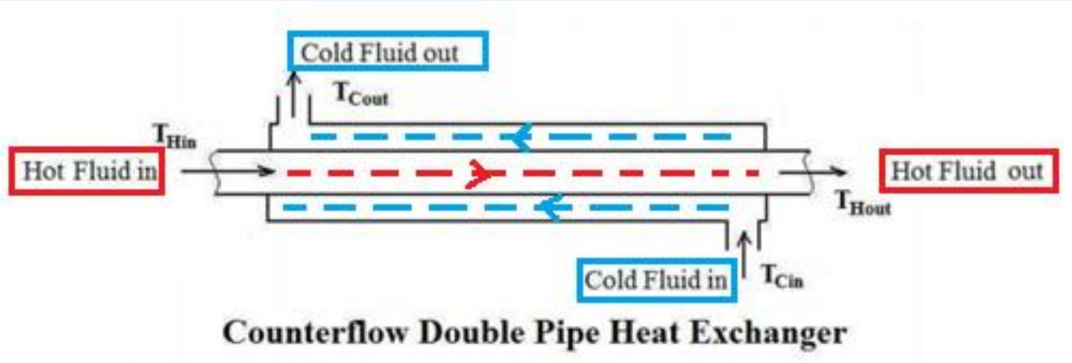
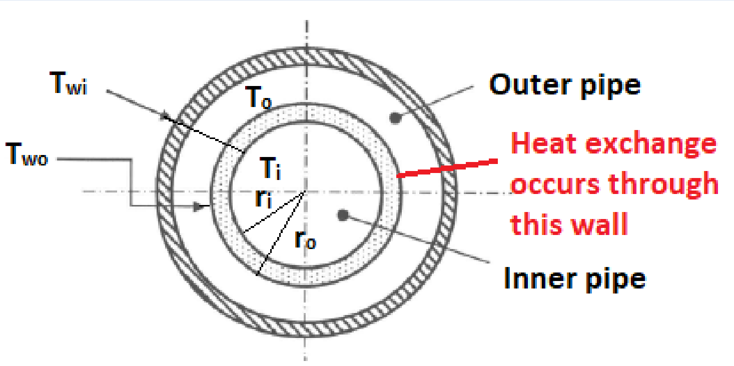
Convection

Heat transfer between two fluids separated by a cylindrical wall

- Consider two concentric pipes such that two fluids, one hot and one cold pass through the inner and outer pipes, respectively
- The two fluids are not in real contact but in ‘thermal contact’ and heat can be transferred from the hot to the cold fluid
- This is a practical device used in industrial applications and is known as a ***double-pipe heat exchanger***





- In case of a cylinder the heat transfer is in the radial direction – the wall separating the two fluids has a variable area along the radius
- The inner and outer radii of the inner pipe is r_i and r_o
- Heat transfer coefficient in the film in the inner and outer side of the inner pipe are h_i and h_o
- The inner and outer surface temperatures of the pipe wall are T_{wi} and T_{wo}
- The bulk temperatures of hot and cold fluids in the inner and outer pipe are are T_i and T_o
- Thermal conductivity of the material of the pipe wall is k_w
- Length of the pipe considered is L
- The area of the inner wall of the inner pipe is, $A_i = 2\pi r_i L$ and the area of the outer wall of the inner pipe is, $A_o = 2\pi r_o L$

- A section of the assembly of length L over which the temperatures T_i and T_o remain approximately constant is considered

Rate of convective heat transfer from the hot fluid to the inner surface (at $r = r_i$) $= Q = A_i h_i (T_i - T_{wi})$

Rate of conductive heat transfer through the pipe wall = $Q = \frac{2\pi k_w L (T_{wi} - T_{wo})}{ln\left(r_o/r_i\right)}$

Rate of convective heat transfer from the outer surface of the pipe (at $r = r_o$) to the cold fluid $= Q = A_o h_o (T_{wo} - T_o)$

$(T_i - T_{wi}) = \frac{Q}{A_i h_i} = \frac{Q}{2\pi r_i L h_i}$; $\quad (T_{wi} - T_{wo}) = \frac{Q}{2\pi k_w L / ln\left(r_o/r_i\right)}$; $\quad (T_{wo} - T_o) = \frac{Q}{A_o h_o} = \frac{Q}{2\pi r_o L h_o}$

- Adding the three equations leads to,

$$Q = \frac{(T_i - T_o)}{\frac{1}{A_i h_i} + \frac{ln\left(r_o/r_i\right)}{2\pi k_w L} + \frac{1}{A_o h_o}}$$

As the inner and outer surface areas of the inner pipe are different, two different overall heat transfer coefficients are defined

$$Q = U_i A_i (T_i - T_o) = U_o A_o (T_i - T_o)$$

$$U_i = \frac{1}{\frac{1}{h_i} + \frac{A_i ln\left({}^{r_o}/_{r_i}\right)}{2\pi k_w L} + \frac{A_i}{A_o h_o}} = \frac{1}{\frac{1}{h_i} + \frac{r_i ln\left({}^{r_o}/_{r_i}\right)}{k_w} + \frac{r_i}{r_o h_o}}$$

$$\frac{1}{U_i} = \frac{1}{h_i} + \frac{r_i ln\left({}^{r_o}/_{r_i}\right)}{k_w} + \frac{r_i}{r_o h_o}$$

$$U_o = \frac{1}{\frac{A_o}{A_i h_i} + \frac{A_o ln\left({}^{r_o}/_{r_i}\right)}{2\pi k_w L} + \frac{1}{h_o}} = \frac{1}{\frac{r_o}{r_i h_i} + \frac{r_o ln\left({}^{r_o}/_{r_i}\right)}{k_w} + \frac{1}{h_o}}$$

$$\frac{1}{U_o} = \frac{r_o}{r_i h_i} + \frac{r_o ln\left({}^{r_o}/_{r_i}\right)}{k_w} + \frac{1}{h_o}$$

and $U_i A_i = U_o A_o$

Problem

A saturated refrigerant at - 40°C flows through a copper pipe of 10 mm inside diameter and wall thickness of 2 mm. A layer of 40 mm thermocole is provided on the outer surface of the pipe to reduce the heat flow. Assume the internal and external heat transfer coefficients to be 500 and 5 W/m²°C. The ambient temperature is 40°C. The thermal conductivities of copper and thermocole is 400 W/mK and 0.03 W/mK, respectively.

(a) Determine the overall heat transfer coefficient both for covered pipe and bare pipe.

(b) Determine the heat leakage to the refrigerant per m length of pipe

(c) Find the amount of refrigerant vaporized per h per m length of the pipe when the pipe is covered and when it is bare. (λ of refrigerant = 1390 kJ/kg)

(d) Also, calculate the skin temperature of the insulation

(a) For a metal pipe covered by insulation

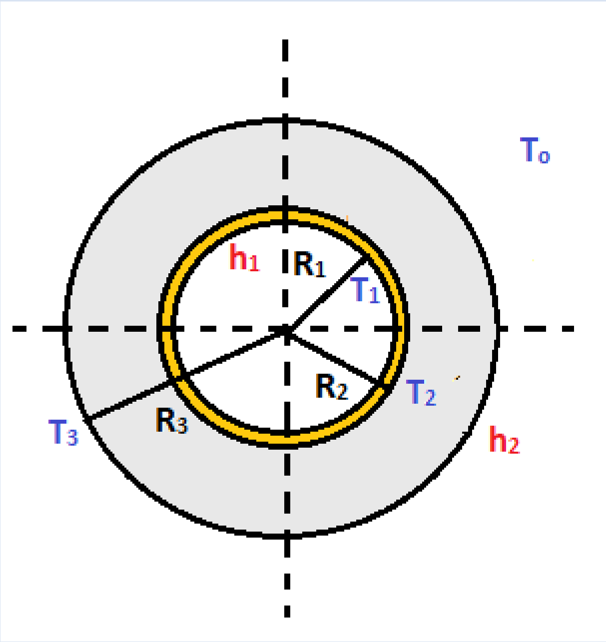
$$\frac{1}{U_i} = \frac{1}{h_i} + \frac{R_1 ln\left(R_2/R_1\right)}{k_{Cu}} + \frac{R_1 ln\left(R_3/R_2\right)}{k_{Th}} + \frac{R_1}{R_3 h_o}$$

Here, $R_1 = 0.005$ m, $R_2 = 0.007$ m, $R_3 = 0.007 + 0.04 = 0.047\ m$,

$$\frac{1}{U_i} = \frac{1}{500} + \frac{0.005 ln\left(0.007/0.005\right)}{400} + \frac{0.005 ln\left(0.047/0.007\right)}{0.03} + \frac{0.005}{0.047 \times 5}$$

$$= 2 \times 10^{-3} + 4.206 \times 10^{-6} + 0.3174 + 0.02128 = 0.3407$$

$U_i = 2.935$ W/m²K



(a) For a bare metal pipe

$$\frac{1}{U_i} = \frac{1}{h_i} + \frac{R_1 ln\left({R_2}/{R_1}\right)}{k_{Cu}} + \frac{R_1}{R_2 h_o}$$

Here, $R_1 = 0.005$ m,
$R_2 = 0.007$ m,

$$\frac{1}{U_i} = \frac{1}{500} + \frac{0.005 ln\left({0.007}/{0.005}\right)}{400} + \frac{0.005}{0.007 \times 5} = 2 \times 10^{-3} + 4.206 \times 10^{-6} + 0.1429 = 0.1449$$

$U_i = 6.903$ W/m²K

(b) For a metal pipe covered by insulation, $Q = U_i A_i (T_1 - T_o) = 2.935 \times 2 \times \pi \times 0.005 \times 1 \times (-40 - 40) = -7.376\ W$

For a bare metal pipe, $Q = U_i A_i (T_1 - T_o) = 6.903 \times 2 \times \pi \times 0.005 \times 1 \times (-40 - 40) = -17.35\ W$

Negative sign implies that heat flows into the pipe

(c) For a metal pipe covered by insulation, heat leakage per hour = $-7.376 \times 3600 = -26553.6\ J = -26.553\ kJ$

Refrigerant vaporized per hour = $\frac{25.553}{1390} = 0.0191$ kg

For a bare metal pipe, heat leakage per hour = $-17.35 \times 3600 = -62460\ J = -62.46\ kJ$

Refrigerant vaporized per hour = $\frac{62.46}{1390} = 0.045$ kg

(d) In case of the insulated pipe, the skin temperature (temperature of the outer surface of the insulation) can be determined by

$$Q = h_o A_3 (T_3 - T_o)$$

$$Q = h_2 (2\pi R_3 L)(T_3 - T_o)$$

It was earlier determined that, for the metal pipe covered by insulation, $Q = -7.376\ W$

Therefore,

$$-7.376 = 5 \times 2 \times \pi \times 0.047 \times 1 \times (T_3 - 40)$$

$$-7.376 = 1.4765 \times (T_3 - 40)$$

$$-7.376 = 1.4765 T_3 - 59.06$$

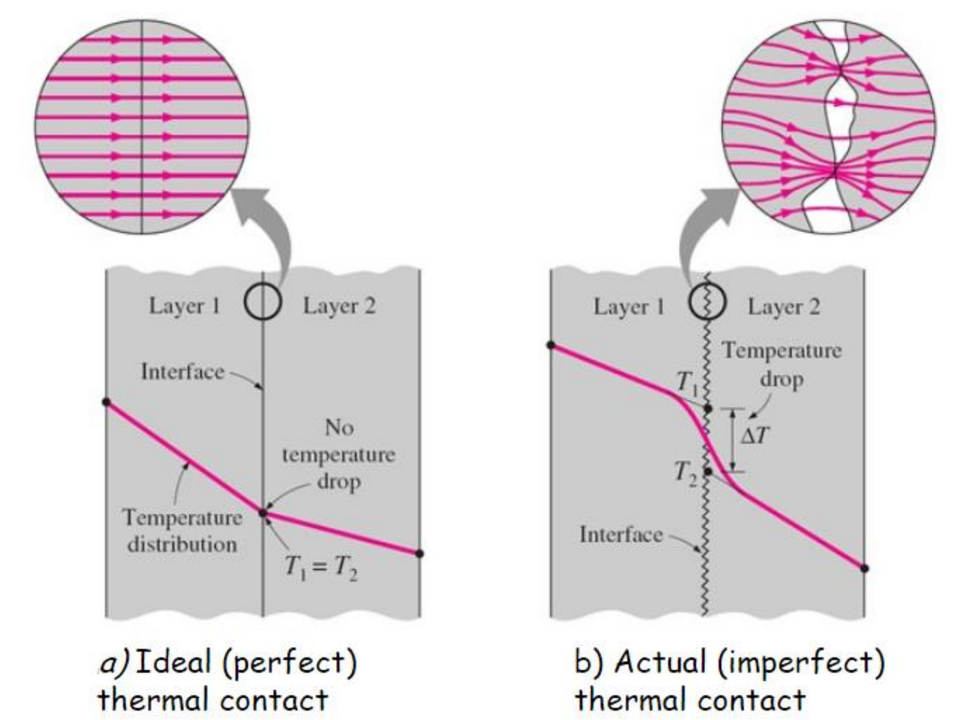
$$1.4765 T_3 = 51.684$$

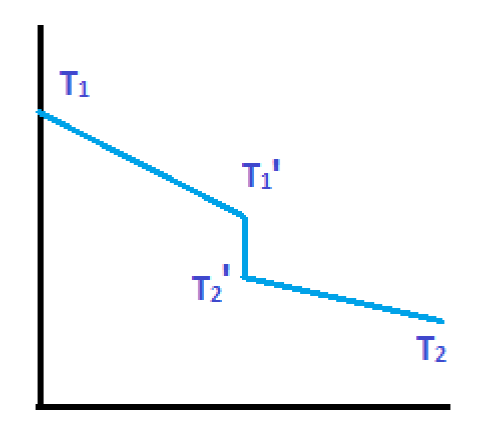
$$T_3 = 35℃$$

Thermal contact resistance

- Heat is conducted through a composite wall of two different solids in contact with each other and the rate of heat transfer is the same through both the solids under steady state
- Under *ideal conditions, there is no resistance to heat transfer at the interface*
- In *actual situations, however, it is seen that there is often a temperature drop at the interface*
- This happens because no two surfaces are absolutely smooth
- When two solids are in contact it is observed that air spaces exist between them at microscopic levels
- This leads to a heat transfer resistance at the interface called the ***thermal contact resistance*** $\left(\frac{1}{h_c A}\right)$
- Therefore, rate of heat flow is given as,

$$Q = \frac{(T_1 - T_2)}{\frac{l_1}{k_1 A} + \frac{1}{h_c A} + \frac{l_2}{k_2 A}}$$





Problem

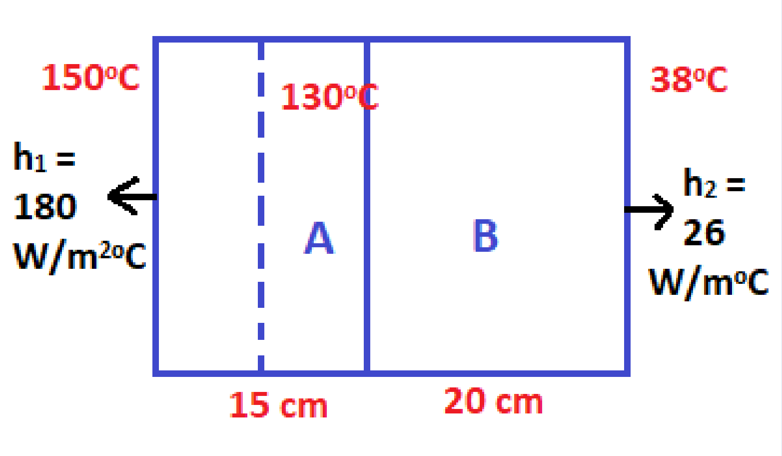
A composite wall consists of material A (thickness = 15 cm, $k_A = 10$ W/m°C) and material B (thickness = 20 cm, $k_B = 16$ W/m°C). The exposed surface of A is in contact with a hot fluid at 150°C ($h_1 = 180$ W/m²°C) and that of B is in contact with air at 38°C ($h_2 = 26$ W/m²°C). The mid-plane temperature of A (i.e., 7.5 cm away from the exposed surface) at steady state is measured to be 130°C.

(a) Is there any contact resistance at the junction of A and B? If so, what is the magnitude?

(b) Calculate the temperature jump at the interface.

(c) If there was no contact resistance, by what percentage (%) should the thickness of slab A be increased to get the same heat flux (keeping the thickness of B unchanged)?

(a) The rate of heat transfer can be calculated using the temperature of the hot fluid and the mid-point temperature in the wall



$$Q = \frac{(T_h - T_m)}{\frac{1}{h_h A} + \frac{l_A}{k_A A}} = \frac{(150 - 130)}{\frac{1}{180} + \frac{0.075}{10}} = 1531.9\ W$$

At steady state, this is the heat flow through the hot fluid, slab A, slab B and air

Estimating the temperature of the interface of A and B (T_{Ai})

$$Q = \frac{(T_m - T_{Ai})}{\frac{l_A}{k_A A}} \quad \Rightarrow \quad 1531.9 = \frac{(130 - T_{Ai})}{\frac{0.0075}{10}} \quad \Rightarrow T_{Ai} = 118.51°\text{C}$$

The interface temperature at the junction of B and A (T_{Bi}) can be estimated as,

$$Q = \frac{(T_{Bi} - T_a)}{\frac{l_B}{k_B A} + \frac{1}{h_a A}} \quad \Rightarrow \quad 1531.9 = \frac{(T_{Bi} - 38)}{\frac{0.2}{16} + \frac{1}{26}} \quad \Rightarrow T_{Bi} = 116.07℃$$

As there is a difference between temperatures calculated starting from slab A and when calculated starting from slab B, it can be concluded that contact resistance exists

Magnitude of this resistance can be estimated from,

$$Q = \frac{(T_1 - T_2)}{\frac{1}{h_h A} + \frac{l_A}{k_A A} + R_c + \frac{l_B}{k_B A} + \frac{1}{h_a A}}$$

$$1531.9 = \frac{(150 - 38)}{\frac{1}{180} + \frac{0.15}{10} + R_c + \frac{0.20}{16} + \frac{1}{26}}$$

$$R_c = \frac{112}{1531.9} - 0.071517 = 0.001595 \ ^oC/W$$

(b) Temperature jump = $118.51℃ - 116.07℃ = 2.44℃$

(c) In the absence of contact resistance,

$$Q = \frac{(T_1 - T_2)}{\frac{1}{h_h A} + \frac{l_A}{k_A A} + R_c + \frac{l_B}{k_B A} + \frac{1}{h_a A}}$$

$$1531.9 = \frac{(150 - 38)}{\frac{1}{180} + \frac{l'_A}{10} + \frac{0.20}{16} + \frac{1}{26}}$$

$$0.056517 + \frac{l'_A}{10} = 0.0731118$$

$$l'_A = 0.166 \text{ m}$$

Percentage (%)increase in thickness of slab A = $\frac{0.166-0.15}{0.15} \times 100 = 10.67\%$