# CONTROL SYSTEM DESIGN BY FREQUENCY RESPONSE

Chapter 16

- The purpose of this chapter

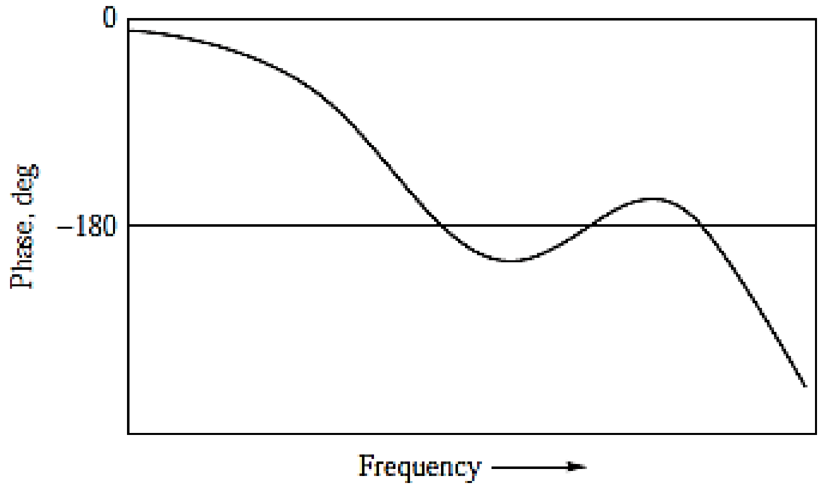
First, it is indicated that the stability of a control system can usually be determined from the Bode diagram of its open-loop transfer function.

Second, The methods are presented for rational selection of controller parameters based on this Bode diagram.

## THE BODE STABILITY CRITERION

- *Control system is unstable if the open-loop frequency response exhibits an AR exceeding unity at the frequency for which the phase lag is* 180 °.
- This frequency is called the *crossover frequency.*
- The rule is called the *Bode stability criterion.*
- Fortunately, most process control systems can be analyzed with the simple Bode criterion, and it therefore finds wide application.

- Phase behavior of a complex system for which the Bode criterion is not applicable.




## GAIN AND PHASE MARGINS

The crossover frequency, at which the phase lag is 180°, is noted as $w_{co}$ on the Bode diagram.
At this frequency, the AR is *A.*
If *A* exceeds unity, we know from the Bode criterion that the system is unstable and that we have made a poor selection of $G_c(s)$.
*A* is less than unity and therefore the system is stable.

- It is necessary to ascertain to what degree the system is stable.
- Intuitively, if *A* is only slightly less than unity, the system is "almost unstable" and may be expected to behave in a highly oscillatory manner even though it is theoretically stable.

Open-loop Bode diagram for a typical control system.

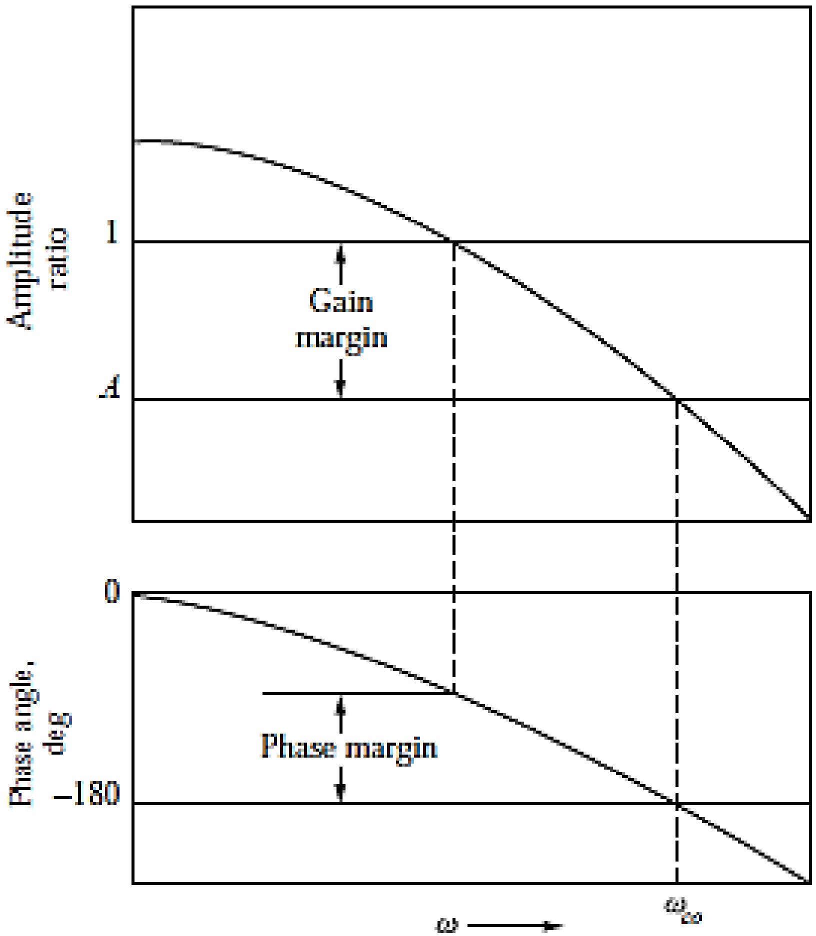
Furthermore, the constant *A* is determined by the physical parameters of the system, such as time constants.
Hence, a design for which *A* is close to unity does not have an adequate safety factor.
To assign some quantitative measure to these considerations, the concept of gain margin (GM) is introduced.

$$\text{Gain margin} = \frac{1}{A} = \frac{1}{AR_{\phi=-180^\circ}}$$

*Typical specifications for design are that the gain margin should be greater than* 1.7.
This means that the AR at crossover could increase by a factor of 1.7 over the design value before the system became unstable.

$$AR_{\phi=-180°} = \frac{1}{GM} = \frac{1}{1.7} = 0.59$$

- Gain margin is really a safety factor that maintains the AR a "safe distance" away from AR= 1 at $w_{co}$ .
- *As such, its value varies considerably with the application and designer.*
- A gain margin of unity or less indicates an unstable system.
- Another margin frequently used for design is the phase margin.
- It is the difference between 180° and the phase lag at the frequency for which the gain is unity.




$$\text{Phase margin (PM)} = 180° - \phi_{LAG}\big|_{AR=1}$$

- The phase margin therefore represents the additional amount of phase lag required to destabilize the system, just as the gain margin represents the additional gain for destabilization.
- *Typical design specifications are that the phase margin must be greater than* 30 °.
- A negative phase margin indicates an unstable system.

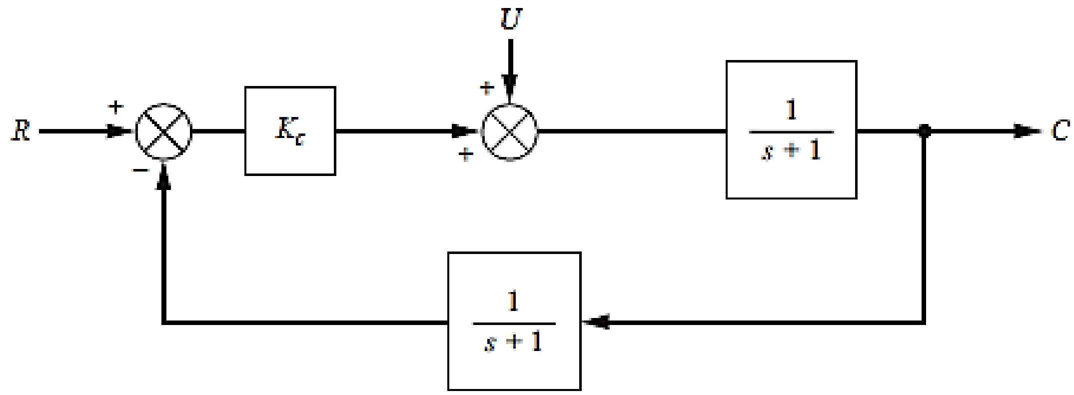
**Example 16.1.** Find a relation between relative stability (see below) and the phase margin for the control system of Fig. 16–7. A proportional controller is to be used.

This block diagram corresponds to the stirred-tank heater system, for which the block diagram has been given in Fig. 12–17. The particular set of constants is

$$\tau = \tau_m = 1$$

$$\frac{1}{wC} = 1$$

These are to be regarded as fixed, while the proportional gain $K_c$ is to be varied to give a satisfactory phase margin.




The *closed-loop* transfer function for this system is given by Eq. rewritten for our particular case as

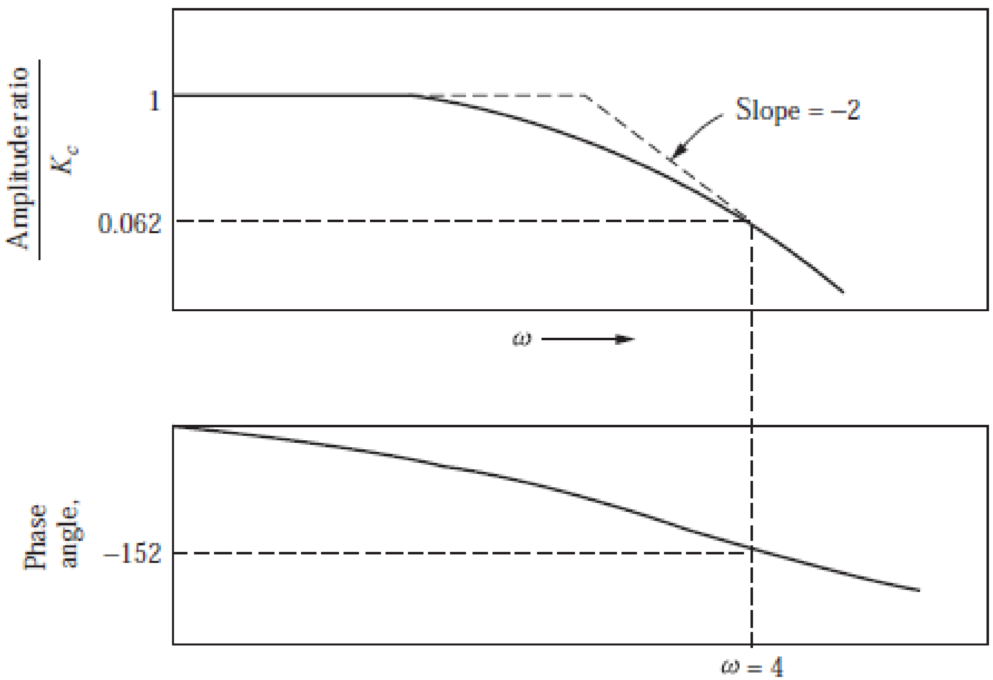
$$\frac{C}{R} = \frac{K_c}{1 + K_c} \frac{s + 1}{\tau_2{}^2 s^2 + 2\tau_2 \zeta_2 s + 1}$$

$$\tau_2 = \sqrt{\frac{1}{1 + K_c}} \qquad \zeta_2 = \sqrt{\frac{1}{1 + K_c}}$$

- Since the closed-loop system is second-order, it can never be *unstable.*
- The lower $\zeta_2$ is made, the more oscillatory and hence the "less stable" will be the response.
- Therefore, a relationship between phase margin and $\zeta_2$ will give the relation between phase margin and relative stability.

- To find this relation the open-loop Bode diagram is prepared and is shown in Fig.
- The simplest way to proceed from this diagram is as follows: Consider a typical frequency $\omega = 4$.
- If the open-loop gain were 1 at this frequency, then since the phase angle is 152 °, the phase margin would be 28 °.
- To make the open-loop gain 1 at $\omega = 4$, it is required that

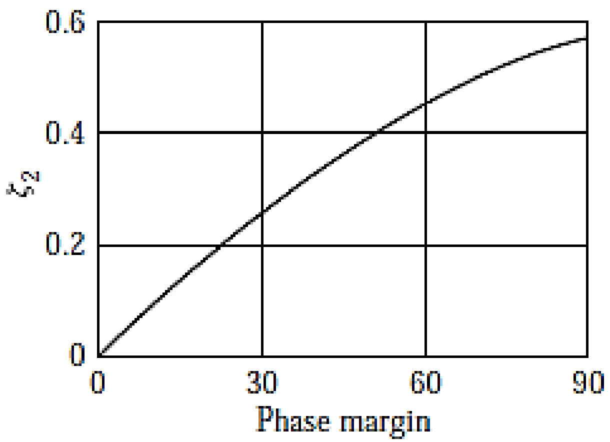
Open-loop Bode diagram for system of Example 16.1, $G = 1/(s+1)^2$.

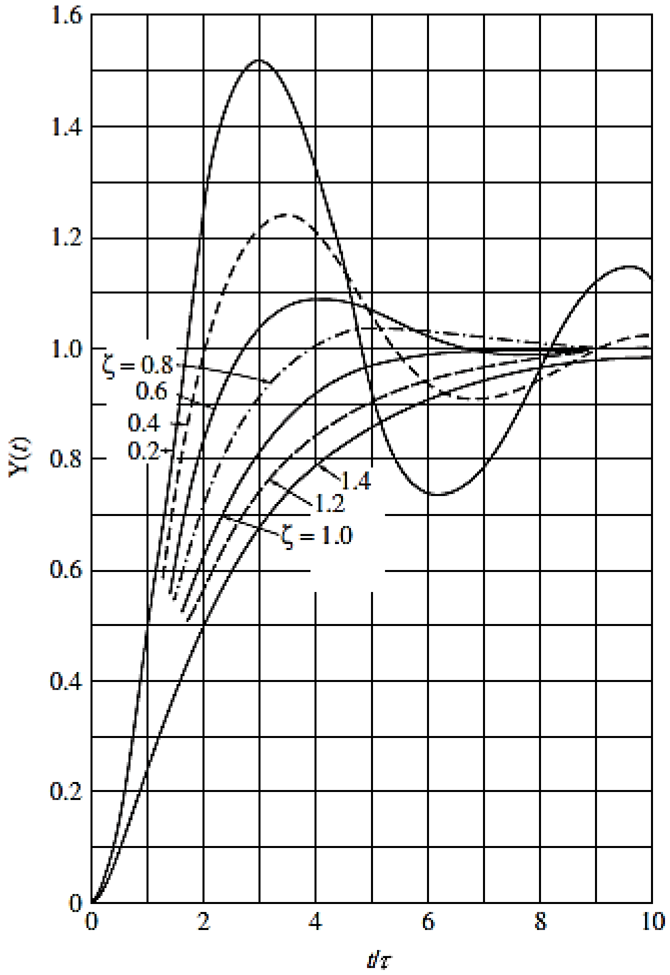


$$K_c = \frac{1}{0.062} = 16.1 \qquad \zeta_2 = \sqrt{\frac{1}{1+K_c}} = 0.24$$

Hence, a point on the curve of $\zeta_2$ versus phase margin is

$$\zeta_2 = 0.24 \qquad \text{phase margin} = 28°$$



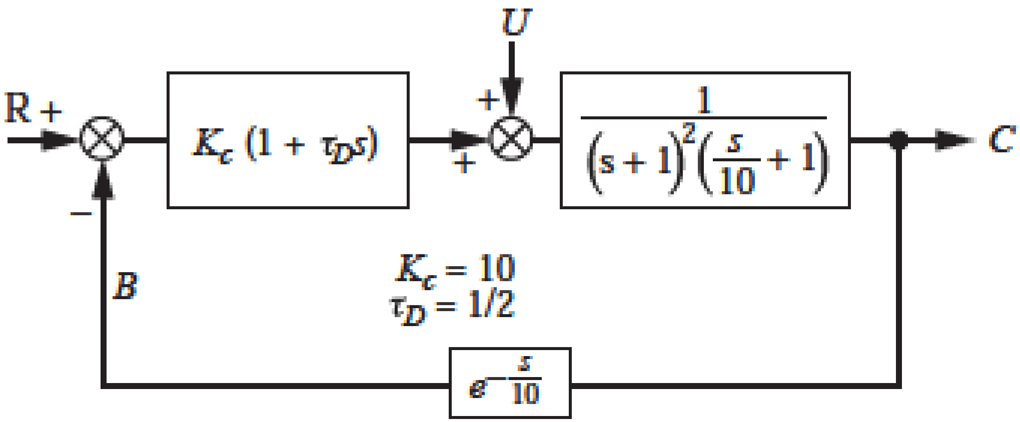




From this figure it is seen that $\zeta_2$ decreases with decreasing phase margin.

If the phase margin is less than 30 °, then $\zeta_2$ is less than 0.26.

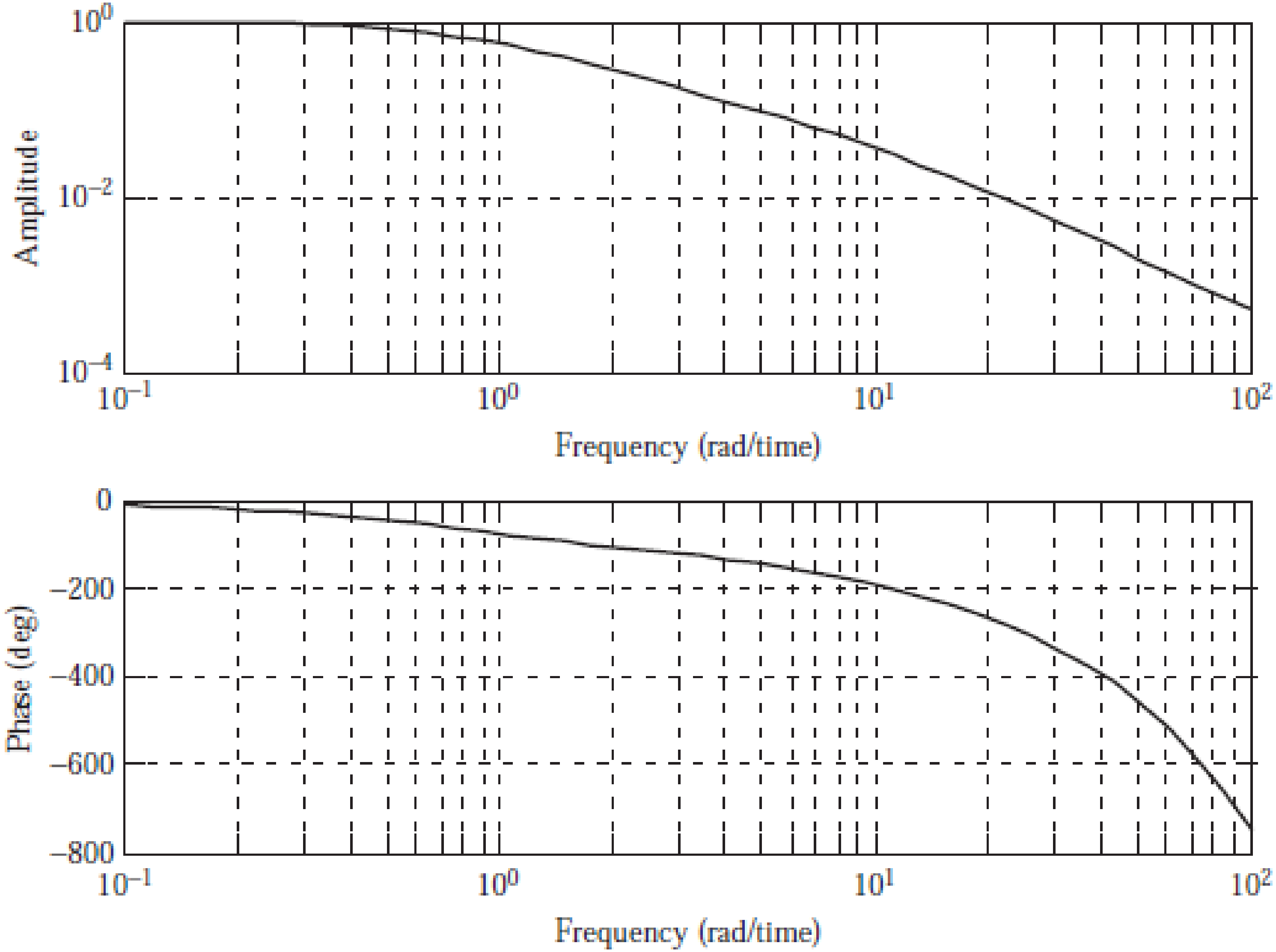
it can be seen that the response of this system for $\zeta_2 < 0.26$ is highly oscillatory, hence relatively unstable, compared with a response for the system with phase margin 50 ° and $\zeta_2 =$ 0.4.

- Thus, the phase margin is a useful design tool for application to systems of higher complexity.
- To repeat, the rule of thumb is that the phase margin must be greater than 30 °
- A similar statement can be made about the gain margin.
- As the gain margin is increased, the system response generally becomes less oscillatory, hence more stable.
- A control system designer will often try to make *both* the gain and phase margins equal to or greater than specified minimum values, typically 1.7 and 30 °.
- However, the phase margin requirement of 30 ° necessitates that $\zeta_2 > 0.26$, hence $K_c < 14$, which means that an offset of 1/15 must be accepted.




**The gain is to be specified for the two cases:**

| | Proportional gain | $\tau_D$ | Open-loop transfer function |
|---|---|---|---|
| Case 1 (PD control) | $K_c$ | 0.5 min | $G(s) = \frac{K_c(0.5s+1)e^{-s/10}}{(s+1)^2(0.1s+1)}$ |
| Case 2 (P control only) | $K_c$ | 0 min | $G(s) = \frac{K_c e^{-s/10}}{(s+1)^2(0.1s+1)}$ |



$10^0$
$10^{-2}$
$10^{-4}$
Amplitude
$10^{-1}$
$10^0$
$10^1$
$10^2$
Frequency (rad/time)
0
−200
−400
−600
−800
Phase (deg)
$10^{-1}$
$10^0$
$10^1$
$10^2$
Frequency (rad/time)

**Case 1.** Consider first the gain margin.

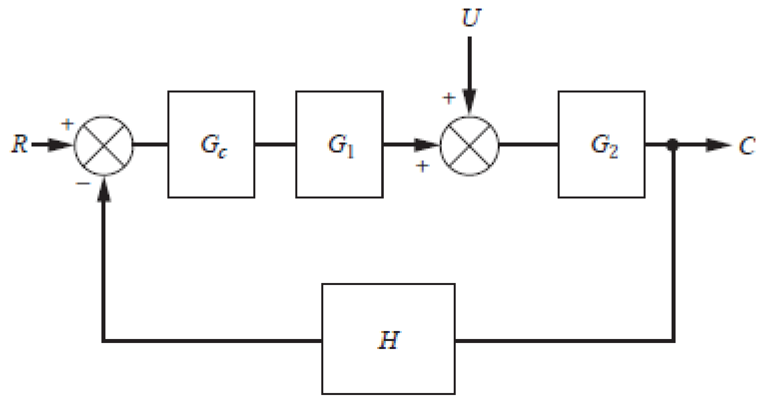
- The crossover frequency for the curve with derivative action is 8.62 rad/min.
- At this frequency, the open-loop gain is 0.0445 if the value of *Kc* is unity. (Including the factor of 1 /10 in the ordinate is actually equivalent to plotting the case *Kc* = 1.)
- Therefore, according to the Bode criterion, the value of *Kc* necessary to destabilize the loop is 1/0.0445, or 22.5.
- To achieve a gain margin of 1.7, *K c* must be taken as 22.5/1.7, or 13.2.
- To achieve proper phase margin, note that the frequency for which the phase lag is 150 ° (phase margin is 30 ° ) is 5.52 rad/min.
- At this frequency, a value for *K c* of 1/0.0815, or 12.3, will cause the open-loop gain to be unity.
- Since this is lower than 13.2, we use 12.3 as the design value of *K c* .
- The resulting gain margin is then 1.83.

In almost every situation, the designer faces this conflict between speed of response and degree of oscillation.

In addition, if integral action is not used, the amount of the offset must be considered.

- The concepts of gain and phase margin are useful in selecting *Kc* for proportional action.
- However, for additional modes of control such as PD, these concepts are difficult to apply in practice.
- A typical design procedure is to select the value of $\tau_D$ for which the value of *Kc* resulting in a 30° phase margin is maximized.
- The motivation for this choice is that the offset will be minimized.
- However, the procedure is clearly trial and error.
- In the case of three-mode control, there are two parameters, $\tau_I$ and $\tau_D$ , which must be varied by trial to meet various design criteria.
- Fortunately, for this case and others there are simple rules for directly establishing values of the control parameters that usually give satisfactory gain and phase margins.
- These are the Ziegler-Nichols rules,

# ZIEGLER-NICHOLS CONTROLLER SETTINGS




Consider selection of a controller $G_c$ for the general control system of Fig.

- We first plot the Bode diagram for the final control element, the process, and the measuring element in series $G_1G_2H\,(jw)$.
- *It should be emphasized that the controller is omitted from this plot.*
- As noted on the figure, the crossover frequency for these three components in series is $\omega_{co}$ .
- At the crossover frequency, the overall amplitude ratio is *A,* as indicated.
- According to the Bode criterion, then, the gain of a proportional controller which would cause the system to be on the verge of instability is 1/ *A.*

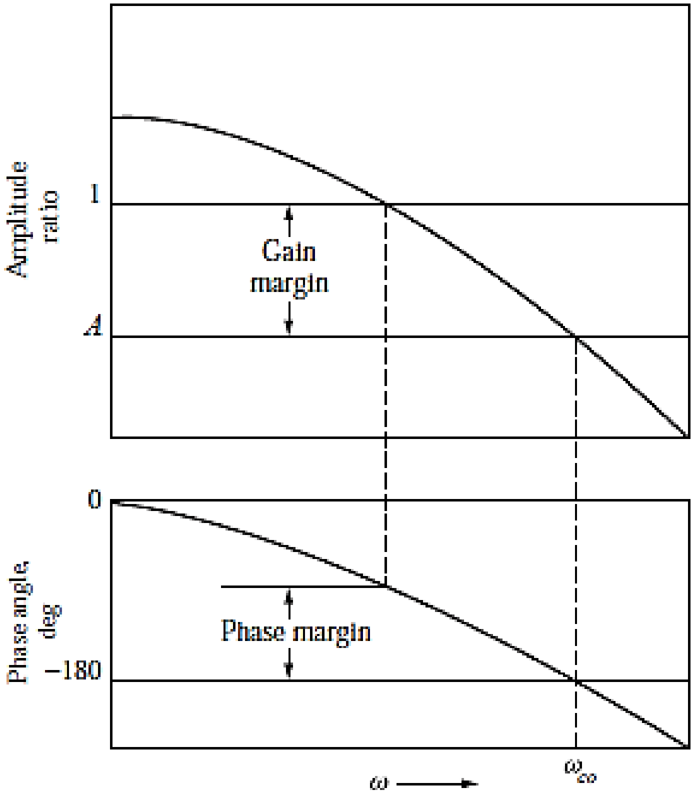
We define this quantity to be the ultimate gain *Ku*

$$K_u = \frac{1}{A}$$

*The ultimate period Pu is defined as the period of the sustained cycling that would occur if a proportional controller with gain Ku were used.*

$$P_u = \frac{2\pi}{\omega_{co}} [=] \frac{\text{radians / cycle}}{\text{radians / sec}} [=] \frac{\text{sec}}{\text{cycle}}$$

The factor of 2 $\pi$ appears, so *Pu* will be in units of time per cycle rather than time per radian.
It should be emphasized that *Ku* and *Pu* are easily determined from the Bode diagram



- The Ziegler-Nichols settings for controllers are determined directly from *Ku* and *Pu* according to the rules summarized in Table

**Ziegler-Nichols controller settings**

| Type of control | $G_c(S)$ | $K_c$ | $\tau_I$ | $\tau_D$ |
|---|---|---|---|---|
| Proportional | $K_c$ | $0.5K_u$ | | |
| Proportional-integral (PI) | $K_c\left(1+\frac{1}{\tau_I s}\right)$ | $0.45K_u$ | $\frac{P_u}{1.2}$ | |
| Proportional-integral-derivative (PID) | $K_c\left(1+\frac{1}{\tau_I s}+\tau_D s\right)$ | $0.6K_u$ | $\frac{P_u}{2}$ | $\frac{P_u}{8}$ |

Unfortunately, specifications of $K_c$ and $\tau_D$ for PD control cannot be made using only *Ku* and *Pu* .

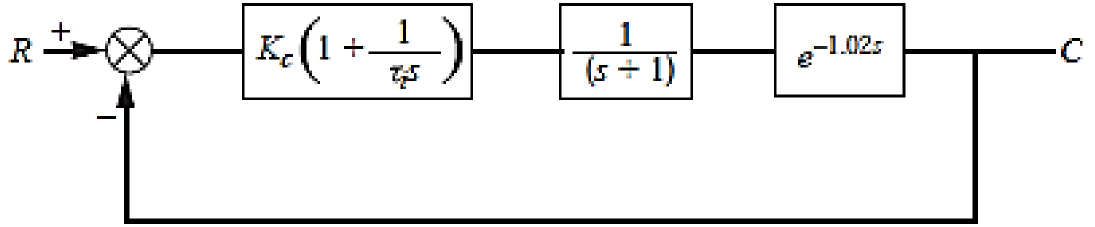
In general, the values 0.6 *Ku* and *Pu* /8, which correspond to the limiting case of no integral action in a three mode controller, are too conservative.

That is, the resulting system will be too stable.

There exist methods for this case which are in principle no more difficult to use than the Ziegler-Nichols rules.

- One of these is selection of $\tau_D$ for maximum *Kc* at 30 ° phase margin, which was discussed above.
- Another method, which utilizes the step response and avoids trial and error.
- The reasoning behind the Ziegler-Nichols selection of values of *Kc* is relatively clear.
- In the case of proportional control only, a gain margin of 2 is established.
- The addition of integral action introduces more phase lag at all frequencies hence a lower value of *Kc* is required to maintain roughly the same gain margin.
- Adding derivative action introduces phase lead.
- Hence, more gain may be tolerated.
- However, by and large the Ziegler-Nichols settings are based on experience with typical processes and should be regarded as first estimates.

Using the Ziegler-Nichols rules, determine *Kc* and $\tau_I$ for the control system shown in Fig.




We first obtain the crossover frequency by applying the Bode stability criterion.

$$-180° = -\tan^{-1}\omega - \left(\frac{180}{\pi}\right)(1.02)(\omega)$$

The value $180/\pi = 57.3$ converts radians to degrees. Solving this equation by trial and error gives for the crossover frequency $\omega_{co} = 2$ rad/min. The amplitude ratio AR at the crossover frequency for the open loop can be written as

$$\text{AR} = K_c \frac{1}{\sqrt{1+\omega^2}}(1) = \frac{K_c}{2.24}$$

According to the Bode criterion, the AR is 1.0 at the crossover frequency when the system is on the verge of instability.

Inserting AR= 1 into the above equation and solving for *Kc* gives *Kcu=* 2.24.

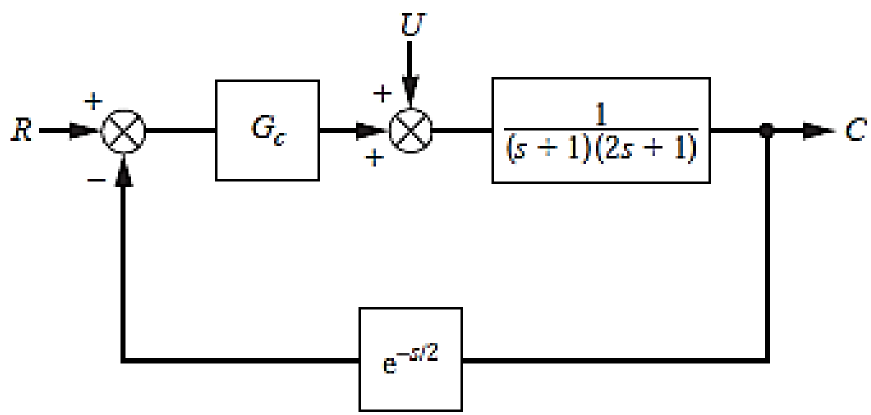
From the Ziegler-Nichols rules of Table

$$K_c = 0.45 K_{cu} = (0.45)(2.24) = 1.01$$

$$K_c = 0.45K_{cu} = (0.45)(2.24) = 1.01$$

$$\tau_I = \frac{P_u}{1.2} = \frac{2\pi / \omega_{co}}{1.2} = \frac{2\pi / 2}{1.2} = 2.62 \text{ min}$$

Using the Ziegler-Nichols rules, determine controller settings for various modes of control of the two-tank chemical-reactor system




For convenience, the process gain *K* and the controller gain *Kc* are combined into an overall gain *K*1 . The equivalent controller transfer function is regarded as

$$G_c = K_1\left(1 + \frac{1}{\tau_I s} + \tau_D s\right)$$

where $K_1$ (as well as $t_I$ and $t_D$) is to be selected by the Ziegler-Nichols rules. The required value of $K_c$ is then easily determined as

$$K_c = \frac{K_1}{K}$$ where $K$ = 0.09

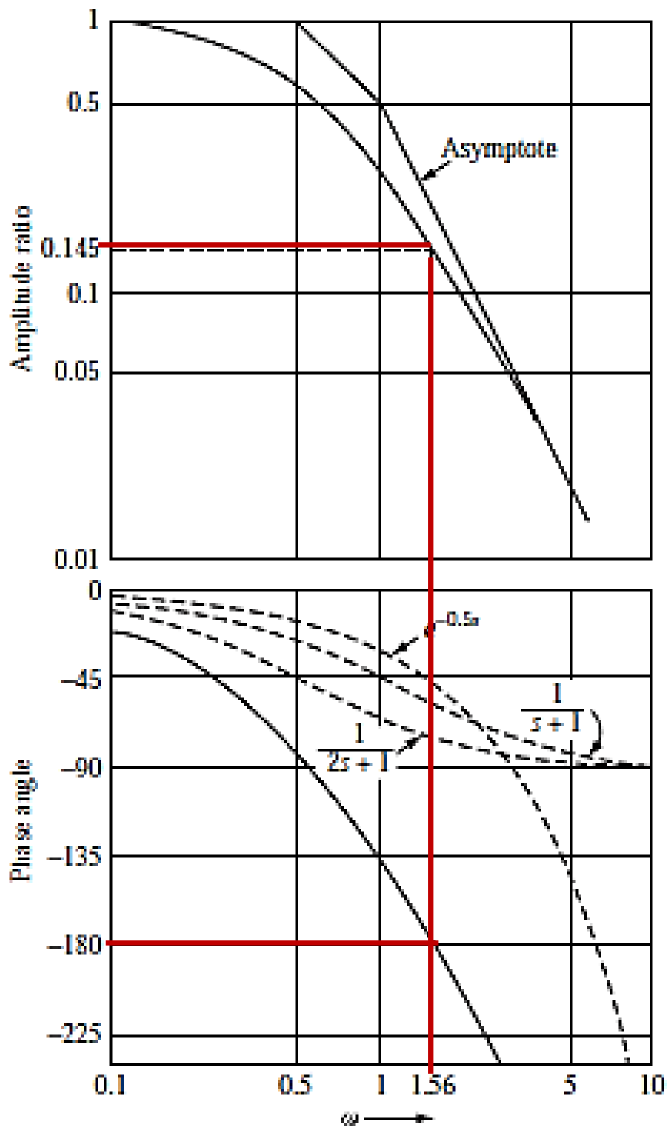
The Bode diagram for the transfer function *without the controller*

$$\frac{e^{-0.5s}}{(s+1)(2s+1)}$$

$$\omega_{co} = 1.56 \text{ rad / min}$$

$$K_{1_u} = \frac{1}{0.145} = 6.9$$

$$P_u = \frac{2\pi}{1.56} = 4.0 \text{ min / cycle}$$




constants determined from Table

**Ziegler-Nichols controller settings**

| Type of control | $G_c(S)$ | $K_c$ | $\tau_I$ | $\tau_D$ |
|---|---|---|---|---|
| Proportional | $K_c$ | $0.5K_u$ | | |
| Proportional-integral (PI) | $K_c\left(1 + \frac{1}{\tau_I s}\right)$ | $0.45K_u$ | $\frac{P_u}{1.2}$ | |
| Proportional-integral-derivative (PID) | $K_c\left(1 + \frac{1}{\tau_I s} + \tau_D s\right)$ | $0.6K_u$ | $\frac{P_u}{2}$ | $\frac{P_u}{8}$ |

- A plot comparing the open-loop frequency responses including the controller for the three cases, using the controller constants of Table is given in Fig.
- This figure shows quite clearly the effect of the phase lead due to the derivative action.
- The resulting gain and phase margins are listed in NEXT Table.
- From this table it may be seen that the margins are adequate and generally conservative.
- Note that to obtain the Bode diagram for systems including the PID controller, the controller transfer function is rewritten as

$$K_c\left(1+\frac{1}{\tau_I s}+\tau_D s\right)=K_c\frac{\tau_D\tau_I s^2+\tau_I s+1}{\tau_I s}$$

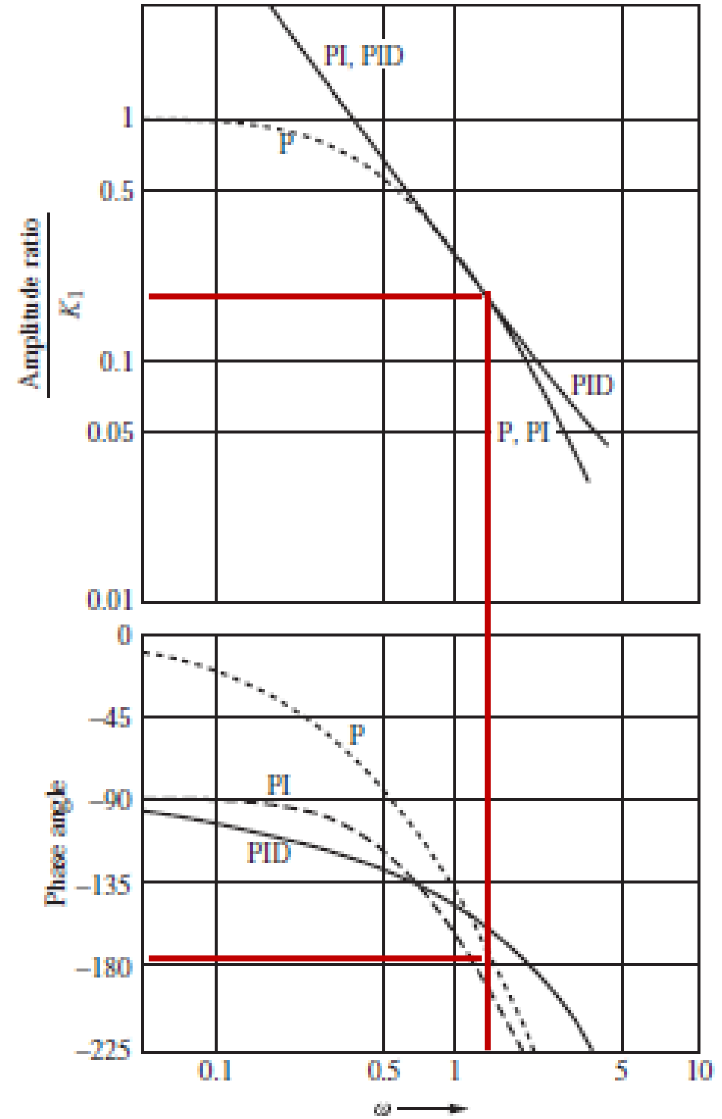
For the Ziegler- Nichols settings it is seen from Table that $\tau_I = 4\ \tau_D$. Making this substitution

$$G_c = K_c\frac{4\tau_D{}^2 s^2+4\tau_D s+1}{4\tau_D s}=\frac{K_c(2\tau_D s+1)^2}{4\tau_D s}$$

**Control constants for Example 16.4**

| Control | $K_1$ | $\tau_I$ | $\tau_D$ |
|---|---|---|---|
| P | 3.5 | | |
| PI | 3.1 | 3.3 | |
| PID | 4.2 | 2.0 | 0.50 |

$$\text{Gain margin} = \frac{1}{A} = \frac{1}{AR_{\phi=-180°}}$$

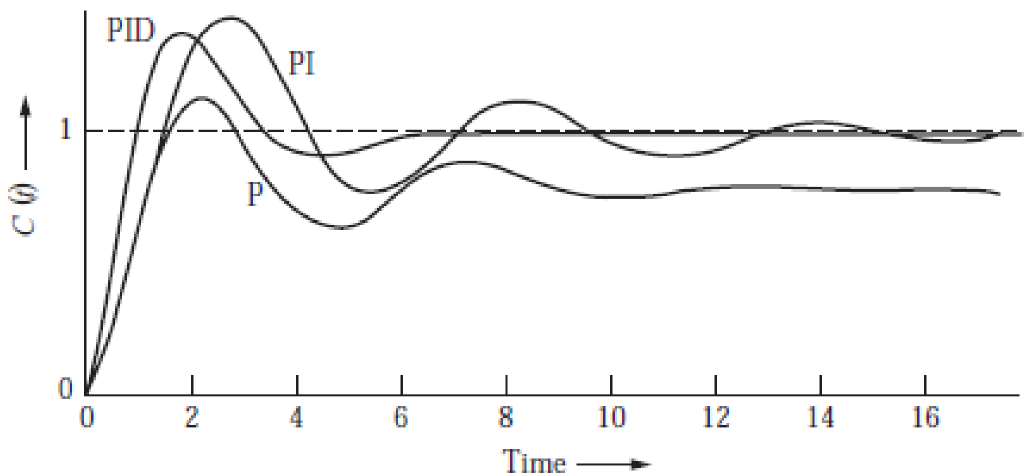



## Gain and Phase margins

| Control | Gain margin | Phase margin |
|---|---|---|
| P | 2.0 | 45° |
| PI | 1.9 | 33° |
| PID | 2.6 | 34° |

# Transient Responses

- Responses of $C(t)$ to a unit-step change in $R(t)$ are shown in Fig.
- These responses were obtained using the Ziegler-Nichols controller settings determined



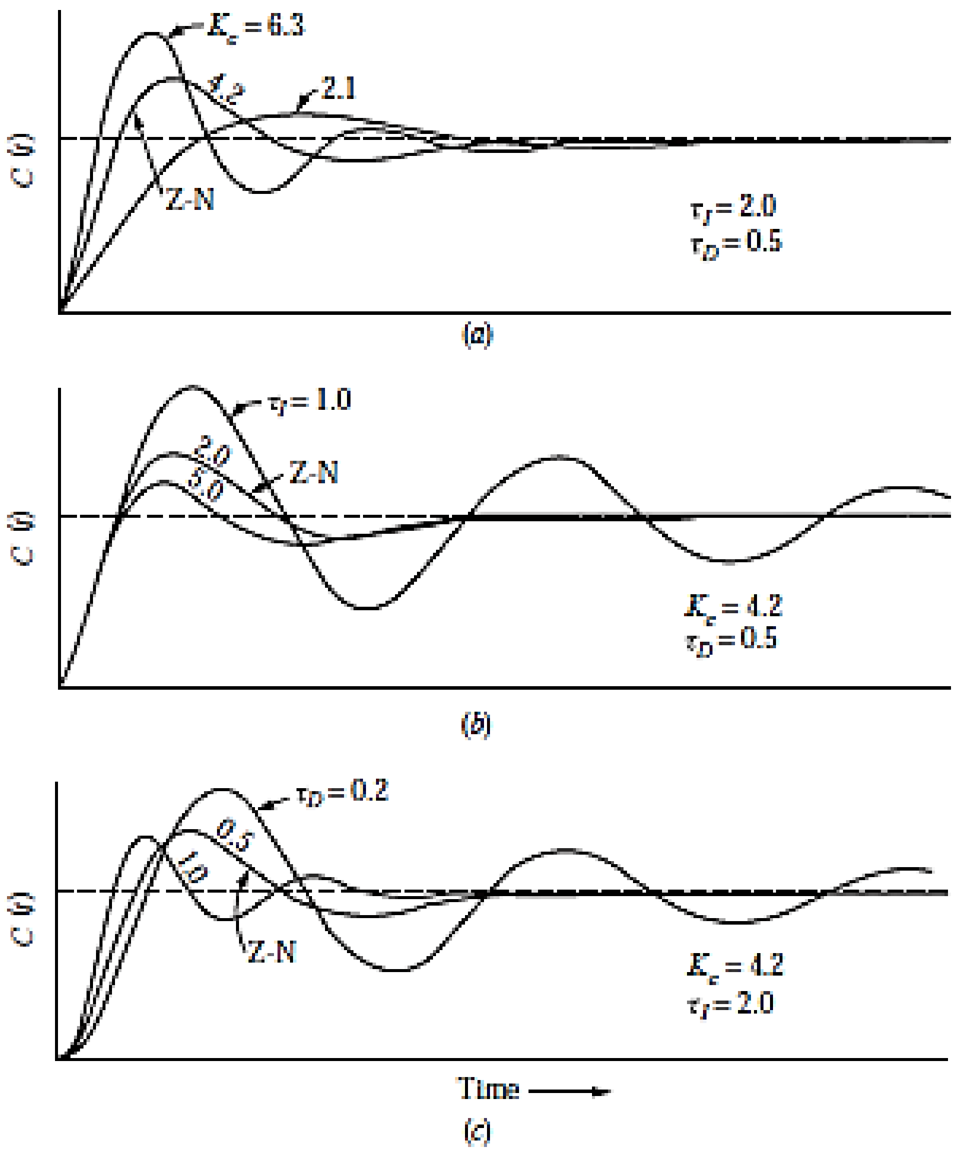

**Parameters for response of control system of Fig. 16–15 with Z-N settings**

| Control | Overshoot | Decay ratio | Rise time, min | Response time, min | Period of oscillation, min | Offset |
|---|---|---|---|---|---|---|
| P | 0.49 | 0.26 | 1.3 | 10.4 | 5.0 | 0.21 |
| PI | 0.46 | 0.29 | 1.5 | 11.8 | 5.5 | 0 |
| PID | 0.42 | 0.05 | 0.9 | 4.9 | 5.0 | 0 |

- that addition of integral action eliminates offset at the expense of a more oscillatory response.
- When derivative action is also included, the response is much faster (lower rise time) and much less oscillatory (lower response time).
- The large overshoots realized in all three cases are characteristic of systems with relatively large time delays

# Effect of varying controller settings

(Z-N indicates response using Ziegler-Nichols settings.)




- A possible combination, which should be tried, is to reduce *Kc* slightly and to increase $\tau_I$ and $\tau_D$ moderately.
- These changes would probably be tried on the actual reactor system when it is put into operation.
- Such adjustments from the preliminary settings are usually made by experienced control engineers, using trial procedures that are more art than science.

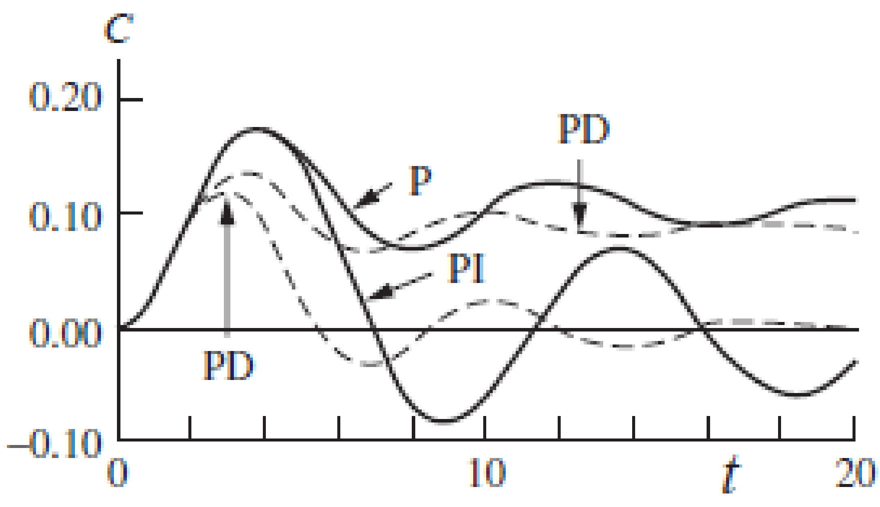
For this reason, we leave the problem of adjustment at this point

- We introduced the concepts of Bode stability criterion as well as gain margin and phase margin for determining appropriate controller settings to obtain the desired system response, while maintaining system stability.
- We also studied the use of Ziegler-Nichols controller settings as initial estimates for controller tuning.

# CONTROLLER TUNING

- The adjustment of the controller parameters to achieve satisfactory control is called *tuning.*
- The selection of the controller parameters is essentially an optimization problem in which the designer of the control system attempts to satisfy some criterion of optimality.
- The process of tuning can vary from a trial-and-error attempt to find suitable control parameters for good control to an elaborate optimization calculation.
- In many applications, there is no model of the process, and the criterion for good control is only vaguely defined.
- Methods for determining the model of a process from experimental tests will be described.
- Determining the model of a process experimentally is referred to as *process identification.*

- A typical criterion for good control is that the response of the system to a step change in set point or load have minimum overshoot and a one-quarter decay ratio.
- Other criteria may include minimum rise time and minimum settling time.




**PROPORTIONAL CONTROL.**

Proportional control produces an overshoot followed by an oscillatory response, which levels out at a value that does not equal the set point; this ultimate displacement from the set point is the offset.

**PROPORTIONAL-DERIVATIVE CONTROL.**

- Response exhibits a smaller overshoot and a smaller period of oscillation compared to the response for proportional control.
- The offset that still remains is less than that for proportional control.

**PROPORTIONAL-INTEGRAL CONTROL.**

- Response has about the same overshoot as proportional control, but the period is larger; however, the response returns to the set point (offset 0) after a relatively long settling time.
- The most beneficial influence of the integral action in the controller is the elimination of offset.

**PROPORTIONAL-INTEGRAL-DERIVATIVE CONTROL.**

- The use of PID control combines the beneficial features of PD and PI control. The response has lower overshoot and returns to the set point more quickly than the responses for the other types of controllers.
- Integral action, which is present in PI and PID controllers, eliminates offset.
- The addition of derivative action speeds up the response by contributing to the controller output a component of the signal that is proportional to the rate of change of the process variable.

- For simple, low-order (first- or second-order) processes that can tolerate some offset, P or PD control is satisfactory.
- For processes that cannot tolerate offset and are of low order, PI control is required.
- For processes that are of high order (those with transport lag or many first-order lags in series), PID control is needed to prevent large overshoot and long settling time.
- There is probably little justification to select a P or PD controller for most processes.
- The PI controller is often the choice because it eliminates offset and requires only two parameter adjustments.
- Tuning a PID controller is more difficult because three parameters must be adjusted.
- The presence of derivative action can also cause the controller output to be very jittery if there is much noise in the signals.

# Criteria for Good Control

- We now turn our attention to some of the criteria for good control that are used to judge whether a control system is well tuned.
- For example, a response that gives minimum overshoot and ¼ decay ratio is often considered as a satisfactory response.
- In many cases, tuning is done by trial and error until such a response is obtained.
- To compare different responses that use different sets of controller parameters, a criterion that reduces the entire response to a single number, or a *figure of merit, is desirable.*
- *The figure of merit provides a means* of "keeping score" for the different control parameters, and as we shall see, the low "score" generally wins.
- It is dangerous, however, to rely solely on the score to determine the best choice for the control parameters.
- The control system designer should examine the nature of the response in conjunction with the requirements for the process to determine the "best" choice of settings.

- One criterion that is often used to evaluate a response of a control system is the integral of the square of the error (ISE) with respect to time.
- The definition of ISE is as follows:
- Integral of the square of the error (ISE)

$$\text{ISE} = \int_0^\infty e^2\, dt$$

where *e is the usual error (i.e., set point – control variable).*
*For a stable system for which* there is no offset [i.e., *e(∞) = 0],*
*produces a single number as a figure of merit.*
The objective of the designer is to obtain the minimum value of ISE by proper choice of control parameters.
A response that has large errors and persists for a long time will produce a large ISE.
For the cases of P and PD control, where offset occurs, the integral given by Eq. does not converge.
In these cases, one can use a modified integr and, which replaces the error
*r(t)- c(t) by c(∞) - c(t).*
*Since c(∞) - c(t) does approach zero* as *t goes to infinity, the integral will converge and serve as a figure of merit.*

Two other criteria often used in process control are defined as follows:

- **Integral of the absolute value of error (IAE)**

$$\mathrm{IAE} = \int_0^\infty |e|\,dt$$

- **Integral of time-weighted absolute error (ITAE)**

$$\mathrm{ITAE} = \int_0^\infty |e|\,t\,dt$$

Each of the three figures of merit, given by Eqs. has different purposes.

The ISE will penalize (i.e., increase the value of ISE) the response that has large errors, which usually occur at the beginning of a response, because the error is squared.

The ITAE will penalize a response that has errors that persist for a long time.

The IAE will be less severe in penalizing a response for large errors and treat all errors (large and small) in a uniform manner.

- The ISE figure of merit is often used in optimal control theory because it can be used more easily in mathematical operations (e.g., differentiation) than the figures of merit, which use the absolute value of error.
- In applying the tuning rules to be discussed in the next section, these figures of merit can be used in comparing responses that are obtained with different tuning rules.

# TUNING RULES

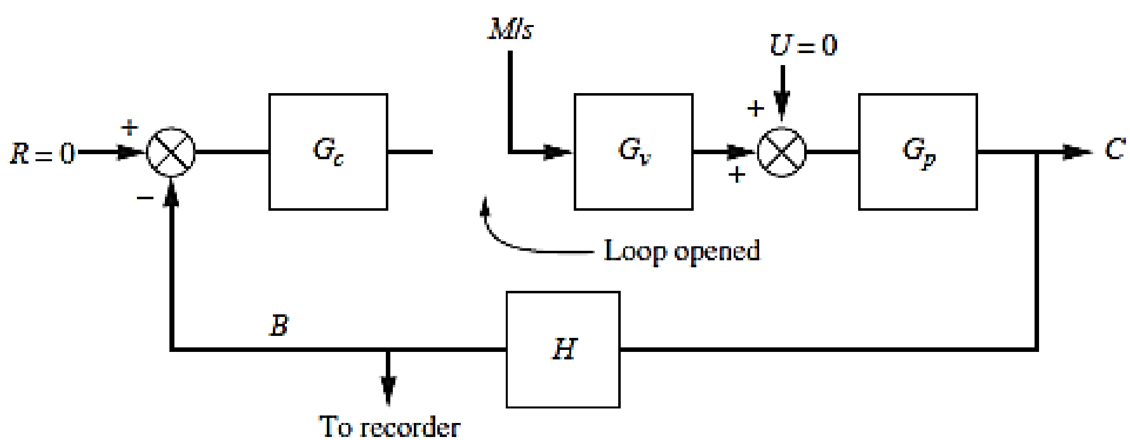
## Ziegler-Nichols (Z-N) Rules

- After the process reaches steady state at the normal level of operation, remove the integral and derivative modes of the controller, leaving only proportional control.
- On some PID controllers, this requires that the integral time $\tau_I$ *be set to its maximum* value and the derivative time $\tau_D$ *to its minimum value.*
- *On computer-based* controllers, the integral and derivative modes can be removed completely from the controller.
- Select a value of proportional gain *Kc*, disturb the system, and observe the transient response.
- Select a higher value of *Kc* and again observe the response of the system.
- Continue increasing the gain in small steps until the response first exhibits a sustained oscillation.
- The value of gain and the period of oscillation that correspond to the sustained oscillation are the ultimate gain *Kcu* and the ultimate period *Pu*.

- From the values of *Kcu* and *Pu* found in step 2, use the Ziegler-Nichols rules given In Table to determine controller settings ($Kc$, $\tau_I$, $\tau_D$).
- Fine-tuning of the controller settings is usually required to get an improved control response.

# Cohen and Coon (C-C) Rules

- This method was proposed by Cohen and Coon (1953) and is often used as an alternative to the Z-N method
- The method of tuning to be discussed is an *open-loop method,* in which the control action is removed from the controller by placing it in manual mode.
- The open-loop transient is induced by a step change in the signal to the valve.



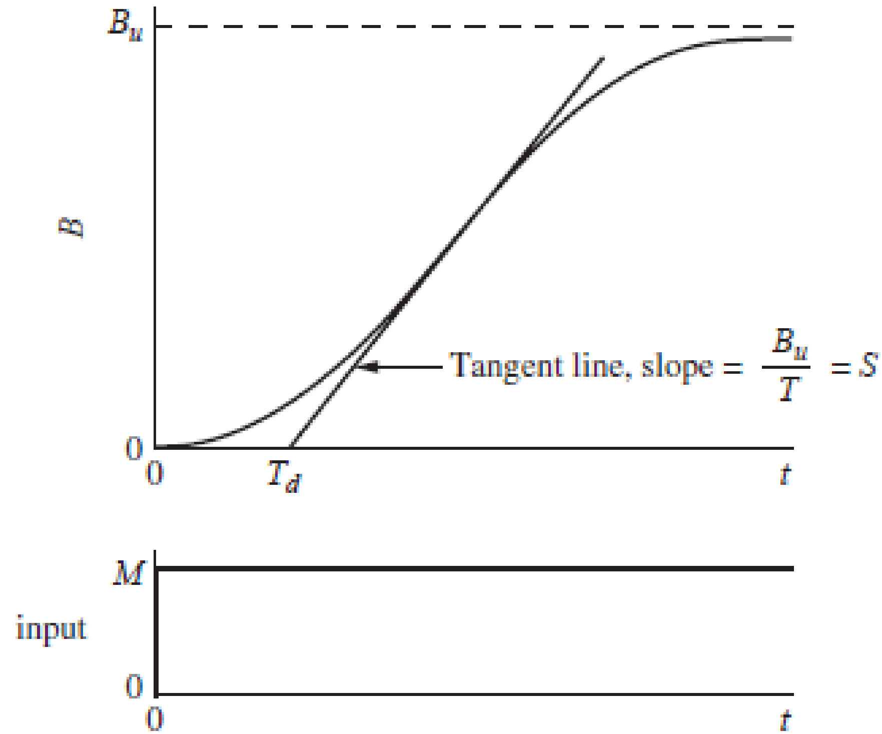
- Figure shows a typical control loop in which the control action is removed and the loop opened for the purpose of introducing a step change ($M/s$) to the valve.
- The step response is recorded at the output of the measuring element.
- The step change to the valve is conveniently provided by the output from the controller, which is in manual mode.
- The response of the system (including the valve, process, and measuring element) is called the *process reaction curve;* exhibits an S shape
- After we present the Cohen and Coon method of tuning, The basis for their recommendations will be discussed.
- The C-C method is summarized in the following steps:

1. After the process reaches steady state at the normal level of operation, switch the controller to manual.

In a modern controller, the controller output will remain at the same value after switching as it had before switching.

(This is called “bumpless” transfer.)

**2.** With the controller in manual, introduce a small step change in the controller output that goes to the valve and record the transient, which is the process reaction curve




**3.** Draw a straight line tangent to the curve at the point of inflection.
The intersection of the tangent line with the time axis is the apparent transport lag *Td*; the apparent first-order time constant *T* is obtained from

$$T = \frac{B_u}{S}$$

where $B_u$ is the ultimate value of *B* at large *t* and *S* is the slope of the tangent line.

The steady-state gain that relates *B* to *M* in Fig.

$$K_p = \frac{B_u}{M}$$

The attempt to model the process reaction curve by the method shown in Fig. is crude and does not give a very good fit. Finding the point of inflection and drawing a tangent line at this point are quite difficult, especially if the data for the process reaction curve are not accurate and if they scatter.

**4.** Using the values of *Kp*, *T*, and *Td* from step 3, the controller settings are found from the relations given in Table.

- All the controller settings are a function of the dimensionless group *Td* /*T*, the ratio of the apparent transport lag to the apparent time constant.
- Also $K_c$ is inversely proportional to $K_p$.
- Their computations required that the response have 1/ 4decay ratio, minimum offset, minimum area under the load-response curve, and other favorable properties.

**Cohen-Coon controller settings**

| Type of control | Parameter setting |
|---|---|
| Proportional (P) | $K_c = \frac{1}{K_p}\frac{T}{T_d}\left(1 + \frac{T_d}{3T}\right)$ |
| Proportional-integral (PI) | $K_c = \frac{1}{K_p}\frac{T}{T_d}\left(\frac{9}{10} + \frac{T_d}{12T}\right)$ |
| | $\tau_I = T_d\frac{30 + 3T_d/T}{9 + 20T_d/T}$ |
| Proportional-derivative (PD) | $K_c = \frac{1}{K_p}\frac{T}{T_d}\left(\frac{5}{4} + \frac{T_d}{6T}\right)$ |
| | $\tau_D = T_d\frac{6 - 2T_d/T}{22 + 3T_d/T}$ |
| Proportional-integral-derivative (PID) | $K_c = \frac{1}{K_p}\frac{T}{T_d}\left(\frac{4}{3} + \frac{T_d}{4T}\right)$ |
| | $\tau_I = T_d\frac{32 + 6T_d/T}{13 + 8T_d/T}$ |
| | $\tau_D = T_d\frac{4}{11 + 2T_d/T}$ |

A better method for fitting the process reaction curve to a first-order with transport lag model is to perform a least-squares fit of the data. Some computer software, such as the LOOP-PRO package (see www.controlstation.com), provide an easy means of performing the fitting process.

The rationale for the C-C tuning method begins with the representation of the S-shaped process reaction curve by a first-order with transport lag mode

- Thus

$$G_p(s) = \frac{K_p e^{-T_d s}}{Ts + 1}$$

- To understand the basis for the graphical procedure, consider the response of the transfer function of Eq. to a step change in input.
- After *t = Td*, the response is a first order response.
- The point of inflection of the curve occurs at *t = Td*, and the slope of the tangent line at this point is related to the time constant by the relation

$$S = \frac{B_u}{T}$$