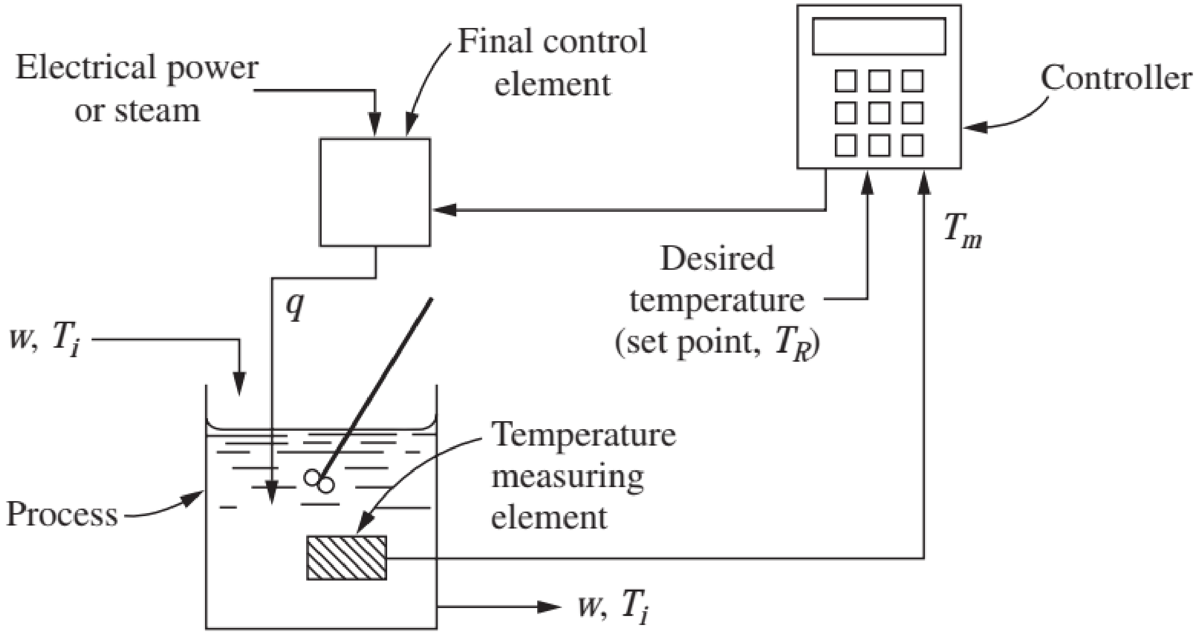
LINEAR CLOSED-LOOP SYSTEMS

Chapter 8

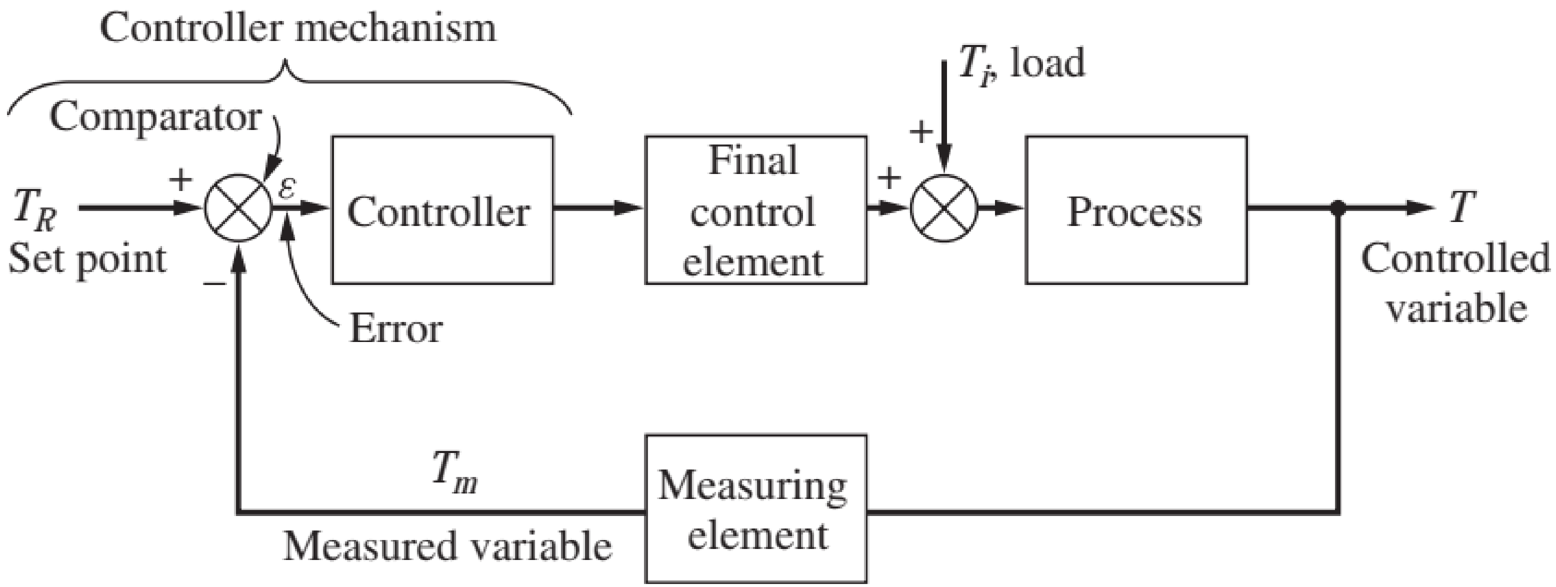


A liquid stream at a temperature T_i enters an insulated, well-stirred tank at a constant flow rate w (mass/time).
It is desired to maintain (or control) the temperature in the tank at T_R by means of the controller.
If the measured tank temperature T_m differs from the desired temperature T_R, the controller senses the difference or *error* $\varepsilon = T_R - T_m$ and changes the heat input in such a way as to reduce the magnitude of ε.
If the controller changes the heat input to the tank by an amount that is proportional to ε, we have *Proportional control.*

COMPONENTS OF A CONTROL SYSTEM

- The system shown in Figure may be divided into the following components:
  1. Process (stirred-tank heater).
  2. Measuring element (thermometer).
  3. Controller.
  4. Final control element (variable transformer or control valve).

BLOCK DIAGRAM



- Diagram makes it much easier to visualize the relationships among the various signals.
- New terms, which appear in Fig, are *set point* and *load.*
- The set point is a synonym for the desired value of the controlled variable.
- The load refers to a change in any variable that may cause the controlled variable of the process to change.
- The inlet temperature *Ti* is a load variable. Other possible loads for this system are changes in flow rate and heat loss from the tank.
- The control system is called a *closed-loop* system or a feedback system.
- The measured value of the controlled variable is returned or “feed back” to a device called the *comparator.*
- In the comparator, the controlled variable is compared with the desired value or *set point.*
- If there is any difference between the measured variable and the set point, an error is generated.
- This error enters a *controller,* which in turn adjusts the *final control element* to return the controlled variable to the set point.

Negative Feedback Versus Positive Feedback

- Negative feedback ensures that the difference between T_R and T_m is used to adjust the control element so that the tendency is to reduce the error.
- If the load *Ti* should increase, *T* and *Tm* would start to increase, which would cause the error ε to become negative.
- The decrease in error would cause the controller and final control element to *decrease* the flow of heat to the system, with the result that the flow of heat would eventually be reduced to a value such that *T* approaches T_R
- If the signal to the comparator were obtained by adding T_R and T_m, we would have a *positive feedback* system, which is inherently unstable.
- If *Ti* were to increase, *T* and *Tm* would increase, which would cause the signal from the comparator (ε) to increase.
- Result that the heat to the system would increase. However, this action, which is just the opposite of that needed, would cause *T* to increase further.
- It should be clear that this situation would cause *T* to “run away” and control would not be achieved.
- For this reason, positive feedback would never be used intentionally in the system.
- However, in more complex systems it may arise naturally.

Servo Problem Versus Regulator Problem

- The control system can handle either of two types of situations.
- In the first situation, which is called the servomechanism-type (or servo) problem, we assume that there is no change in load *Ti* .
- We are interested in changing the bath temperature according to some prescribed function of time.
- For this problem, the set point T_R would be changed in accordance with the desired variation in bath temperature.
- If the variation is sufficiently slow, the bath temperature may be expected to follow the variation in T_R very closely.
- There are occasions when a control system in the chemical industry will be operated in this manner.
- For example, one may be interested in varying the temperature of a reactor according to a prescribed time-temperature pattern.
- However, the majority of problems that may be described as the servo type come from fields other than the chemical industry.

- The tracking of missiles and aircraft and the automatic machining of intricate parts from a master pattern are well-known examples of the servo-type problem.
- The servo problem can be viewed as trying to follow a moving target (i.e., the changing set point).
- The desired value T_R is to remain fixed, and the purpose of the control system is to maintain the controlled variable at T_R in spite of changes in load T_i.
- This problem is very common in the chemical industry, and a complicated industrial process will often have many self contained control systems, each of which maintains a particular process variable at a desired value.
- These control systems are of the regulator type.

DEVELOPMENT OF BLOCK DIAGRAM

- In block diagram representations of control systems, the variables selected are *deviation variables,* and inside each block is placed the transfer function relating the input-output pair of variables.
- Finally, the blocks are combined to give the overall block diagram.

- **Process**
  Consider first the block for the process. This block will be seen that two input variables are present;
- However, the procedure for developing the transfer function remains the same.

An unsteady-state energy balance around the tank gives

$$q + wC(T_i - T_o) - wC(T - T_o) = \rho CV \frac{dT}{dt}$$

where *To* is the reference temperature.

At steady state, *dT/dt* is zero,

$$q_s + wC(T_{i_s} - T_o) - wC(T_s - T_o) = 0$$

$$q - q_s + wC[(T_i - T_{i_s}) - (T - T_s)] = \rho CV \frac{d(T - T_s)}{dt}$$

Notice that the reference temperature *To* cancels in the subtraction.
If we introduce the deviation variables

$$T_i' = T_i - T_{i_s}$$

$$Q = q - q_s$$

$$T' = T - T_s$$

$$Q + wC(T_i' - T') = \rho CV \frac{dT'}{dt}$$

$$Q(s) + wC[T_i'(s) - T'(s)] = \rho CVsT'(s)$$

$$T'(s)\left(\frac{\rho V}{w}s + 1\right) = \frac{Q(s)}{wC} + T_i'(s)$$

The gain for $Q(t)$ is

Stirred heater transfer function $\boxed{T'(s) = \frac{1/wC}{\tau s + 1}Q(s) + \frac{1}{\tau s + 1}T_i'(s)}$

$$\tau = \frac{\rho V}{w}[=]\frac{\mathrm{kg}}{\mathrm{kg/min}}[=]\mathrm{min}$$

The gain for $Q(t)$ is

$$\frac{1}{wC}[=]\frac{1}{\left(\frac{\mathrm{kg}}{\mathrm{min}}\right)[\mathrm{kJ}/(kg \cdot {}^\circ C)]}[=]\frac{{}^\circ C}{kJ/\mathrm{min}}$$

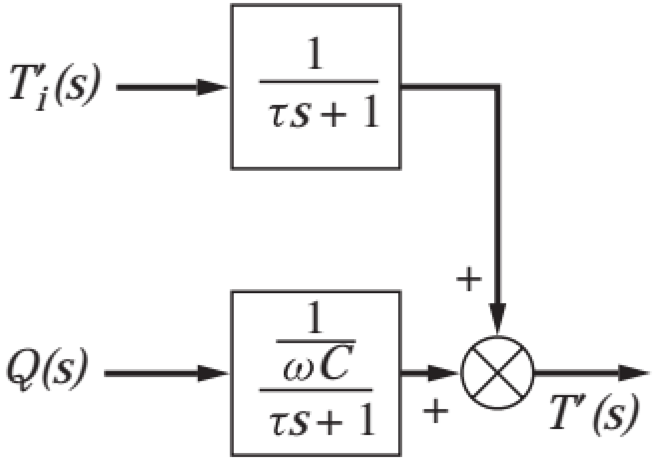
Stirred heater transfer function $\boxed{T'(s) = \frac{1/wC}{\tau s + 1} Q(s) + \frac{1}{\tau s + 1} T_i'(s)}$

- If there is a change in $Q(t)$ only, then $T_i(t) = 0$ and the transfer function relating T to Q is

$$\frac{T'(s)}{Q(s)} = \frac{1/wC}{\tau s + 1}$$

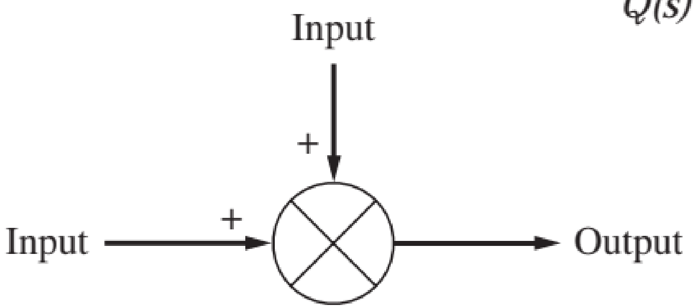
If there is a change in $T_i'(t)$ only, then $Q(t) = 0$ and the transfer function relating T' to T_i' is

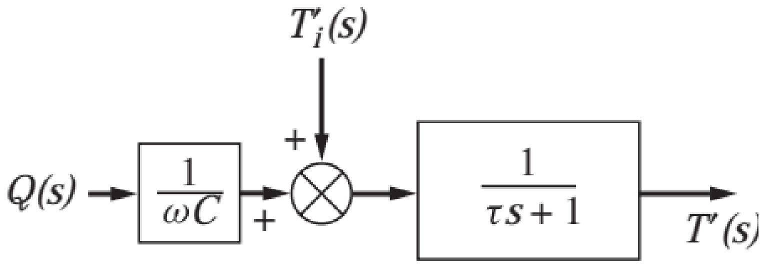
$$\frac{T'(s)}{T_i'(s)} = \frac{1}{\tau s + 1}$$

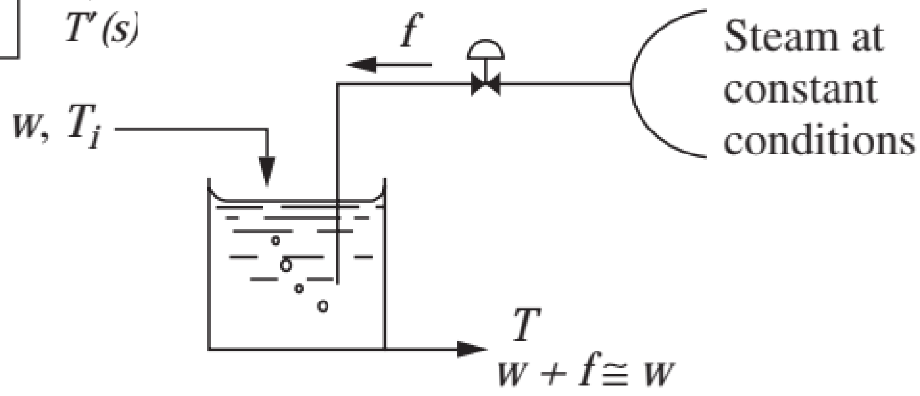


- Superposition makes this representation possible. Which is called a *summing junction.*
- Subtraction can also be indicated with this symbol by placing a minus sign at the appropriate input.
- The summing junction was used previously as the symbol for the comparator of the controller

$$T'(s) = \left[\frac{1}{wC} Q(s) + T_i'(s)\right]\frac{1}{\tau s + 1}$$







Physically the two inputs (heat and inlet temperature) have independent effects on the outlet temperature.

If the flow of steam, *f* (lb/time) is small compared with the inlet flow *w,* the total
outlet flow is approximately equal to *w.*
When the system is at steady state, the heat balance may be written

$$wC(T_{is} - T_o) - wC(T_s - T_o) + f_s(H_g - H_{ls}) = 0$$

where T_o = reference temperature used to evaluate enthalpy of all streams entering and leaving tank
H_g = specific enthalpy of steam supplied, a constant
H_{ls} = specific enthalpy of condensed steam flowing out at T_s, as part of total stream

The term H_{ls} may be expressed in terms of heat capacity and temperature (assuming no phase change occurs between *Ts* and *To*).

$$H_{ls} = C(T_s - T_o)$$

Now consider an *unsteady-state* operation in which *f* is much less than *w* and the temperature *T* of the bath does not deviate significantly from the steady-state temperature T_s.
we may write the unsteady-state balance approximately;

$$wC(T_i - T_o) - wC(T - T_o) + f(H_g - H_{ls}) = \rho CV \frac{dT}{dt}$$

$$q = f(H_g - H_{ls})$$

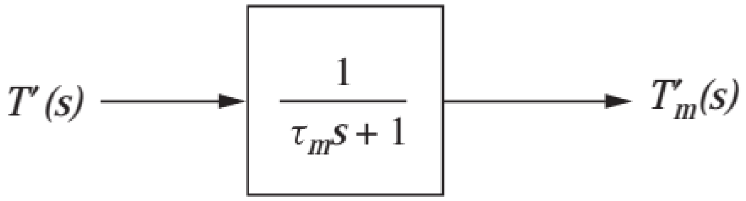
- **Measuring Element**
  The temperature measuring element, which senses the bath temperature *T* and transmits a signal *Tm* to the controller, may exhibit some dynamic lag.
- From the discussion of the mercury thermometer observed this lag to be first-order.
- In this example, we will assume that the temperature measuring element is a first-order system, for which the transfer function is

Measuring element transfer function $$\boxed{\frac{T'_m(s)}{T'(s)} = \frac{1}{\tau_m s + 1}}$$

where the input-output variables *T* and *T'm* are deviation variables, defined as

$$T' = T - T_s$$

$$T'_m = T_m - T_{ms}$$



The model for the sensor is first-order. If *T* (*s*)= 1/ *s,* a unit-step change, then the sensor response is

$$T'_m(s) = \frac{1}{s}\left(\frac{1}{\tau_m s + 1}\right)$$

$$T'_m(t) = (1 - e^{t/\tau m})$$

Since the ultimate value of T'_m is 1, we know from the manufacturer's specifications that we can expect the response to be 90 percent complete at *t*= 45 s, to determine t_m .

$$0.9 = (1 - e^{45\text{ s}/\tau m})$$

$$\frac{45\text{ s}}{\tau m} = 2.303$$

$$\tau m = 19.5\text{ s} = 0.33\text{ min}$$

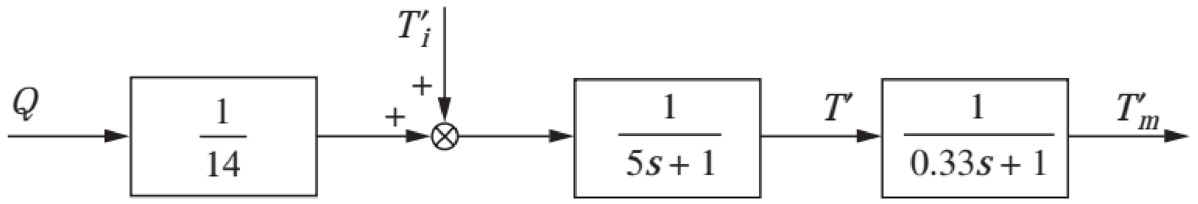
Therefore, the transfer function relating the actual temperature in the tank *T'* to the measured or indicated temperature T'_m is

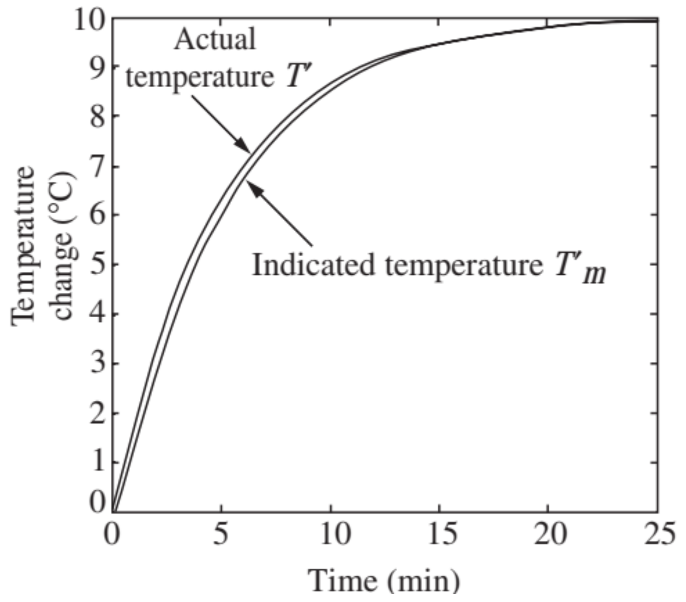
$$\frac{T'_m(s)}{T'(s)} = \frac{1}{0.33s + 1}$$

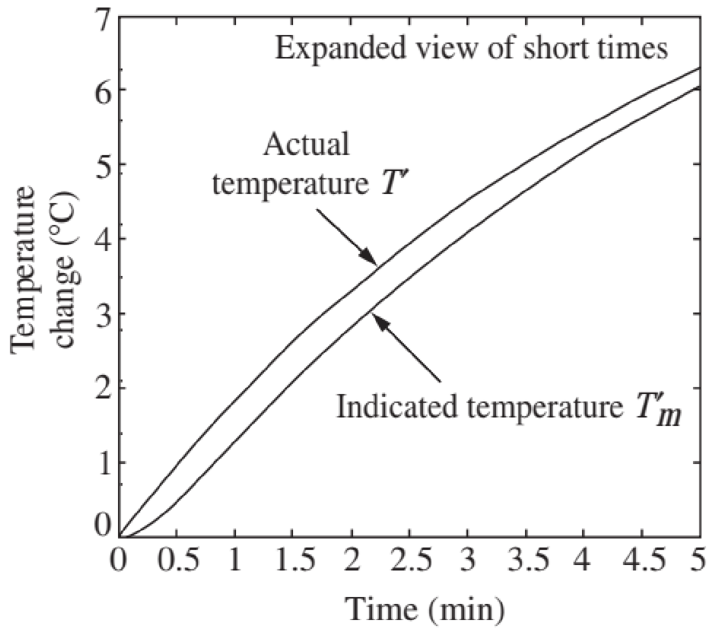
$$T'(s) = \frac{1/wC}{\tau s + 1}\, Q(s) + \frac{1}{\tau s + 1} T'_i(s)$$

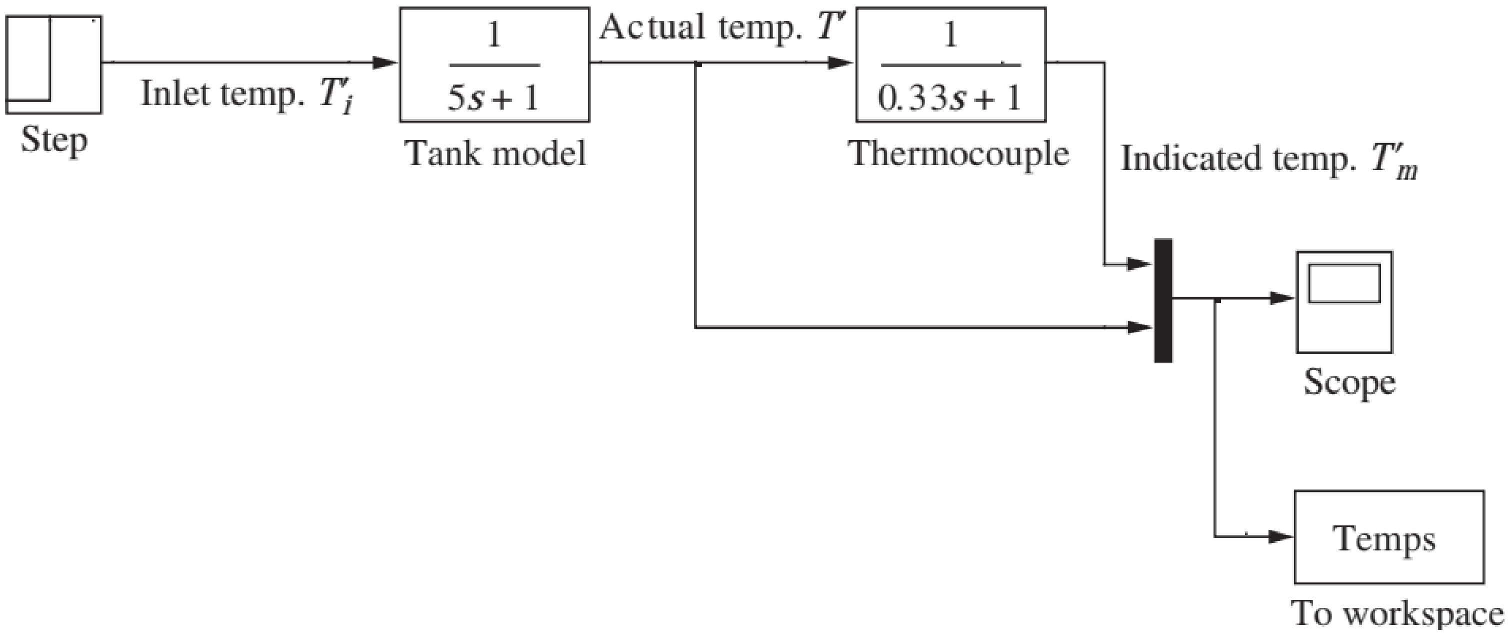
$$T'_m(s) = \frac{10}{s(0.33s + 1)(5s + 1)} = \frac{0.71}{s + 3.03} - \frac{10.71}{s + 0.2} + \frac{10}{s}$$

$$T'_m(t) = 10 + 0.71e^{-3.03t} - 10.71e^{-0.2t}$$









Controller and Final Control Element

For convenience, the blocks representing the controller and the final control element are combined into one block.
In this way, we need be concerned only with the overall response between the error in the temperature and the heat input to the tank.
Also, it is assumed that the controller is a proportional controller.

The relationship for a proportional controller is

$$q = K_c \varepsilon + A$$

$$\varepsilon = T_R - T_M$$

T_R = set point temperature
K_c = proportional sensitivity or controller gain
A = heat input when $\varepsilon = 0$, also called the *bias* value (shortly we will show that $A = q_s$, the steady-state heat input)

At steady state, it is assumed that the set point, the process temperature, and the measured temperature are all equal to one another; thus

$$T_{R_s} = T_s = T_{m_s}$$

Let ε' be the deviation variable for error; thus

$$\varepsilon' = \varepsilon - \varepsilon_s$$

where $\varepsilon_s = T_{R_s} - T_{m_s}$.

$$\text{Since } T_{R_s} = T_{m_s},\ \varepsilon_s = 0$$

$$\varepsilon' = \varepsilon - 0 = \varepsilon$$

This result shows that ε is itself a deviation variable.
Since at steady state there is no error, and *A* is the heat input for zero error,

$$q_s = K_c \varepsilon_s + A = 0 + A = A$$

The steady-state heat input *qs* is the heat required to raise the steady-state inlet temperature from T_is to T_Rs , the desired steady-state set point temperature.
The steady state output from the controller/heater is termed the *bias value.*
It is the output from the controller when the error is zero (i.e., steady state).
It is simply calculated in this case

$$q = K_c \varepsilon + q_s \qquad \text{where } Q = q - q_s.$$

$$Q = K_c \varepsilon$$

Proportional controller transfer function $\boxed{Q(s) = K_c \varepsilon(s)}$

Note that ε, which is also equal to ε', may be expressed as

$$\varepsilon = T_R - T_{R_s} - (T_m - T_{m_s})$$

$$\varepsilon = T'_R - T'_m$$

From the definition of ε and the fact that T_Rs -T_ms

Error definition for stirred heater $\boxed{\varepsilon(s) = T'_R(s) - T'_m(s)}$