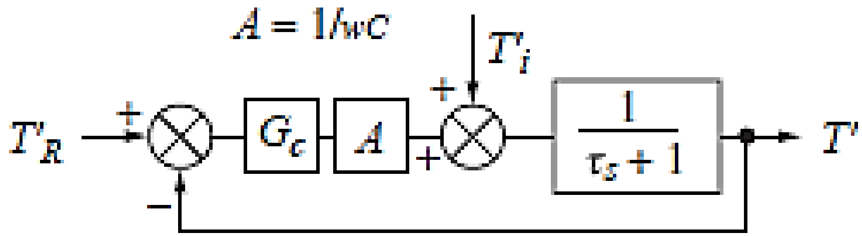
$$\text{Offset} = -\frac{1}{1 + K_c A}$$

As for the case of a step change in set point, the absolute value of the offset is reduced as controller gain K_c is increased.



PROPORTIONAL-INTEGRAL CONTROL FOR LOAD CHANGE

- For PI control, we replace G_c in Fig. by $K_c\,(1 + 1/\,t_I s)$.
- The overall transfer function for load change is therefore



$$\frac{T}{T_i} = \frac{1/(\tau s + 1)}{1 + \dfrac{K_c(1 + 1/\tau_I s)A}{\tau s + 1}}$$

$$\frac{T'}{T_i'} = \frac{\tau_I s}{(\tau s + 1)(\tau_I s) + K_c A(\tau_I s + 1)}$$

$$\frac{T'}{T_i'} = \frac{\tau_I s}{\tau\tau_I s^2 + (K_c A \tau_I + \tau_I)s + K_c A}$$

- Since the denominator contains a quadratic expression,
- Transfer function may be written in the standard form of a second-order system to give

$$\frac{T'}{T_i'} = \frac{(\tau_I/K_cA)s}{(\tau\tau_I/K_cA)s^2 + \tau_I(1 + 1/K_cA)s + 1}$$

$$\frac{T'}{T_i'} = \frac{A_1 s}{\tau_1{}^2 s^2 + 2\zeta\tau_1 s + 1}$$

$$A_1 = \frac{\tau_I}{K_cA}$$

$$\tau_1 = \sqrt{\frac{\tau\tau_I}{K_cA}}$$

$$\zeta = \frac{1}{2}\sqrt{\frac{\tau_I}{\tau}}\frac{1 + K_cA}{\sqrt{K_cA}}$$

For a unit-step change in load, $T_i = 1/s$.

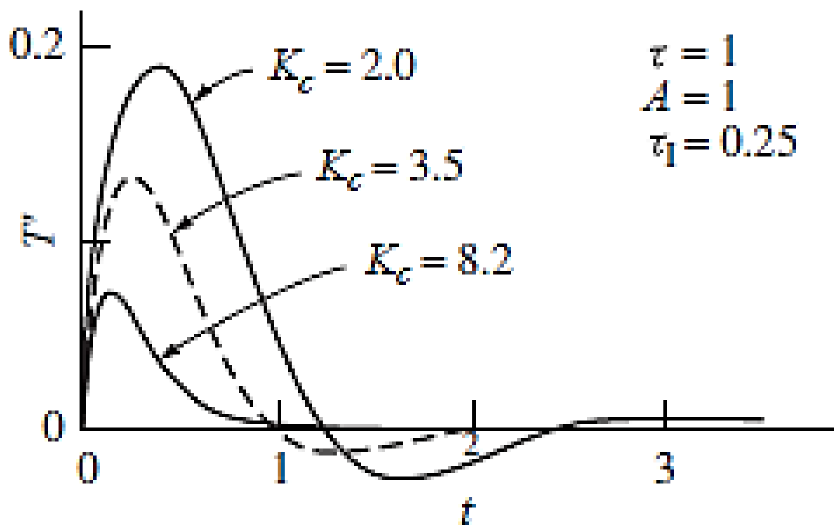
$$T' = \frac{A_1}{\tau_1{}^2 s^2 + 2\zeta\tau_1 s + 1}$$

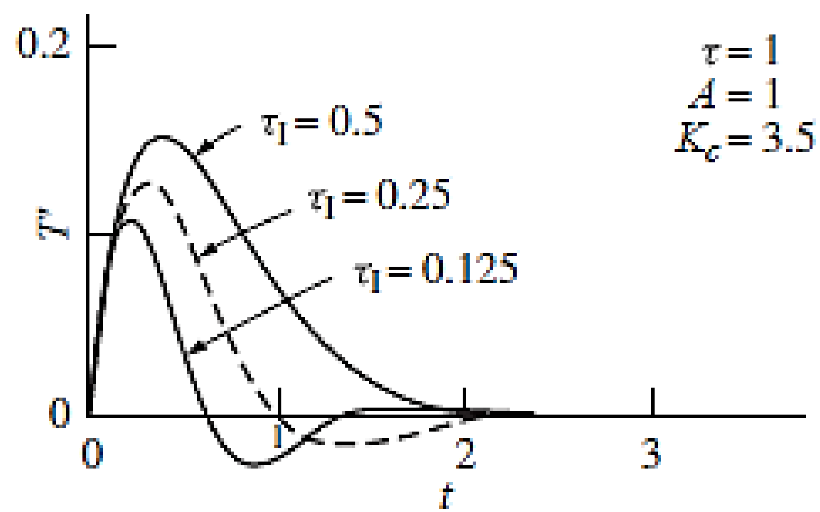
This justifies in part some of our previous work on transients.

The impulse response for this system may be written for $\zeta < 1$ as

$$T' = A_1\left(\frac{1}{\tau_1}\frac{1}{\sqrt{1 - \zeta^2}}e^{-\zeta t/\tau_1}\sin\sqrt{1 - \zeta^2}\,\frac{t}{\tau_1}\right)$$

- We see that an increase in K_c, for a fixed value of τ_I, improves the response by decreasing the maximum deviation and by making the response less oscillatory.
- The formula for ζ in Eq. shows that ζ increases with K_c, which indicates that the response is less oscillatory.
- For a fixed value of K_c, a decrease in τ_I decreases the maximum deviation and period.
- However, a decrease in t_I causes the response to become more oscillatory, which means that ζ decreases.
- This effect of t_I on the oscillatory nature of the response is also given by the formula for ζ in





Summary for Proportional-Integral Control—Response to Step Change in Load

For fixed value of τ_I: As $K_c \uparrow$, $\zeta \uparrow$	Max. deviation $\downarrow$	Oscillations $\downarrow$
For fixed value of K_c: As $\tau_I \downarrow$, $\zeta \downarrow$	Max. deviation $\downarrow$	Oscillations $\uparrow$

$$\begin{aligned}\text{Offset} &= T_R'(\infty) - T'(\infty)\\ &= 0 - 0 = 0\end{aligned}$$

One of the most important advantages of PI control is the elimination of offset.

Key Point **Proportional-Integral Control $\rightarrow$ No offset**