Nyquist Plots

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A Nyquist plot is an alternative way to represent the frequency response characteristics of a dynamic system. It uses the Im $[G(j\omega)]$ as ordinate and Re $[G(j\omega)]$ as abscissa. Figure 17.13 shows the form of a Nyquist plot.

The *Nyquist plot contains the same information as the pair of Bode plots for the same system.* Therefore, its construction is rather easy given the corresponding Bode plots. Let us now construct the Nyquist plots of some typical systems using their Bode plots developed in the preceding section.

A specific value of the frequency ω defines a point on this plot. Thus at point 1 (Figure 17.13) the frequency has a value ω_1 and we observe the following:

1. The distance of the point 1 from the origin (0, 0) is the amplitude ratio at the frequency ω_1:

$$\text{distance} = \sqrt{[\text{Re}\,[G(j\omega_1)]]^2 + [\text{Im}\,[G(j\omega_1)]]^2} = |G(j\omega_1)| = \text{AR}$$

2. The angle ϕ with the real axis is the phase shift at the frequency ω_1:

$$\phi = \tan^{-1}\frac{\text{Im}\,[G(j\omega_1)]}{\text{Re}\,[G(j\omega_1)]} = \arg G(j\omega_1) = \text{phase shift}$$

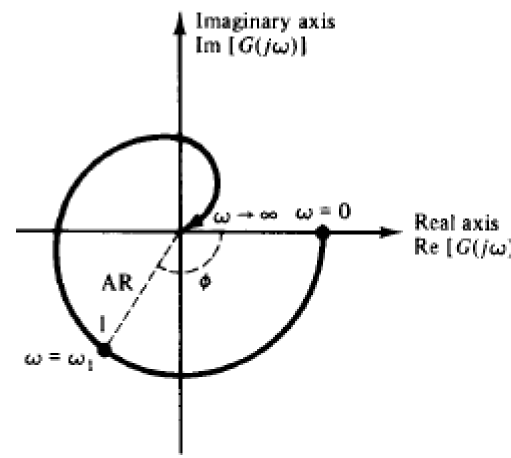


Figure 17.13 Form of a Nyquist plot.

Frequency Response of a Pure Capacitive Process

The transfer function is

$$G(s) = \frac{K_p}{s}$$

Put $s = j\omega$ and take

$$G(j\omega) = \frac{K_p}{j\omega} = \frac{K_p}{j\omega}\frac{j\omega}{j\omega} = 0 - j\,\frac{K_p}{\omega}$$

Consequently, for the ultimate response:

1. The amplitude ratio is

$$\text{AR} = |G(j\omega)| = \frac{K_p}{\omega}$$

2. The phase shift is

$$\phi = \tan^{-1} -\infty = -90^\circ$$

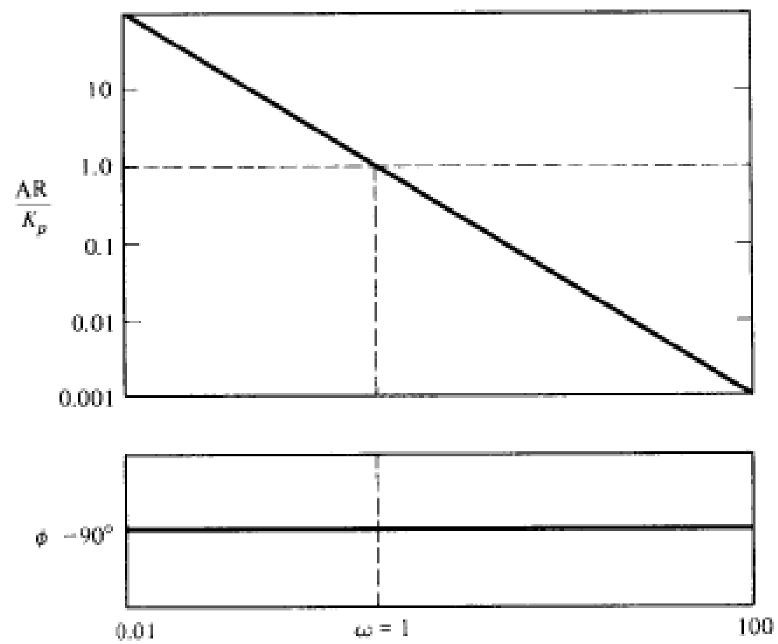
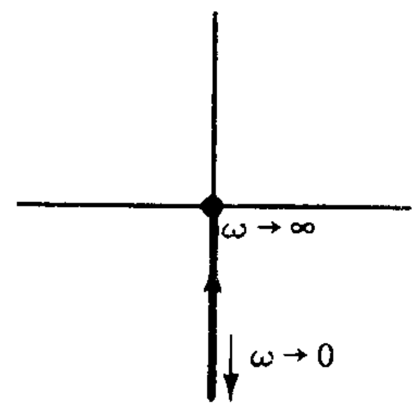


Figure 17.4 Bode plots for pure capacitive process.

that is, the ultimate sinusoidal response of the system *lags behind* the input wave by 90°.



(e)

Frequency Response of a Pure Dead-Time Process

The transfer function is

$$G(s) = e^{-\tau_d s}$$

Put $s = j\omega$ and take

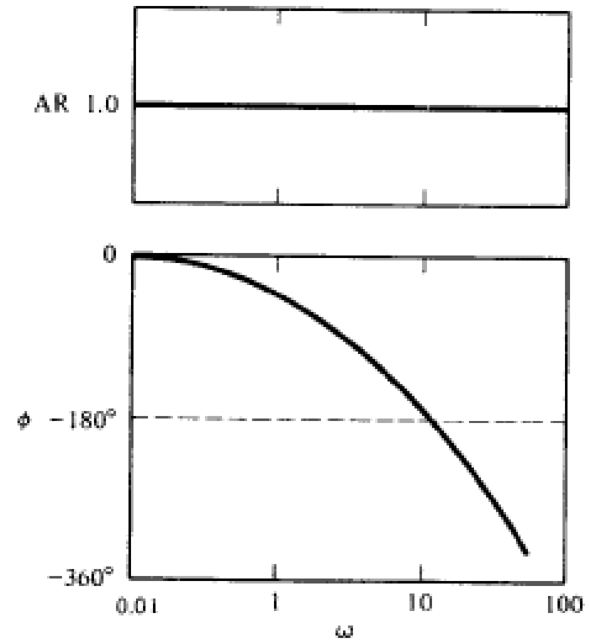
$$G(j\omega) = e^{-j\tau_d \omega}$$

Clearly:

$$\text{amplitude ratio} = |G(j\omega)| = 1$$

$$\phi = \text{phase shift} = \text{argument of } G(j\omega) = -\tau_d \omega$$

that is, a phase lag, since $\phi < 0$.



Frequency Response of Feedback Controllers

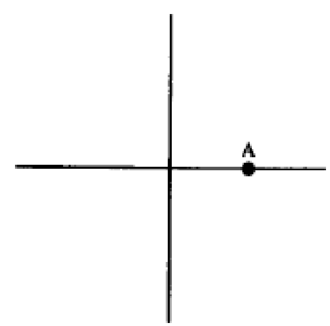
Let us now shift our attention to the various types of feedback controllers.

1. *Proportional controller:* The transfer function is

$$G_c(s) = K_c$$

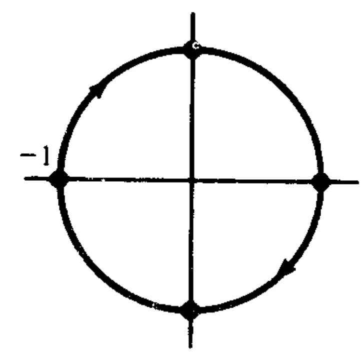
Therefore,

$$\text{AR} = K_c \qquad \text{and} \qquad \phi = 0$$



(a)

Proportional controller



(d)

Pure dead time

2. *Proportional-integral controller:* The transfer function is

$$G_c(s) = K_c\left(1 + \frac{1}{\tau_I s}\right)$$

Therefore,

$$AR = |G_c(j\omega)| = K_c\sqrt{1 + \frac{1}{(\omega\tau_I)^2}}$$

$$\phi = \arg G_c(j\omega) = \tan^{-1}\left(\frac{-1}{\omega\tau_I}\right) < 0$$

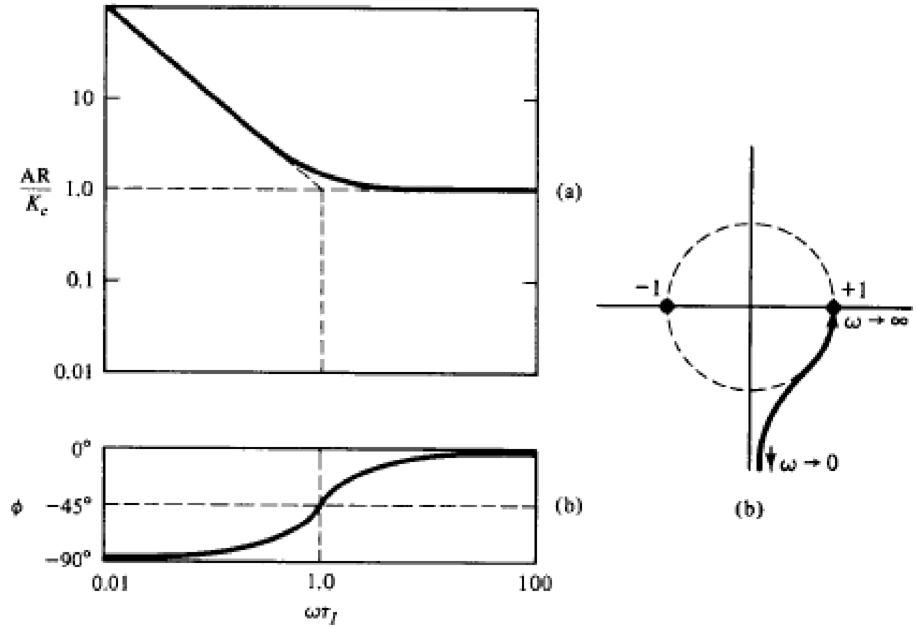


Figure 17.8 Bode plots for PI controller.

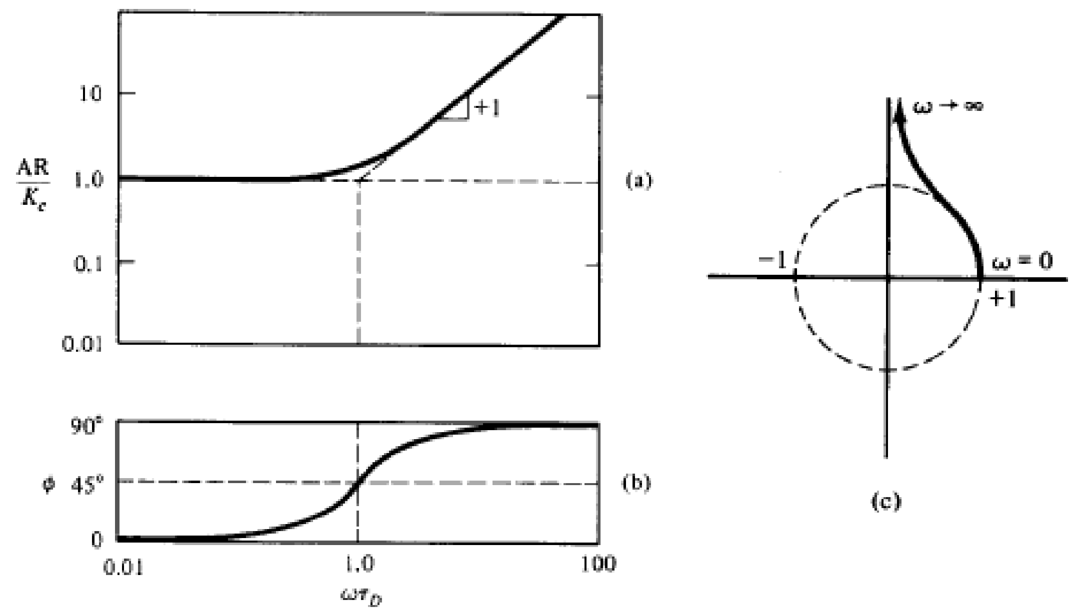
3. *Proportional-derivative controller:* The transfer function is

$$G_c(s) = K_c(1 + \tau_D s)$$

Therefore,

$$AR = |G_c(j\omega)| = K_c\sqrt{1 + \tau_D^2\omega^2}$$

$$\phi = \arg G_c(j\omega) = \tan^{-1}\tau_D\omega > 0$$



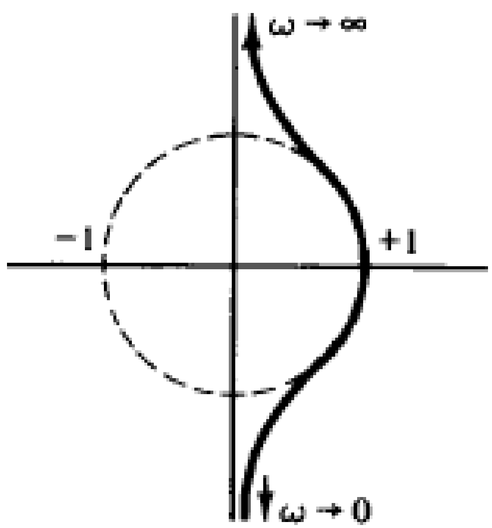
4. *Proportional-integral-derivative controller:* The transfer function is

$$G_c(s) = K_c\left(1 + \frac{1}{\tau_I s} + \tau_D s\right)$$

and it is easy to show that

$$\mathrm{AR} = |G_c(j\omega)| = K_c\sqrt{\left(\tau_D\omega - \frac{1}{\tau_I\omega}\right)^2 + 1}$$

$$\phi = \tan^{-1}\left(\tau_D\omega - \frac{1}{\tau_I\omega}\right)$$



(d)

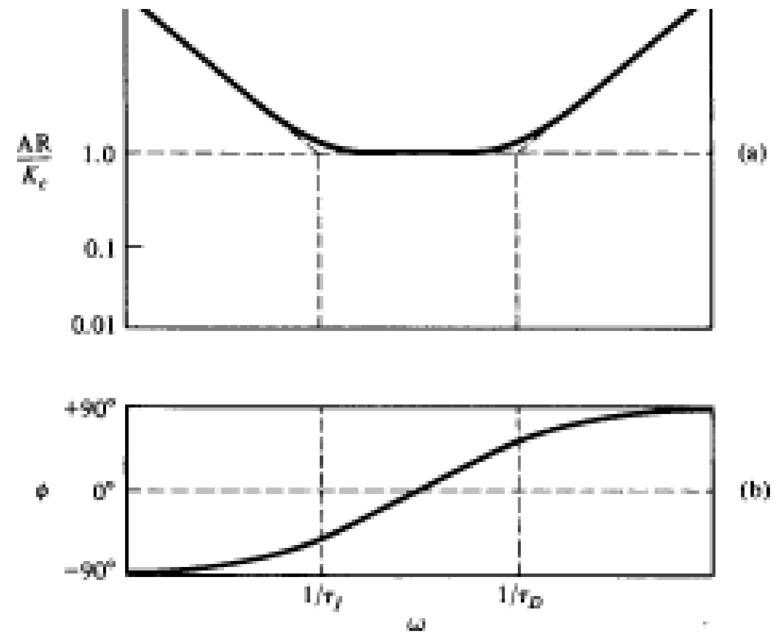


Figure 17.10 Bode plots for PID controller.

Nyquist Plots

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The *Nyquist plot contains the same information as the pair of Bode plots for the same system.* Therefore, its construction is rather easy given the corresponding Bode plots. Let us now construct the Nyquist plots of some typical systems using their Bode plots developed in the preceding section.

A specific value of the frequency ω defines a point on this plot. Thus at point 1 (Figure 17.13) the frequency has a value ω_1 and we observe the following:

1. The distance of the point 1 from the origin (0, 0) is the amplitude ratio at the frequency ω_1:

$$\text{distance} = \sqrt{[\text{Re}\,[G(j\omega_1)]]^2 + [\text{Im}\,[G(j\omega_1)]]^2} = |G(j\omega_1)| = \text{AR}$$

2. The angle ϕ with the real axis is the phase shift at the frequency ω_1:

$$\phi = \tan^{-1}\frac{\text{Im}\,[G(j\omega_1)]}{\text{Re}\,[G(j\omega_1)]} = \arg G(j\omega_1) = \text{phase shift}$$

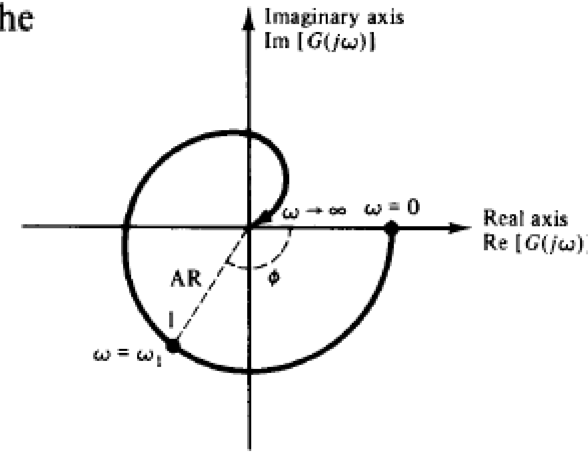


Figure 17.13 Form of a Nyquist plot.

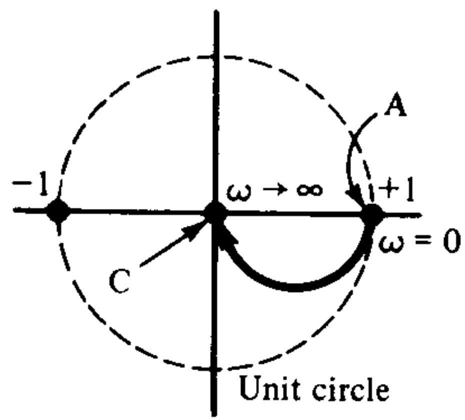
The shape and location of a Nyquist plot are characteristic for a particular system.

First-order system

1. When $\omega = 0$, then AR = 1 and $\phi = 0$. Therefore, the beginning of the Nyquist plot is on the real axis where $\phi = 0$ and at a distance from the origin (0, 0) equal to 1 (see point A in Figure 17.14a).
2. When $\omega \to \infty$, then AR $\to$ 0 and $\phi \to -90°$. Therefore, the end of the Nyquist plot is at the origin where the distance from it is zero (point C in Figure 17.14a).
3. Since for every intermediate frequency

$$0 < AR < 1 \qquad \text{and} \qquad -90° < \phi < 0$$

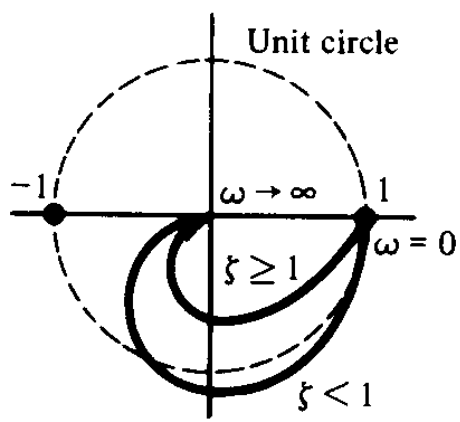
the Nyquist plot will be inside a unit circle and will never leave the first quadrant. Its complete shape and location are shown in Figure 17.14a.



(a)

Second-order system

1. When $\omega = 0$, then AR = 1 and $\phi = 0$. Thus the beginning of the Nyquist plot is on the real axis at a distance equal to 1 from the origin.
2. When $\omega \to \infty$, then AR $\to 0$ and $\phi \to -180°$; that is, the Nyquist plot will end at the origin and will approach it from the second quadrant.
3. When $\zeta \geq 1$, then AR ≤ 1 and the Nyquist plot stays within a unit circle. When $\zeta < 1$, then AR becomes larger than 1 for a range of frequencies. Thus the Nyquist plot goes outside the unit circle for a certain range of frequencies. Figure 17.14b shows the Nyquist plot for a second-order system.



(b)

Third-order system

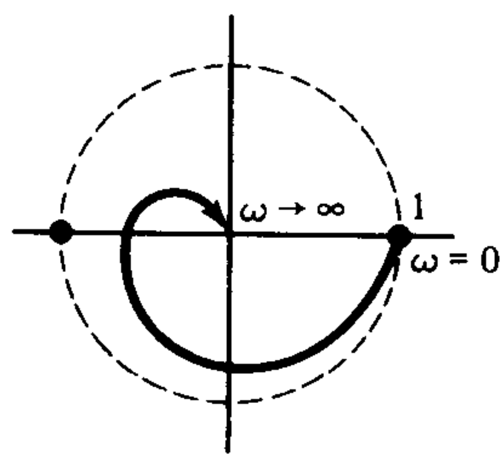
The transfer function is

$$G(s) = \frac{1}{(\tau_1 s + 1)(\tau_2 s + 1)(\tau_3 s + 1)} \qquad \text{with } \tau_1, \tau_2, \tau_3 \text{ real and positive}$$

It is easy to show that:

When $\omega = 0$, then $AR = 1$ and $\phi = 0$, while
When $\omega \to \infty$, then $AR = 0$ and $\phi \to -270°$.

Therefore, the Nyquist plot starts from the real axis at a distance 1 from the origin and ends at the origin, going through the third quadrant (Figure 17.14c).

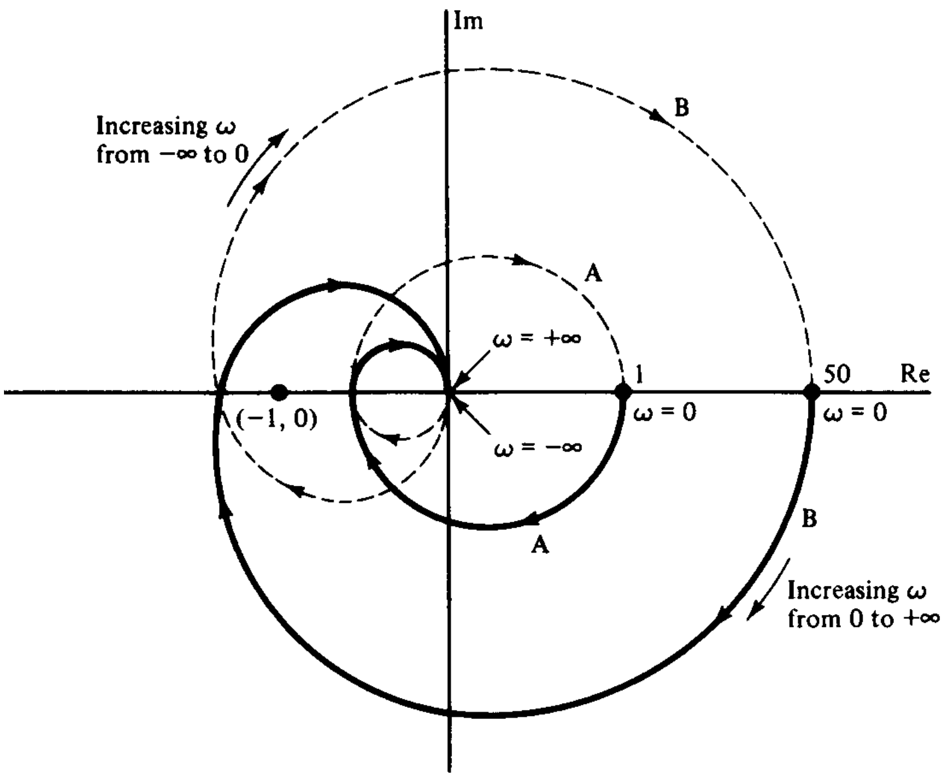


(c)

Nyquist Stability Criterion

The Nyquist stability criterion states that:

If the open-loop Nyquist plot of a feedback system encircles the point (–1, 0) as the frequency ω takes any value from $-\infty$ to $+\infty$, the closed-loop response is unstable.



Consider the open-loop transfer function

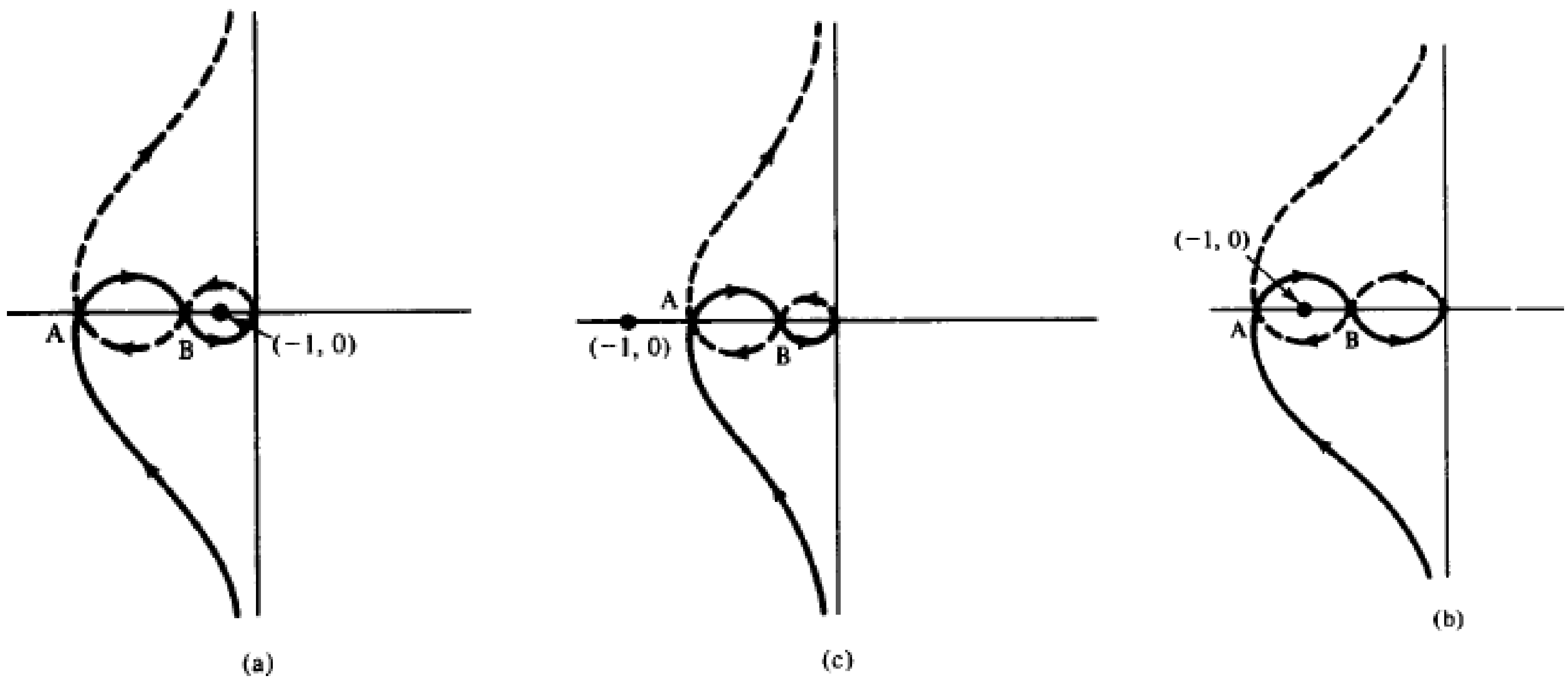
$$G_{OL} = \frac{K_c}{(s+1)(2s+1)(4s+1)}$$

Nyquist plots for G_{OL} when $K_c = 1$ (curve A) and $K_c = 50$ (curve B). For each Nyquist plot the solid line covers the frequency range $0 \le \omega < +\infty$, and the dashed part covers the frequencies from $-\infty$ to 0. The dashed segment of the Nyquist plot is the mirror image of the solid-line segment with respect to the real axis.

shows that curve A *does not encircle* the point (–1, 0), whereas curve B does. Thus, according to the Nyquist criterion, the feedback system with open-loop Nyquist plot the curve A is stable, while curve B indicates an unstable closed-loop system. This in turn implies that for $K_c = 1$ the system is stable, whereas for $K_c = 50$ it is unstable.

Conditional Stability and the Nyquist Criterion

Consider the Nyquist plots shown in Figure 18.8a through c. All correspond to the same open-loop transfer function with different values for the proportional gain K_c. The plots in Figure 18.8a and c do not encircle the point (–1, 0), whereas the Nyquist plot of Figure 18.8b does. Therefore,



Nyquist plots for Example 18.6: (a), (c) stable; (b) unstable.

the feedback systems corresponding to the first and third Nyquist plots have stable closed-loop responses, whereas that of the second is unstable.

From the plots above it is clear that the closed-loop response becomes unstable for a range of values K_c such that the point (–1, 0) is between A and B of the resulting Nyquist plot.

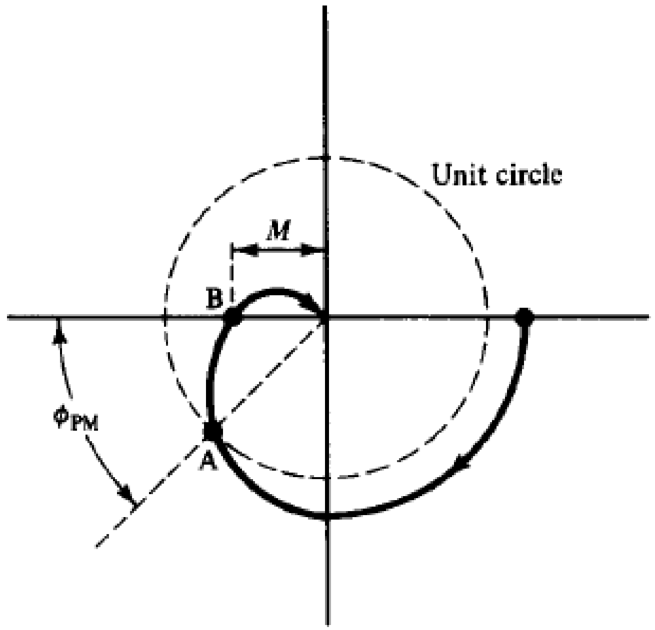
When point (–1, 0) is to the left of A (Figure 18.8c)

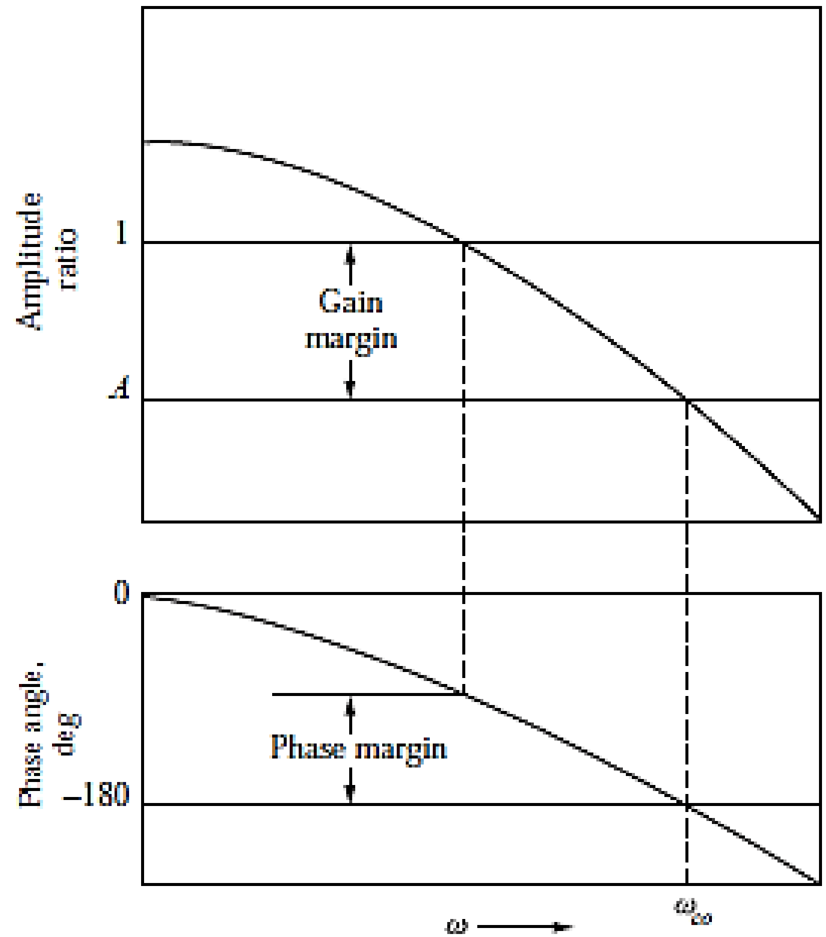
or to the right of B (Figure 18.8a), it is not encircled by the

Nyquist plot and the corresponding closed-loop response is stable.

Computing gain and phase margins from Nyquist plots.

The gain margin and phase margin of an open-loop response can also be computed from a Nyquist plot. This should be expected since Bode and Nyquist plots of a system contain exactly the same information.





At the frequency of point A

the Nyquist plot intersects the unit circle around the origin. Therefore, since the distance of point A from the origin is the amplitude ratio at this frequency, we conclude that the angle ϕ_{PM} represents the phase margin.

Furthermore, at the frequency of point B, the phase lag is equal to 180°. The amplitude ratio at this point is the distance between B and the origin, (i.e., AR = M). Consequently, the gain margin is easily found as $1/M$.

Purely capacitive or pure integrator process definition and it's transfer function

The transfer function of a firstorder process is given by;

$$G(s) = \frac{y(s)}{f(s)} = \frac{k_p}{\tau_p . S + 1}$$

A first-order process with a transfer function given by above eqn. is also known as: first-order lag, linear lag, exponential transfer lag.

If on the other hand, a0 = 0, then from eqn. we take

$$\frac{dy}{dx} = \frac{b}{a1} * f(t) = k_p' . f(t)$$

$$G(s) = \frac{y(s)}{f(s)} = \frac{k_p}{S}$$

In such case the process is called purely capacitive or pure integrator

A pure capacitive process will cause serious control problems, because it cannot balance itself. In the tank of Example 10.3, we can adjust manually the speed of the constant-displacement pump, so as to balance the flow coming in and thus keep the level constant. But any small change in the flow rate of the inlet stream will make the tank flood or run dry (empty). This attribute is known as non-self-regulation. [Pg.457]

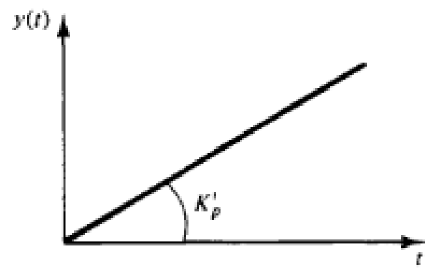


Figure 10.3 Unbounded response of pure capacitive process.

Such response, characteristic of a pure capacitive process, lends the name *pure integrator* because it behaves as if there were an integrator between its input and output.

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Processes with integrating action most commonly encountered in a chemical process are tanks with liquids, vessels with gases, inventory systems for raw materials or products, and so on.