Conduction

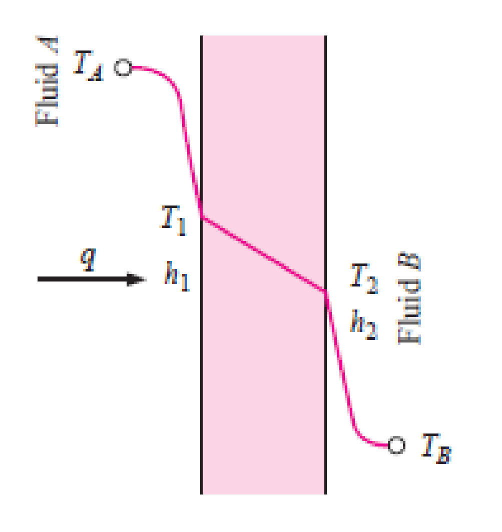
- It has been seen that, $Rate\ of\ heat\ transfer = \frac{Temperature\ difference}{Thermal\ resistance}$
- The resistances considered till now (for eg., in composite solids) were all resistances to conduction
- Suppose we consider heat transfer through a plane wall that is exposed to a *hot fluid A* on one side and a *cooler fluid B* on the other side
- There is resistance to heat transfer in both the hot and cold fluid where heat transfer is by means of *convection*
- In case of convection, $Q = hA\Delta T$
- Thus, heat transfer in case of this wall exposed to hot and cold fluids can be written as,

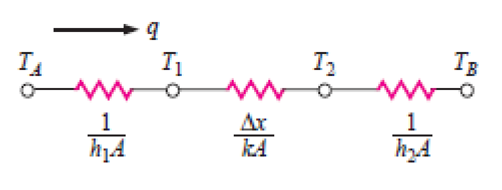
$$Q = h_1 A(T_A - T_1) = \frac{kA}{l}(T_1 - T_2) = h_2 A(T_2 - T_B)$$

- The resistances are now due to both conduction and convection

$$Q = \frac{(T_A - T_B)}{\frac{1}{h_1 A} + \frac{l}{kA} + \frac{1}{h_2 A}}$$

- Similarly, in other cases, if there are other layers of liquids, gases or solids, the resistances are all added up in order to calculate the rate of heat transfer





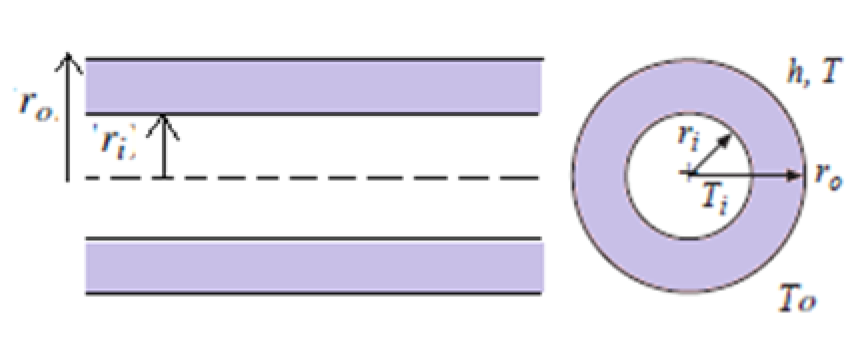
Thermal insulation

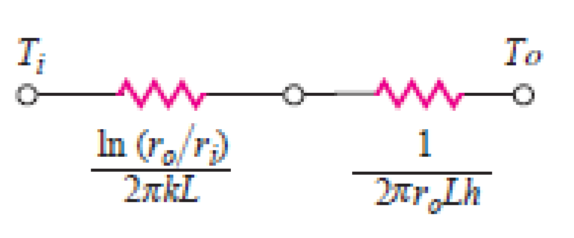
- Process equipment such as reaction vessels, reboilers, distillation columns, evaporators or a steam pipe lose heat to the atmosphere by conduction, convection and radiation
- For all such cases, *conservation of heat is of economic necessity* – this is often achieved by materials known as ***insulators***
- **Insulator is a material having a low thermal conductivity which retards the flow of heat from the system to the surroundings or vice versa with reasonable effectiveness**
- Apart from having a **low thermal conductivity**, insulators must also be **chemically inert, available in a form suitable for application on a surface,** should be **light** and **cheap**
- Ceramic, diatomaceous earth, polymeric materials, fibre glass, mineral wool are used as commercial insulators

- The insulating material is put outside the surface from which heat loss occurs
- In case of a *flat surface (plane wall*), the *thicker the insulation*, the *lesser is the total heat loss*
- However, for a *curved insulation surface*, *using a thicker layer of insulation increases the resistance to heat transfer but it also increases the outer radius and hence the area for heat transfer*
- If the area from which air removes heat increases more rapidly than the resistance, the total heat loss may increase

- When a layer of insulation is put on a curved pipe, there are **two major resistances in series** – the resistance offered by (i) the **layer of insulation** and that offered by (ii) the **gas film on the outside**
- The two effects of *increasing heat transfer resistance* and *increasing surface area* have opposite effects on the rate of heat transfer as the insulation thickness is increased
- The insulation thickness for which the rate of heat transfer will be maximum can be determined

Critical Insulation Thickness





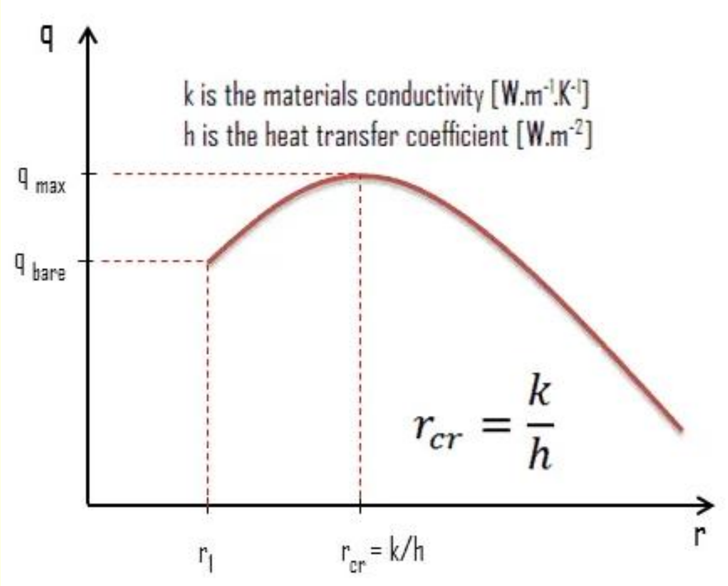
- Let us consider a pipe (cylinder) with a layer of insulation having r_i and r_o as the inner and outer radius of insulation
- The inner temperature is T_i and outer ambient temperature is T_o
- The convective heat transfer coefficient outside the insulation is h and the thermal conductivity of the insulating material is k
- The heat transfer coefficient on the inner side of the insulation is assumed to be large and does not have much resistance
- The rate of heat transfer is given by,

$$Q = \frac{(T_i - T_o)}{\frac{ln(^{r_o}/_{r_i})}{2\pi kL} + \frac{1}{2\pi r_o Lh}} = \frac{2\pi L(T_i - T_o)}{\frac{ln(^{r_o}/_{r_i})}{k} + \frac{1}{r_o h}}$$

- In order to find the condition for maximum heat loss,

$$\frac{dQ}{dr_o} = \frac{-2\pi L(T_i - T_o)\left[\frac{1}{k}\cdot\frac{r_i}{r_o}\times\frac{1}{r_i} - \frac{1}{hr_o^2}\right]}{\left[\frac{ln\left(\frac{r_o}{r_i}\right)}{k} - \frac{1}{r_o h}\right]^2} = 0$$

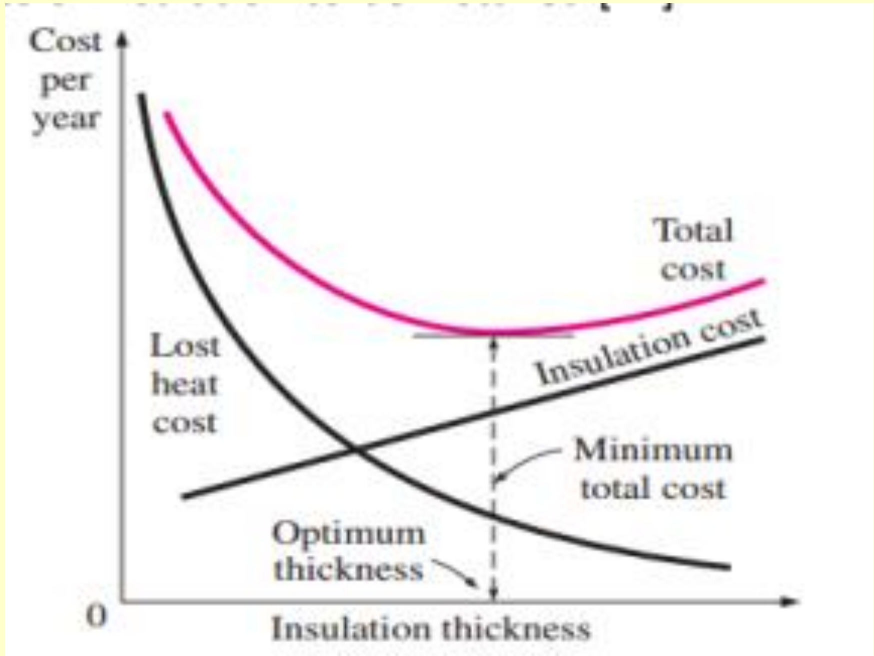
$$\frac{1}{kr_o} - \frac{1}{hr_o^2} = 0 \qquad or, \qquad r_{o,critical} = \frac{k}{h}$$



- The quantity $(r_o - r_i)$ is called the ***critical insulation thickness***
- *When insulation is applied to a pipe to prevent heat loss, its thickness should be above the critical value in order to reduce heat loss*
- *For insulated electric wires, the insulation thickness should be very near the critical value so that the heat loss is maximum*
- If outer radius (r_o) is < the critical value $(r_{o,critical})$, then addition of insulation increases heat loss initially and is maximum at $r_o = r_c$ and then heat loss decreases with increase in insulation thickness
- The ***critical insulation radius for a sphere*** is, $r_{o,critical} = \frac{2k}{h}$
- Critical insulation thickness may not always exist for an insulated pipe - if the value of k and h are such that the ratio $\frac{k}{h}$ $(r_{o,critical})$, is < r_i (bare radius of the pipe), then critical insulation thickness $(r_o - r_i)$ becomes negative and has no physical meaning

Optimum Insulation Thickness

- Optimum insulation thickness is obtained by a purely economic approach
- As the thickness of insulation increases, the heat loss goes down saving energy costs (operating costs) but at the same time the cost of insulation (fixed costs) increases with thickness
- The two opposing factors should be considered in determining the thickness of insulation
- The **optimum insulation thickness is one for which the sum of insulation cost and the cost of heat loss (total cost) is minimum**



Problem

An electric cable 10mm in diameter is to be insulated with rubber (k = 0.14 W/m^{o}C). The insulated cable is exposed to air at 20^{o}C.

(i) What is the optimum thickness of the rubber insulation from heat transfer point of view assuming the cable surface temperature of 60^{o}C in bare as well as in insulated conditions ?

The heat transfer coefficient on the surface of the bare cable or on insulation is 7 W/m^{2o}C

(ii) What is the percentage increase in (a) heat dissipation and (b) current carrying capacity when the most economical insulation is provided?

(i) The optimum thickness from the heat transfer point of view for electric cables and wires is the one which gives the maximum heat dissipation from the surface

The maximum heat loss occurs at the critical insulation radius

Critical insulation radius, $r_{o,critical} = \frac{k}{h} = \frac{0.14}{7} = 0.02\ m$

Critical insulation thickness, $(r_o - r_i) = 0.02 - 0.005 = 0.015\ m$

(ii) Heat dissipation (loss) from the bare electric cable per unit length

$$= \frac{Q_1}{L} = (2\pi r)h(T_i - T_a) = 2 \times \pi \times 0.005 \times 7 \times (60 - 20) = 8.78\ W/m$$

Heat dissipation (loss) per unit length from the electric cable covered with rubber insulation

$$= \frac{Q_2}{L} = \frac{2\pi(T_i - T_o)}{\frac{ln\left(r_o/r_i\right)}{k} + \frac{1}{r_o h}} = \frac{2\times\pi\times(60-20)}{\frac{ln\left(0.02/0.005\right)}{0.14} + \frac{1}{0.02\times 7}} = \frac{251.327}{17.05} = 14.74\ W/m$$

(a) Percentage increase in heat dissipation $= \frac{Q_2 - Q_1}{Q_1} \times 100 = \frac{14.74 - 8.78}{8.78} \times 100 = 67.9\%$

(b) Heat dissipation in case 1 = $Q_1 = I_1^2 R$

Heat dissipation in case 2 = $Q_2 = I_2^2 R$

Resistance is same in both cases, $\frac{Q_1}{Q_2} = \frac{I_1^2}{I_2^2}$

$$\frac{I_1}{I_2} = \sqrt{\frac{Q_1}{Q_2}} = \sqrt{\frac{8.78}{14.74}} = 0.7718$$

Percentage increase in current carrying capacity $= \frac{I_2 - 0.7718 I_2}{0.7718 I_2} \times 100 = 29.57\ \%$